

Chapter 5

Work and Energy

conclusion

Chaper 5 Review: *Work and Energy – Forces and Displacements*

Effect of forces acting over a displacement

Work

$$W = (F \cos \theta) \Delta x$$

Work changes the
Kinetic Energy of a mass

Kinetic Energy

$$K = \frac{1}{2} m v^2$$

Work - Energy Theorem (true always)

$$W = K - K_0$$

Conservative Force

Gravity

Ideal Spring

Potential Energy

$$U_G = mgy$$

$$U_S = \frac{1}{2} kx^2$$

Non-Conservative Forces doing work

W_{NC} Humans, Friction, Explosions

Work - Energy Theorem (w/potential energy U)

$$W_{\text{NC}} = (K - K_0) + (U - U_0) = \Delta E$$

All of these quantities are **scalars**.

(magnitude of a vector is a scalar)

Clicker Question 5.10

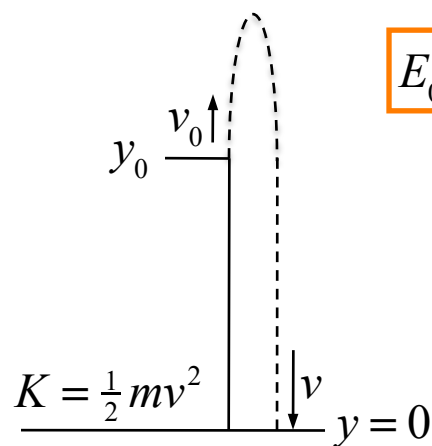
A ball is thrown upward with an initial speed v from the roof of a building. An identical ball is thrown downward with the same initial speed v . When the balls reach the ground, how do the kinetic energies of the two balls compare? Ignore air resistance effects.

- a) The kinetic energies of the two balls are the same.
- b) The first ball has twice the kinetic energy as the second ball.
- c) The first ball has one half the kinetic energy as the second ball.
- d) The first ball has four times the kinetic energy as the second ball.
- e) The first ball has three times the kinetic energy as the second ball.

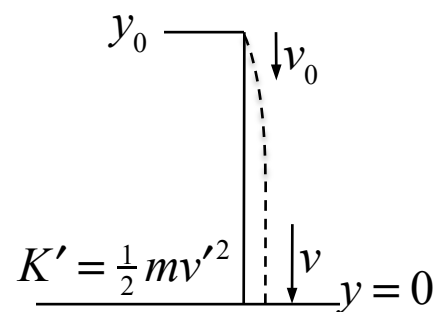
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$$E_0 = K_0 + U_0 = \frac{1}{2}mv_0^2 + mgy_0 \text{ is the same for both cases}$$



$$\begin{aligned} E &= K, E' = K' \\ E &= E_0, E' = E_0 \Rightarrow K = K' \end{aligned}$$

5.6 Power

DEFINITION OF AVERAGE POWER

Average power is the rate at which work is done, and it is obtained by dividing the work by the time required to perform the work, usually generated by a non-conservative force.

$$\bar{P} = \frac{\text{Work}}{\text{Time}} = \frac{W_{\text{NC}}}{t}$$

Power units: joule/s = watt (W)

Note: 1 horsepower = 745.7 watts

Work - Energy Theorem: $W_{\text{NC}} = \Delta E$

$$\bar{P} = \frac{\Delta E}{t} \Rightarrow \Delta E = \bar{P} \Delta t$$

5.6 Power

Example : A 1.0-hp motor powers a boat for 1 minute.

How much energy has it provided ?

$$P = 1.0 \text{ horsepower} = 745.7 \text{ watts} = 745.7 \text{ J/s}$$
$$\Delta E = P\Delta t = (745.7 \text{ J/s})(60 \text{ s}) = 45 \text{ kJ}$$

Also, relating power to a constant force acting on a mass:

$$\bar{P} = \frac{W}{t} = \frac{F_x \Delta x}{t} = F_x \left(\frac{\Delta x}{t} \right)$$
$$= F_x \bar{v}_x$$

Power = (force) (average velocity)

Clicker Question 5.11

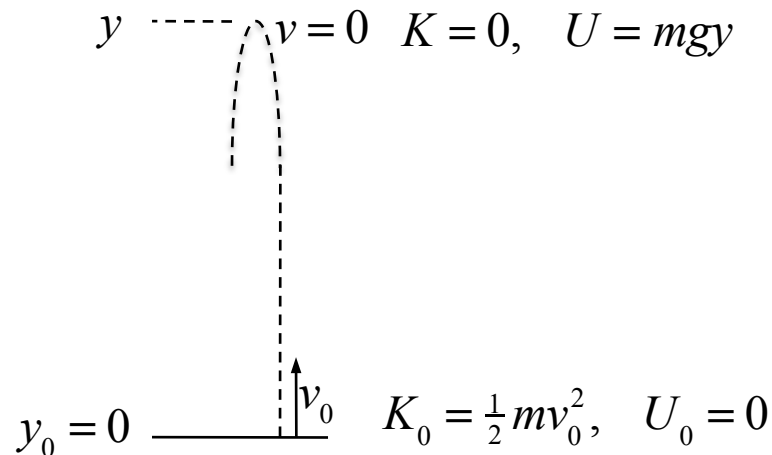
If the amount of energy needed to operate a 100 W light bulb for one minute were used to launch a 2-kg projectile what maximum height could the projectile reach? Ignore air friction. (1 W = 1 J/s)

- a) 20 m
- b) 50 m
- c) 100 m
- d) 200 m
- e) 300 m

Clicker Question 5.11

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Energy available: $E_0 = Pt = (100 \text{ J/s})(60 \text{ s}) = 6000 \text{ J}$

Energy is conserved: $E_0 = E$, and $E = K + U = 0 + mgy$

$$E_0 = mgy$$

$$y = \frac{E_0}{mg} = \frac{6000 \text{ J}}{(2 \text{ kg})(9.81 \text{ m/s}^2)} = 300 \text{ m}$$

5.6 Power

Table of **Human Metabolic Rates^a**

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

^aFor a young 70-kg male.

6.8 *Other Forms of Energy and the Conservation of Energy*

THE PRINCIPLE OF CONSERVATION OF ENERGY

Energy can neither be created nor destroyed, but can only be converted from one form to another.

Heat energy is the kinetic or vibrational energy of molecules. The result of a non-conservative force is often to remove mechanical energy and transform it into heat.

Examples of heat generation: sliding friction, muscle forces.

Chapter 6

Impulse and Momentum

6.1 *The Impulse-Momentum Theorem*

Chapter 6 is about the **COLLISION** of **TWO** masses.

To understand the interaction, both masses must be considered.

Newton's 3rd Law plays a very important part.

Collisions involve two new concepts: **Impulse** and **Momentum**.

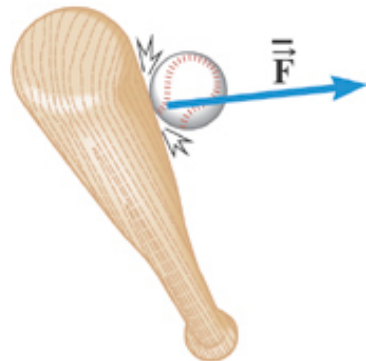
Impulse concept leads to the Momentum definition.

Also applied to two (or more) masses blown apart by an explosion.

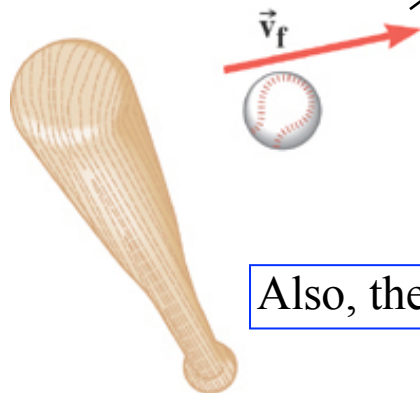
6.1 The Impulse-Momentum Theorem



What is the effect of a force acting over a short time?

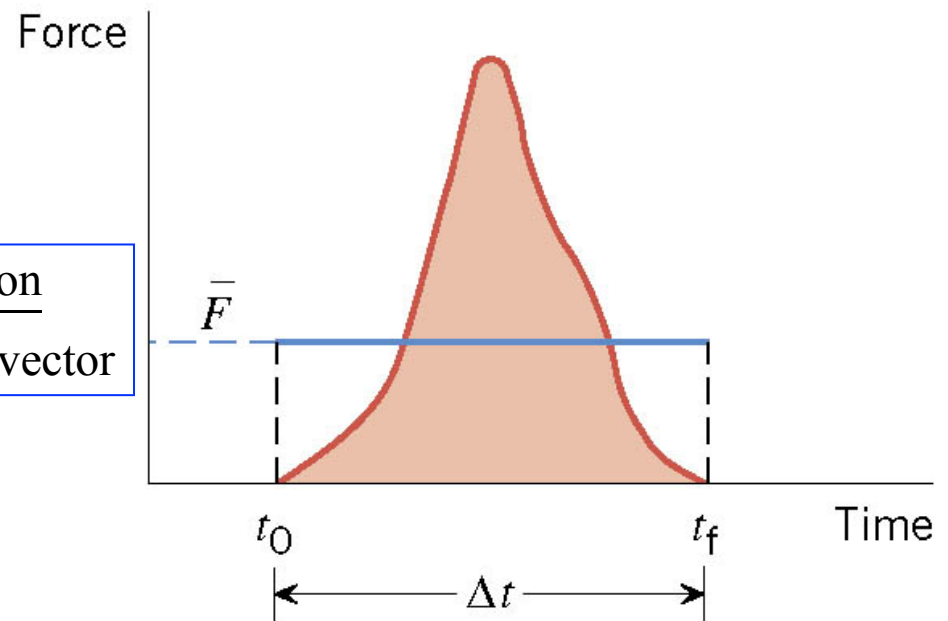


Force changes the direction
of the baseball's velocity vector



The bat/ball force is not constant and the mass makes a very short displacement while it acts.

Also, the reaction force changes the magnitude of the bat's velocity vector



6.1 The Impulse-Momentum Theorem



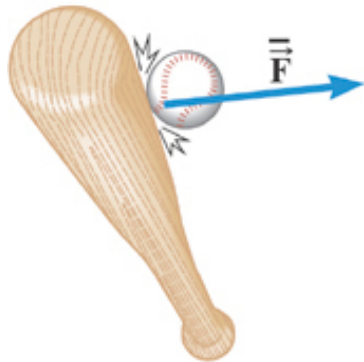
$\vec{F}_{\text{Net}}$ acts on the Baseball
 $m, \vec{v}$, and $\vec{a}$ are of the Baseball

f $\equiv$ final

i $\equiv$ initial

$$\vec{F}_{\text{Net}} = m\vec{a}$$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$



$$\vec{F}_{\text{Net}} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

Momentum

$$\vec{p} = m\vec{v}$$

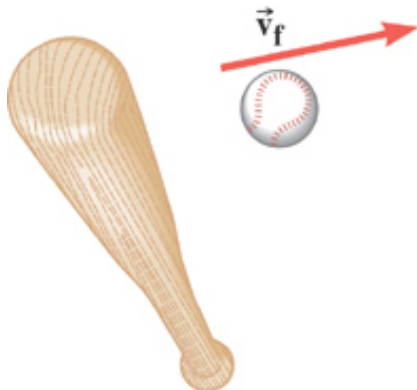
on the BALL

of the BALL

$$\vec{F}_{\text{Net}} \Delta t = \Delta \vec{p}_f$$

Impulse

$$\vec{F}_{\text{Net}} \Delta t$$



Impulse $\Rightarrow$ changes BALL's momentum

reaction Newton's 3rd Law action

$$(\vec{F}_{\text{Net}})_{\text{on the BAT}} = -(\vec{F}_{\text{Net}})_{\text{on the BALL}}$$

6.1 *The Impulse-Momentum Theorem*

DEFINITION OF IMPULSE

The impulse of a force is the product of the average force and the time interval during which the force acts:

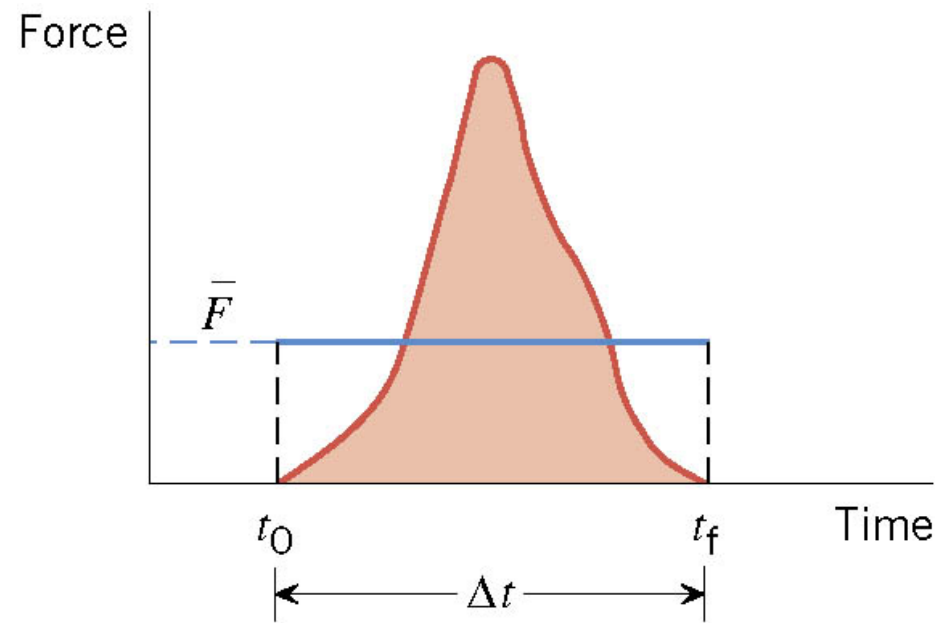
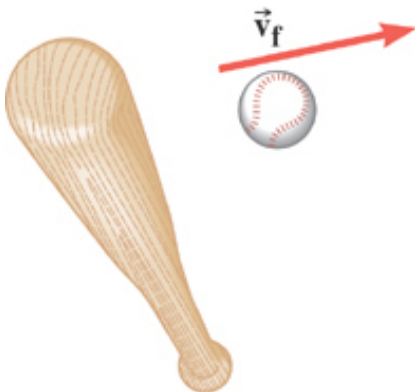
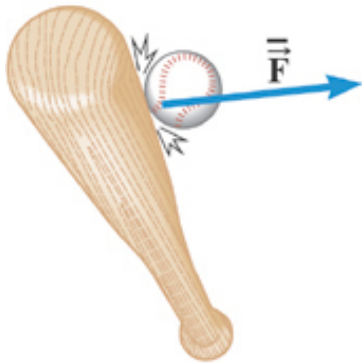
$$\vec{\mathbf{J}} = \vec{\mathbf{F}}_{\text{Net}} \Delta t$$

$$\vec{\mathbf{F}}_{\text{Net}} = \text{average} \\ \text{net force vector}$$

Impulse is a vector quantity and has the same direction as the average force.

newton · seconds (N · s)

6.1 The Impulse-Momentum Theorem



(b)

$$\vec{J} = \vec{F} \Delta t$$

Impulse

6.1 *The Impulse-Momentum Theorem*

DEFINITION OF LINEAR MOMENTUM

The linear momentum of an object is the product of the object's mass times its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Linear momentum is a **vector quantity** and has the same direction as the velocity.

kilogram · meter/second (kg · m/s)

6.1 *The Impulse-Momentum Theorem*

IMPULSE-MOMENTUM THEOREM

When a net force acts on an object, the impulse of this force is equal to the change in the momentum of the object

$$\overset{\text{impulse}}{\vec{\mathbf{F}}_{\text{Net}} \Delta t} = \overset{\text{final momentum}}{m\vec{\mathbf{V}}_f} - \overset{\text{initial momentum}}{m\vec{\mathbf{V}}_i}$$

Time averaged force
acting **on a mass**.

Changes the momentum
of the mass.

6.1 The Impulse-Momentum Theorem

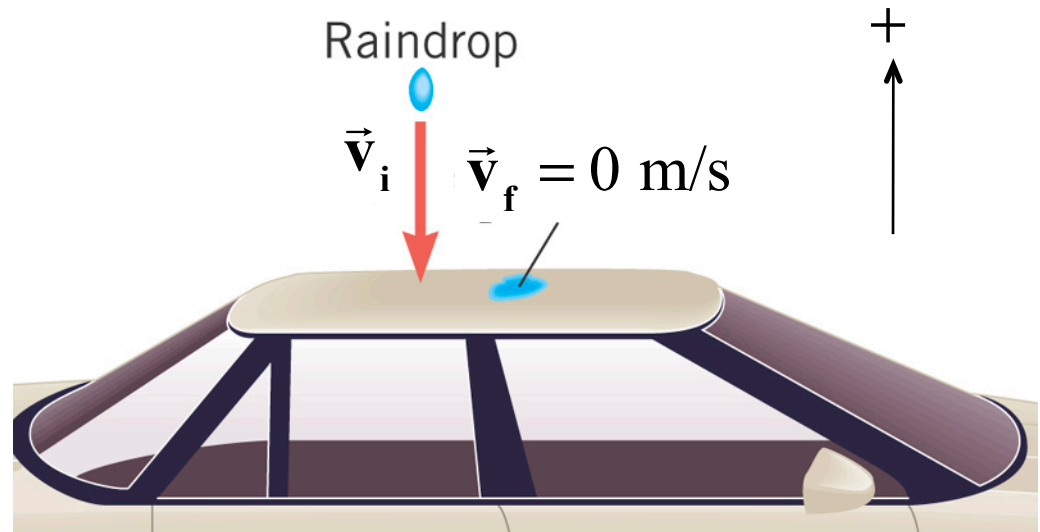
Example: A Rain Storm

Rain comes down with a velocity of -15 m/s and hits the roof of a car. The mass of rain per second that strikes the roof of the car is 0.060 kg/s . Assuming that rain comes to rest upon striking the car, find the **average force** exerted by the rain **on the roof**.

$$\vec{F}_{\text{Net}} \Delta t = m\vec{v}_f - m\vec{v}_i$$

Using this, you will determine the average force **on the raindrops**.

But, using Newton's 3rd law you can get the average force **on the roof**.



6.1 The Impulse-Momentum Theorem

Neglecting the raindrop's weight, the average net force **on the raindrops** caused by the collisions with the roof is obtained.

Impulse of roof
on raindrops

Changes momentum
of the raindrops

$$\vec{F} \Delta t = m\vec{v}_f - m\vec{v}_i$$

$$\vec{v}_f = 0$$

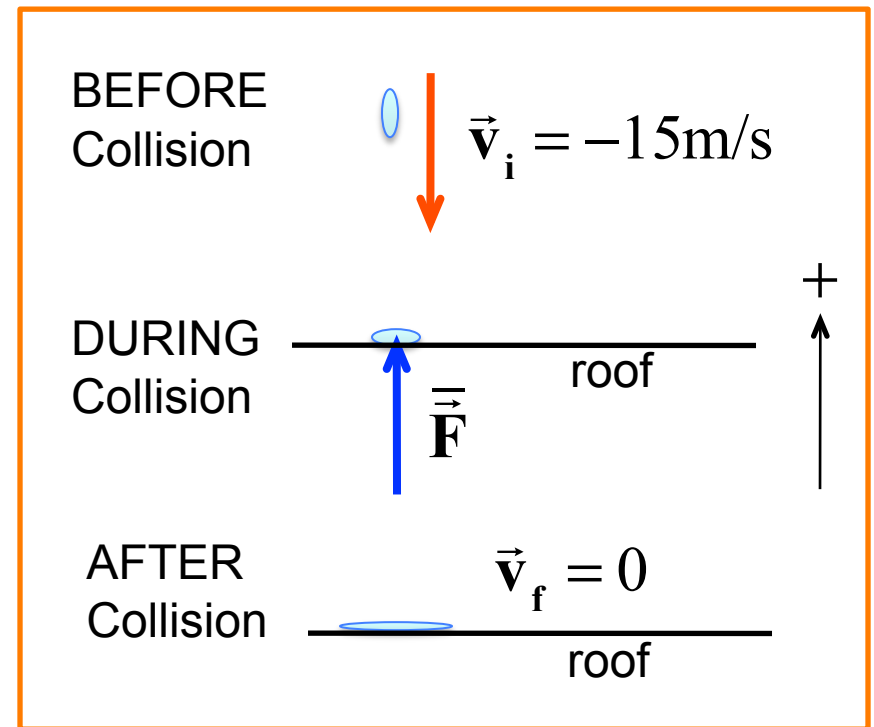
$$\vec{F} = -\left(\frac{m}{\Delta t}\right)\vec{v}_i$$

mass of rain per second $\left(\frac{m}{\Delta t}\right) = 0.060 \text{ kg/s}$

$$\begin{aligned}\vec{F} &= -(0.060 \text{ kg/s})(-15 \text{ m/s}) \\ &= +0.90 \text{ N}\end{aligned}$$

By Newton's 3rd Law average force of raindrops **on the roof** is

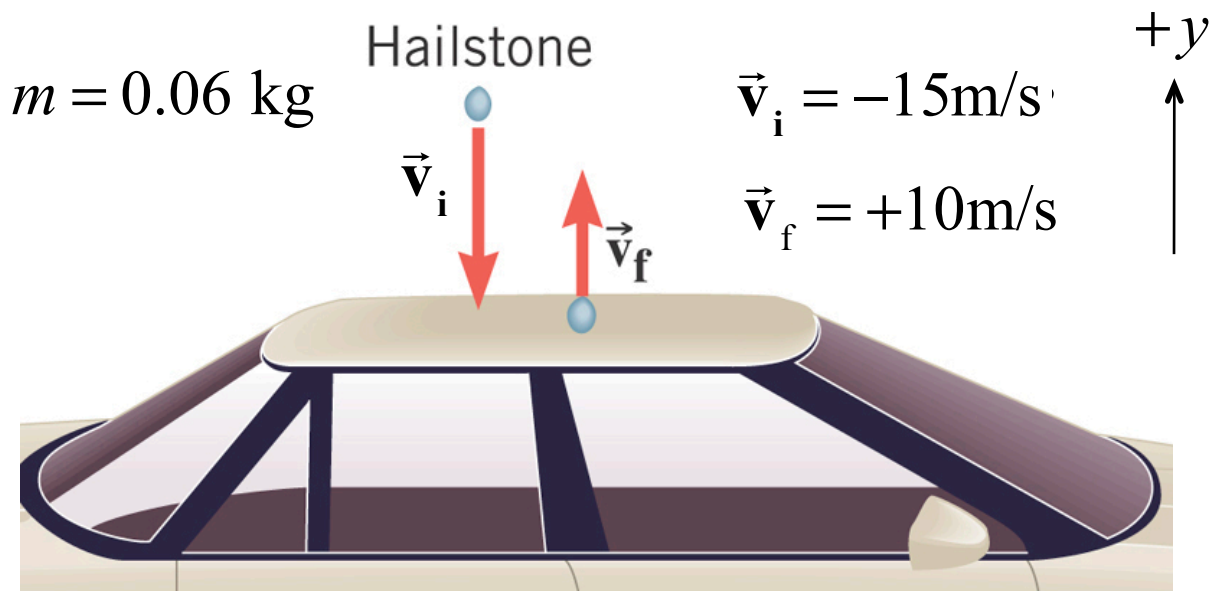
$$\vec{F} = -0.90 \text{ N}$$



Clicker Question 6.1: Hailstones versus raindrops

Instead of rain, suppose hail has velocity of -15 m/s and one hailstone with a mass 0.060 kg hits the roof and bounces off with a velocity of $+10 \text{ m/s}$. In the collision, what is the change in the momentum vector of the hailstone?

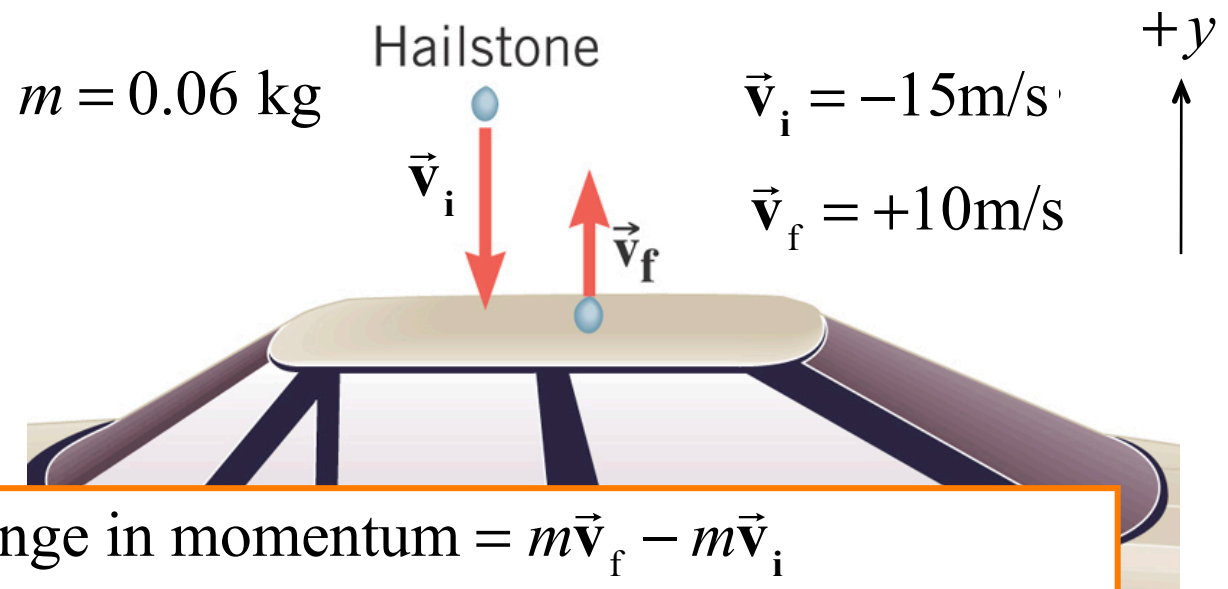
- a) $+0.3 \text{ N} \cdot \text{s}$
- b) $-0.3 \text{ N} \cdot \text{s}$
- c) $0.0 \text{ N} \cdot \text{s}$
- d) $+1.5 \text{ N} \cdot \text{s}$
- e) $-1.5 \text{ N} \cdot \text{s}$



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- c) $0.0 \text{ N} \cdot \text{s}$
- d) $+1.5 \text{ N} \cdot \text{s}$**
- e) $-1.5 \text{ N} \cdot \text{s}$



$$\begin{aligned} \mathbf{F}\Delta t &= \text{change in momentum} = m\vec{v}_f - m\vec{v}_i \\ F_y \Delta t &= m(v_{yf} - v_{yi}) = (0.060 \text{ kg})[+10 \text{ m/s} - (-15 \text{ m/s})] \\ &= +1.5 \text{ kg} \cdot \text{m/s} \end{aligned}$$

6.2 *The Principle of Conservation of Linear Momentum*

WORK-ENERGY THEOREM $\Leftrightarrow$ CONSERVATION OF ENERGY

IMPULSE-MOMENTUM THEOREM $\Leftrightarrow$???

Apply the impulse-momentum theorem to the midair collision between **two objects** while falling due to gravity.

Distinguish the **EXTERNAL** forces and **INTERNAL** forces

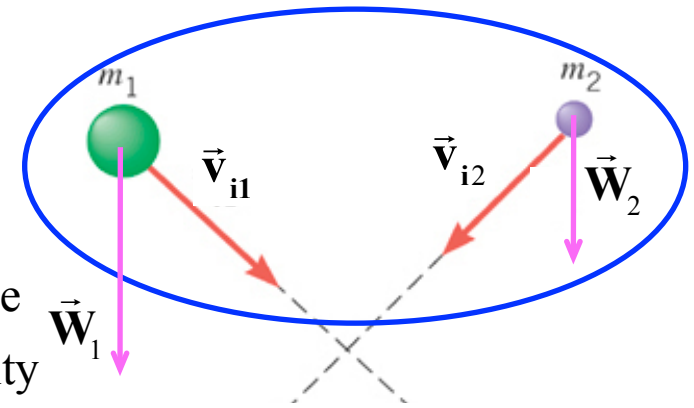
6.2 The Principle of Conservation of Linear Momentum

System of two masses

External forces – Forces exerted on the objects by agents **external** to the system. Net force changes the velocity (& momentum) of the masses.

Newton's 2nd Law

$\vec{W}$ (weight vectors), the **external force** of gravity



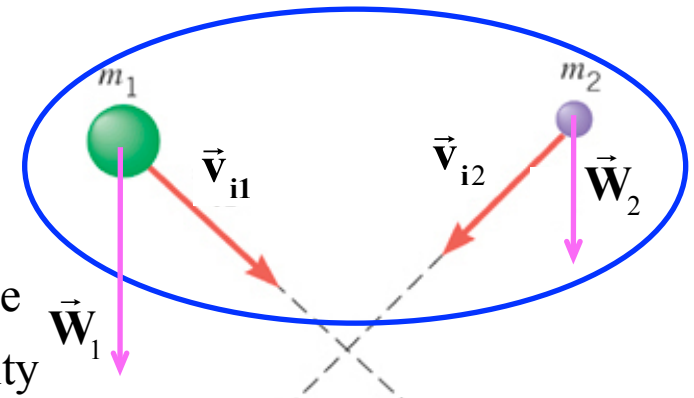
Before the collision

6.2 The Principle of Conservation of Linear Momentum **System of two masses**

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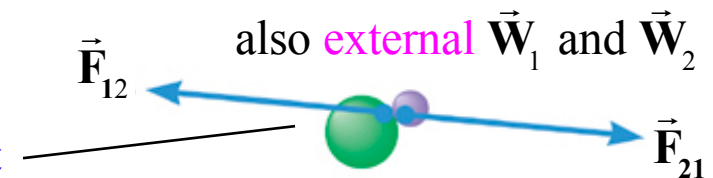


Before the collision

Internal forces – Forces **within the system** that objects exert **on each other**. These forces have equal magnitudes and opposite directions.

Newton's 3rd Law

forces at **contact point**



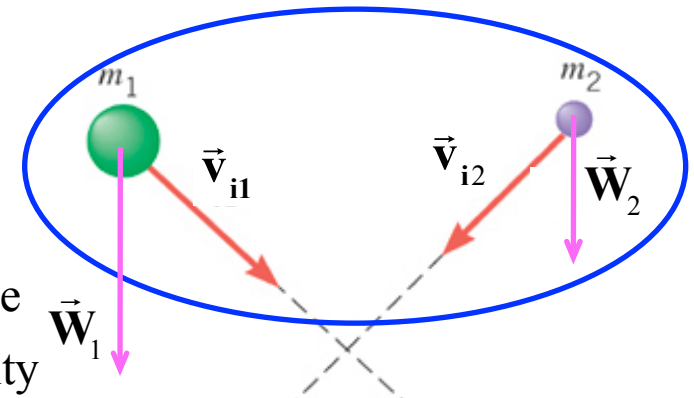
During the collision

6.2 The Principle of Conservation of Linear Momentum System of two masses

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Newton's 2nd Law

$\vec{W}$ (weight vectors), the **external force** of gravity



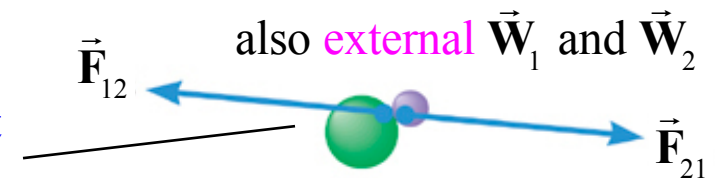
Before the collision

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Newton's 3rd Law

forces at **contact point**

$$\vec{F}_{12} = -\vec{F}_{21}$$

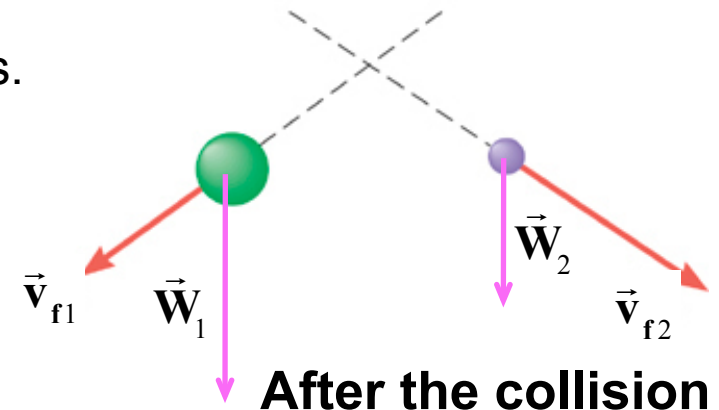


During the collision

External forces – Forces exerted on the objects by agents **external** to the system. Net force changes the velocity (and momentum) of the masses.

Newton's 2nd Law

$\vec{W}$ (weight vectors), the **external force** of gravity



After the collision

6.2 The Principle of Conservation of Linear Momentum

During the collision(Δt)

Impulse-Momentum Theorem

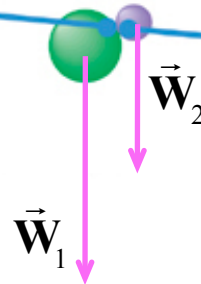
Weight of
mass 1.

Mass 1

$$\left(\vec{W}_1 + \vec{F}_{12}\right)\Delta t = m_1 \vec{v}_{f1} - m_1 \vec{v}_{i1}$$

Force on 1
generated by 2

$\vec{F}_{12}$



Impulse-Momentum Theorem

Weight of
mass 2.

Mass 2

$$\left(\vec{W}_2 + \vec{F}_{21}\right)\Delta t = m_2 \vec{v}_{f2} - m_2 \vec{v}_{i2}$$

Force on 2
generated by 1

$\vec{F}_{21}$

Net effect on the **system** of two masses $\Rightarrow$ add the equations together

6.2 The Principle of Conservation of Linear Momentum

During the collision(Δt)

Impulse-Momentum Theorem

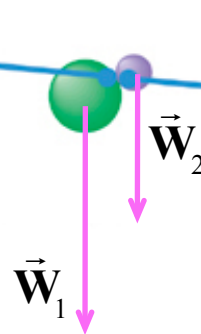
Weight of
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Mass 1

$$\left(\vec{W}_1 + \vec{F}_{12} \right) \Delta t = m_1 \vec{v}_{f1} - m_1 \vec{v}_{i1}$$

Force on 1
generated by 2

$\vec{F}_{12}$



Impulse-Momentum Theorem

Weight of
mass 2.

Mass 2

$$\left(\vec{W}_2 + \vec{F}_{21} \right) \Delta t = m_2 \vec{v}_{f2} - m_2 \vec{v}_{i2}$$

Force on 2
generated by 1

Net effect on the **system** of two masses $\Rightarrow$ add the equations together

$$\left(\vec{W}_1 + \vec{F}_{12} + \vec{W}_2 + \vec{F}_{21} \right) \Delta t = (m_1 \vec{v}_{f1} - m_1 \vec{v}_{i1}) + (m_2 \vec{v}_{f2} - m_2 \vec{v}_{i2})$$

At contact point: $\vec{F}_{12} = -\vec{F}_{21}$

put final values together & initial values together

$$\left(\vec{W}_1 + \vec{W}_2 \right) \Delta t = (m_1 \vec{v}_1 + m_2 \vec{v}_2)_f - (m_1 \vec{v}_1 + m_2 \vec{v}_2)_i$$

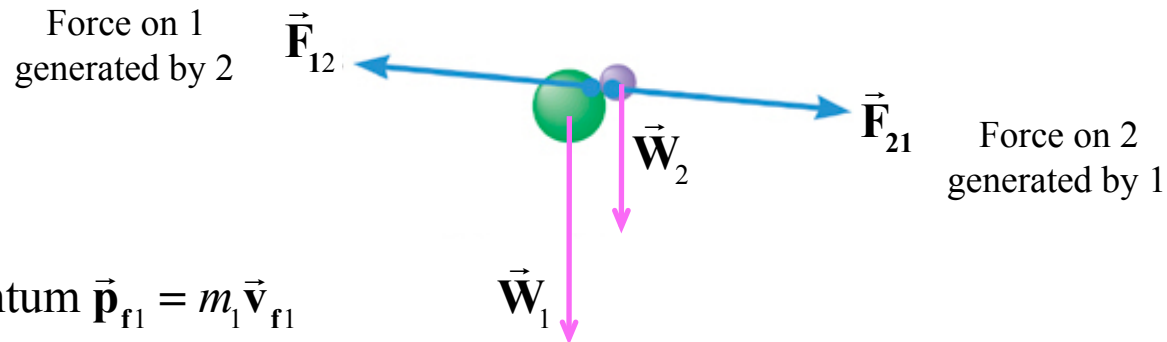
Impulses due only
to **external forces**

Total momentum
in the **final system**

Total momentum
in the **initial system**

6.2 The Principle of Conservation of Linear Momentum

During the collision(Δt)



Using momentum $\vec{p}_{f1} = m_1 \vec{v}_{f1}$

$$(\vec{W}_1 + \vec{W}_2)\Delta t = (\vec{p}_1 + \vec{p}_2)_f - (\vec{p}_1 + \vec{p}_2)_i$$

Impulses due only
to **EXTERNAL** forces

Total momentum
in the **final system**

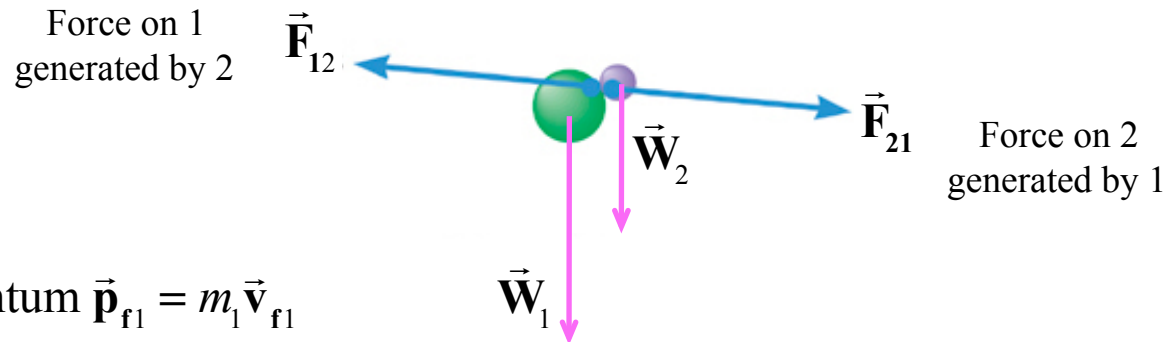
Total momentum
in the **initial system**

Only **EXTERNAL** forces can change the Total Momentum of a system of masses

With only **INTERNAL** forces affecting motion (e.g., if external forces are balanced)

6.2 The Principle of Conservation of Linear Momentum

During the collision(Δt)



Using momentum $\vec{p}_{f1} = m_1 \vec{v}_{f1}$

$$(\vec{W}_1 + \vec{W}_2)\Delta t = (\vec{p}_1 + \vec{p}_2)_f - (\vec{p}_1 + \vec{p}_2)_i$$

Impulses due only
to **external forces**

Total momentum
in the final **system**

Total momentum
in the initial **system**

Only **EXTERNAL** forces can change the Total Momentum of a system of masses

With only **INTERNAL** forces affecting motion (e.g., if external forces are balanced)

$$0 = (\vec{p}_1 + \vec{p}_2)_f - (\vec{p}_1 + \vec{p}_2)_i$$

$$(\vec{p}_1 + \vec{p}_2)_f = (\vec{p}_1 + \vec{p}_2)_i$$

Final value of
total momentum

Initial value of
total momentum

If only **INTERNAL** forces affect motion,
total momentum **VECTOR** of a **system** does not change

6.2 The Principle of Conservation of Linear Momentum

If only INTERNAL forces affect the motion,
total momentum VECTOR of a **system** does not change

$$(\vec{p}_1 + \vec{p}_2 + \dots)_f = (\vec{p}_1 + \vec{p}_2 + \dots)_i$$

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an **isolated system** of masses is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

Most Important example

If there are **NO** external forces affecting the motion, e.g., gravitational forces are balanced by normal forces, the total momentum VECTOR of a system is conserved.

After a collision the total momentum vector does not change.

Clicker Question 6.2

Two hockey pucks bang into each other on frictionless ice. Each puck has a mass of 0.5 kg, and are moving directly toward each other each with a speed of 12 m/s. What is the total momentum vector of the system of two pucks?

- a) $6.0 \text{ N} \cdot \text{s}$
- b) $12 \text{ N} \cdot \text{s}$
- c) $-6.0 \text{ N} \cdot \text{s}$
- d) $-12 \text{ N} \cdot \text{s}$
- e) $0.0 \text{ N} \cdot \text{s}$

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- e) $0.0 \text{ N} \cdot \text{s}$

Clicker Question 6.3

After the pucks collide, what is the total momentum of the system?

- a) $6.0 \text{ N} \cdot \text{s}$
- b) $12 \text{ N} \cdot \text{s}$
- c) $-6.0 \text{ N} \cdot \text{s}$
- d) $-12 \text{ N} \cdot \text{s}$
- e) $0.0 \text{ N} \cdot \text{s}$