Chapter 8

Accelerated Circular Motion

continued

8.3 Angular Variables and Tangential Variables

$\omega =$ angular velocity is the same at all radii

$\alpha =$ angular acceleration is the same at all radii

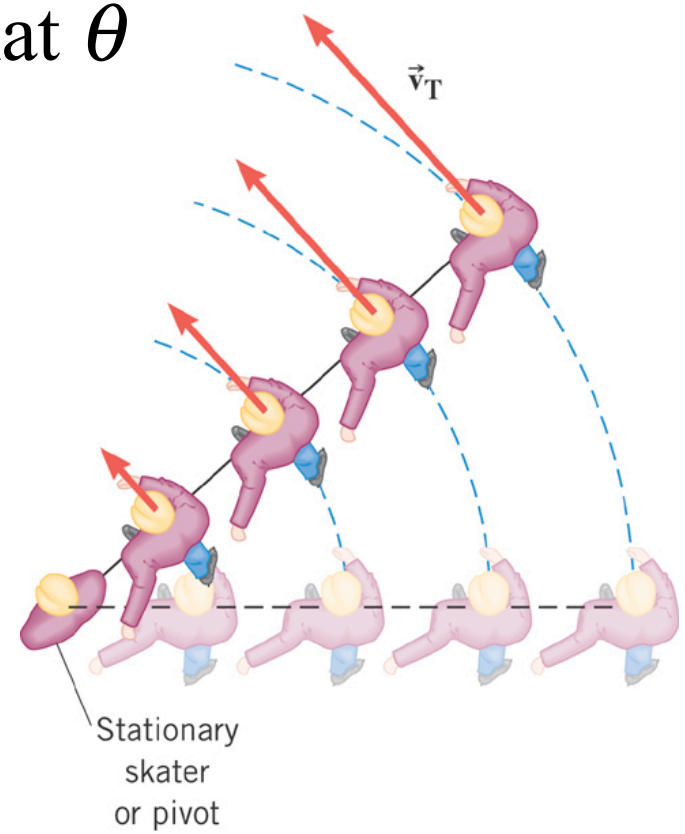
$\vec{\mathbf{v}}_T =$ tangential velocity is different at each radius

$\vec{\mathbf{a}}_T =$ tangential acceleration is different at each radius

Direction is tangent to circle at that θ

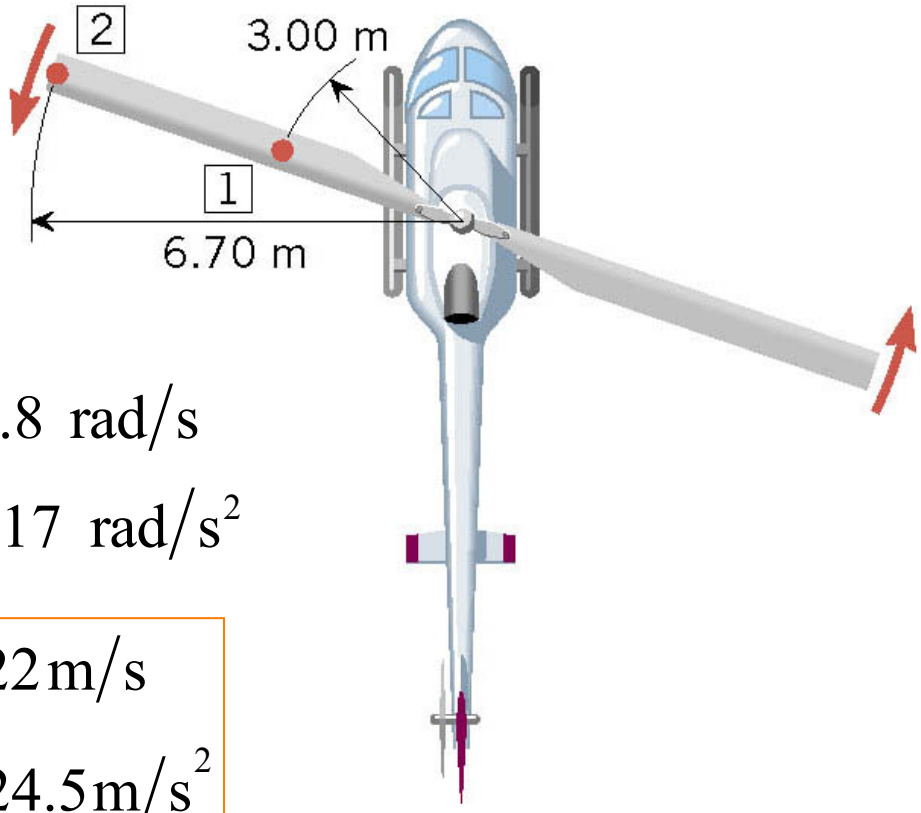
$$\vec{\mathbf{v}}_T = \omega r \qquad \vec{\mathbf{a}}_T = \alpha r$$

$\vec{\mathbf{v}}_T$ (m/s)	$\vec{\mathbf{a}}_T$ (m/s^2)
ω (rad/s)	α (rad/s^2)
r (m)	r (m)



Example: A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2. For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



Convert revolutions to radians

$$\omega = (6.50\ \text{rev/s})(2\pi\ \text{rad/rev}) = 40.8\ \text{rad/s}$$

$$\alpha = (1.30\ \text{rev/s}^2)(2\pi\ \text{rad/rev}) = 8.17\ \text{rad/s}^2$$

$$v_T = \omega r = (40.8\,\text{rad/s})(3.00\,\text{m}) = 122\,\text{m/s}$$

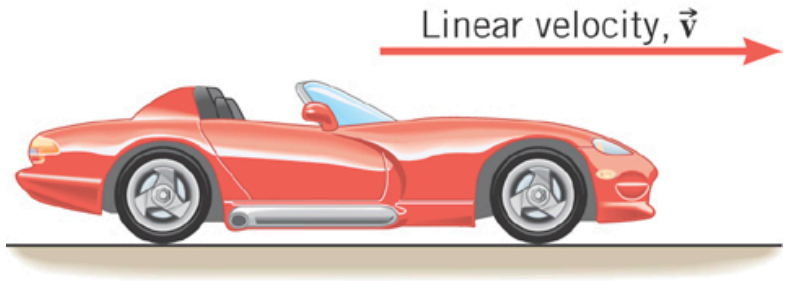
$$a_T = \alpha r = (8.17\,\text{rad/s}^2)(3.00\,\text{m}) = 24.5\,\text{m/s}^2$$

8.3 Rolling Motion

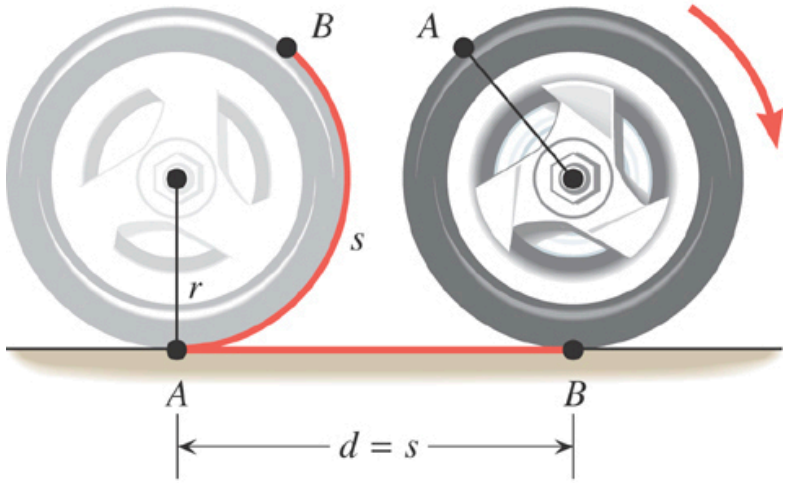
The tangential speed of a point on the outer edge of the tire is equal to the speed of the car over the ground.

$$v = v_T = \omega r$$

$$a = a_T = \alpha r$$



(a)



(b)

Chapter 4.5

Force Generating Uniform Circular Motion

Newton's Second Law

When a net external force acts on an object of mass *m*, the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \qquad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

Vector Equations

Thus, in uniform circular motion there must be a net force to produce the centripetal acceleration.

The centripetal force is the name given to the net force required to keep an object moving on a circular path.

The direction of the centripetal force always points toward the center of the circle and continually changes direction as the object moves.

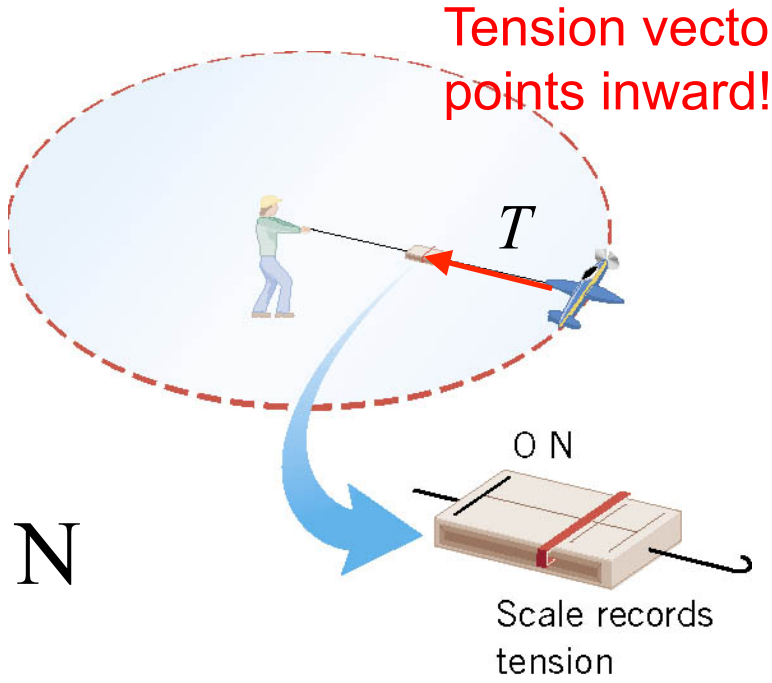
$$F_C = ma_C = m\frac{v^2}{r}$$ Magnitudes

Example: The Effect of Speed on Centripetal Force

The model airplane has a mass of 0.90 kg and moves at constant speed on a circle that is parallel to the ground. The path of the airplane and the guideline lie in the same horizontal plane because the weight of the plane is balanced by the lift generated by its wings. Find the tension in the 17 m guideline for a speed of 19 m/s.

$$T = F_C = m\frac{v^2}{r}$$

$$T = (0.90\ \text{kg})\frac{(19\ \text{m/s})^2}{17\ \text{m}} = 19\ \text{N}$$



Example: The Effect of Speed on Centripetal Force

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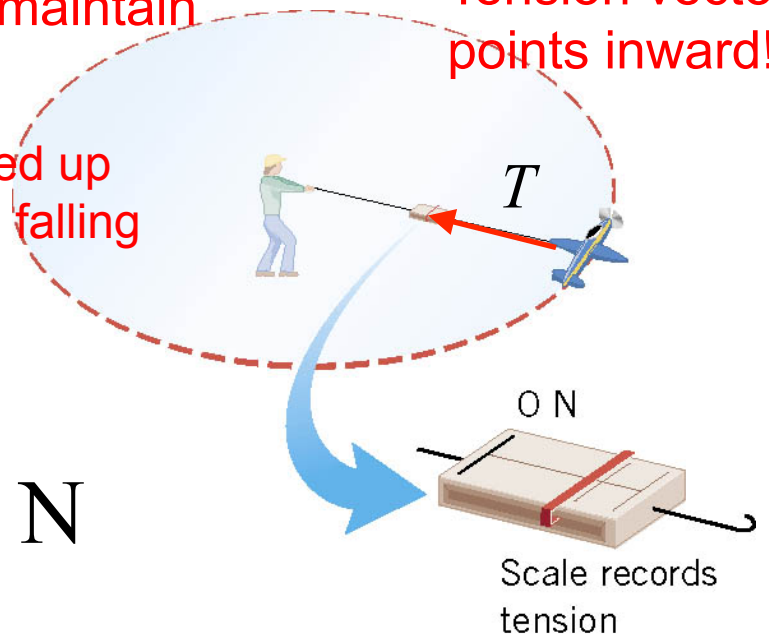
Tension is the centripetal force necessary to maintain airplane in the circle

$$T = F_C = m\frac{v^2}{r}$$

engine keeps speed up
wings keep it from falling

Tension vector points inward!



$$T = (0.90\,\text{kg})\frac{(19\,\text{m/s})^2}{17\,\text{m}} = 19\,\text{N}$$

Conceptual Example: A Trapeze Act

In a circus, a man hangs upside down from a trapeze, legs bent over and arms downward, holding his partner. Is it harder for the man to hold his partner when the partner hangs straight down and is stationary or when the partner is swinging through the straight-down position?

Tension in arms maintains circular motion but also must counter the gravitational force (weight)

Conceptual Example: A Trapeze Act

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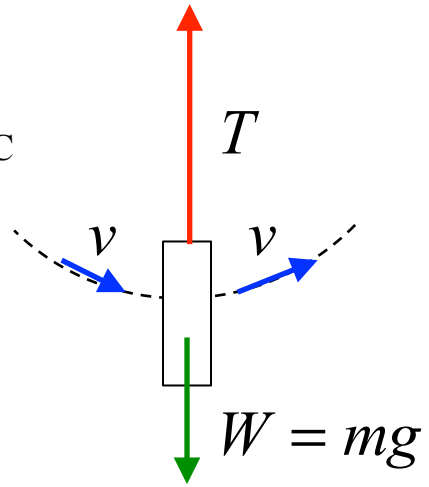
Tension in arms maintains circular motion but also must counter the gravitational force (weight)

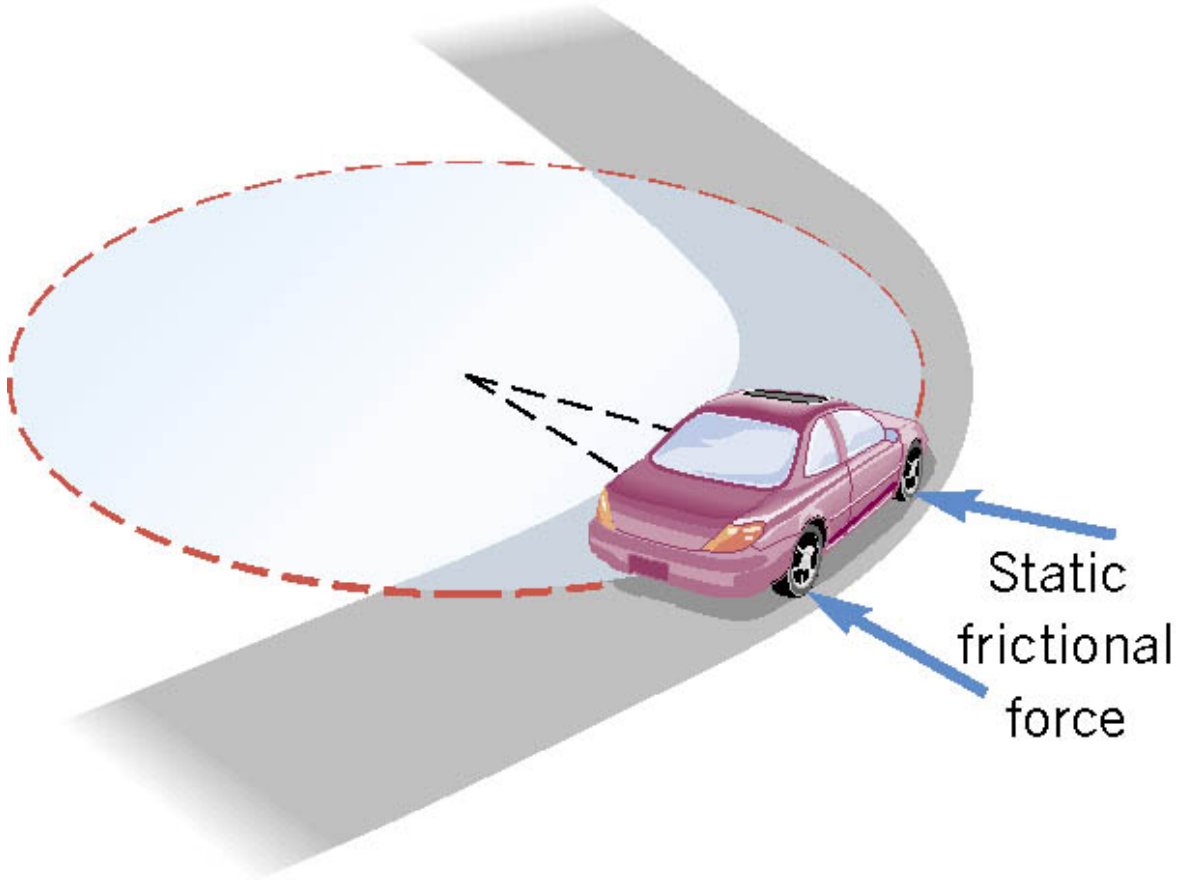
$$\sum \vec{\mathbf{F}} = +T - W = F_C$$

$$T = W + F_C$$



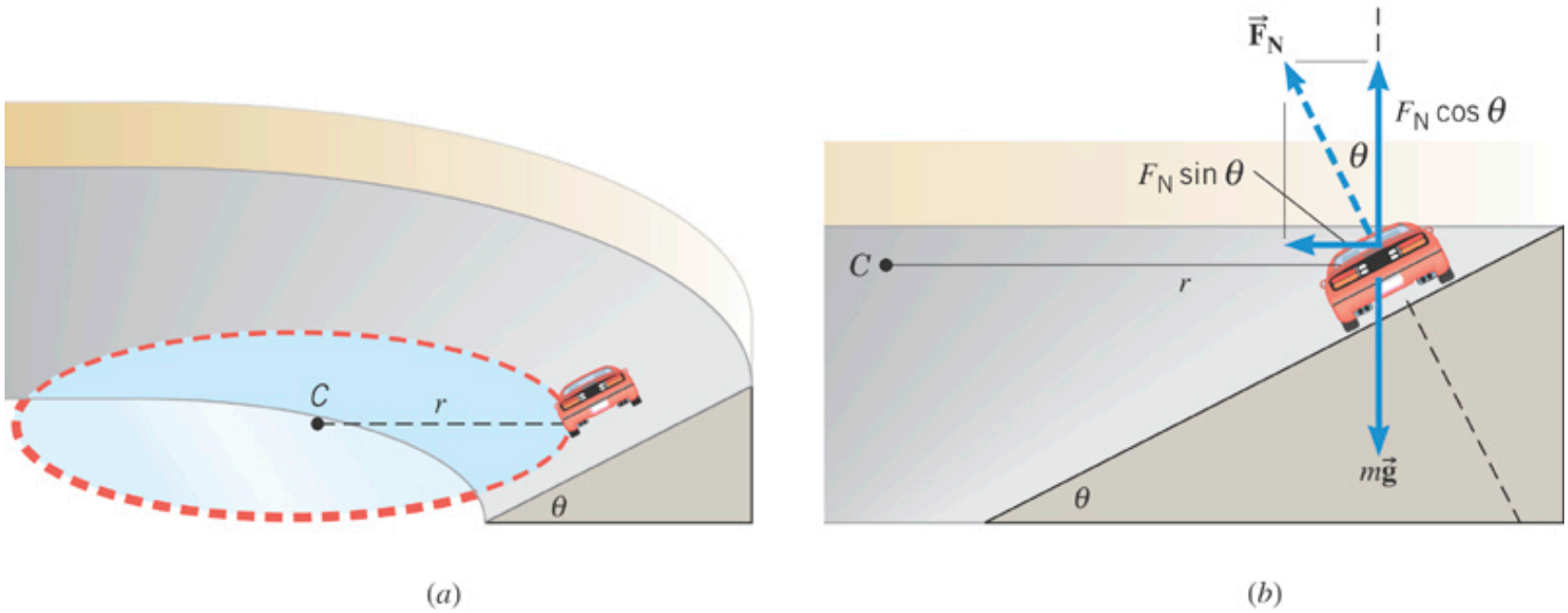
4.5 Banked Curves

On an unbanked curve, the static frictional force provides the centripetal force.



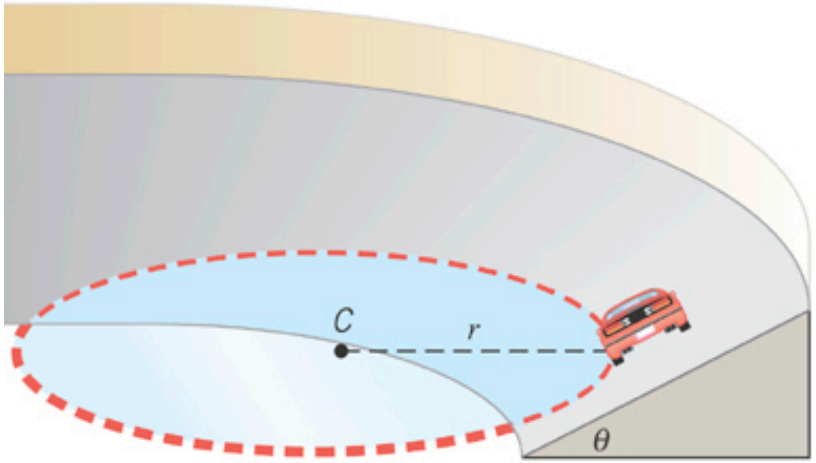
4.5 Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.

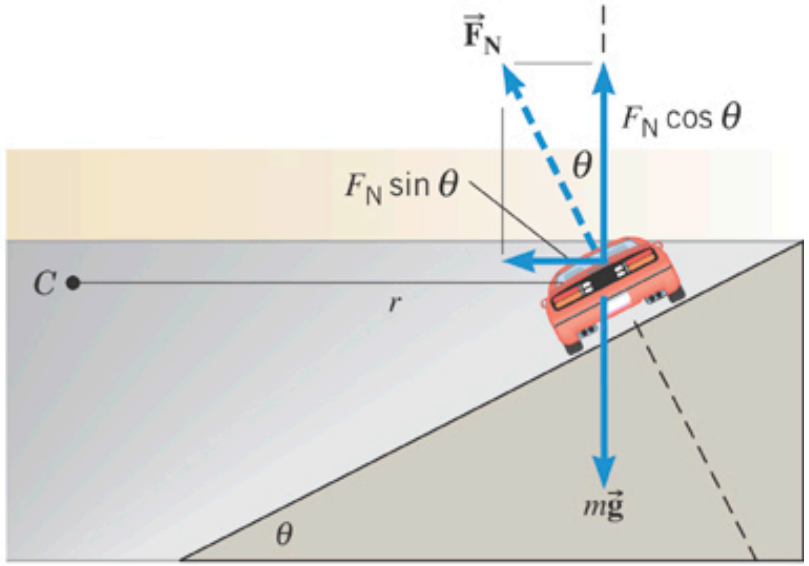


Compression of the banked road provides the normal force. The normal force pushes against the car to 1) support the weight and 2) provides the centripetal force required for the car to move in a circle.

4.5 Banked Curves



(a)



(b)

$$F_C = F_N \sin\theta = m\frac{v^2}{r}$$

$$F_N \cos\theta = mg$$

Combining the two relationships can determine the speed necessary to keep the car on the track with the given angle

Example: Artificial Gravity

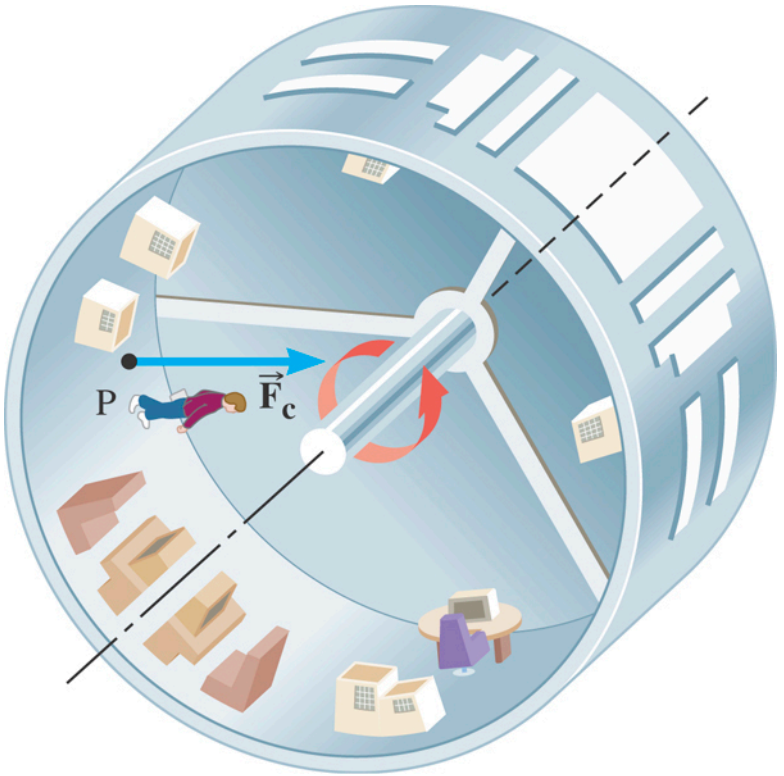
At what speed must the surface of a space station move so that an astronaut experiences a normal force on the feet equal to the weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

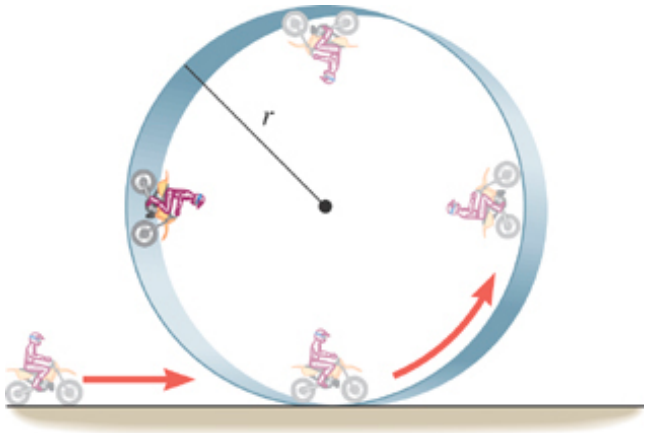
$$= \sqrt{(1700\ \text{m})(9.80\,\text{m/s}^2)}$$

$$= 130\ \text{m/s}$$



4.5 *Vertical Circular Motion*

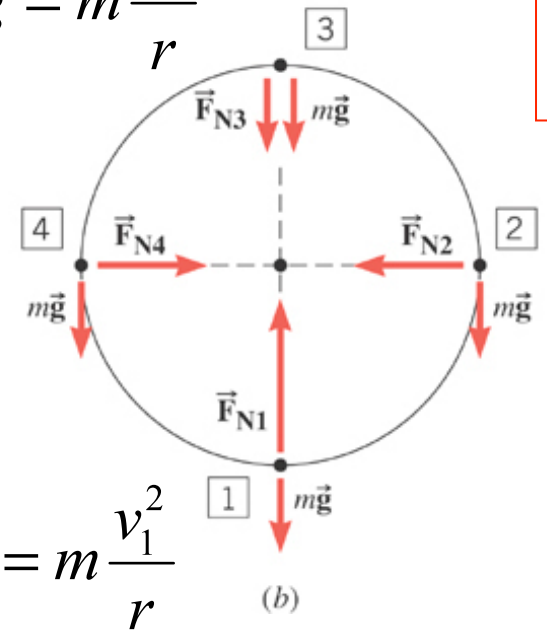
Normal forces are created by stretching of the hoop.



$$\frac{v_3^2}{r} \text{ must be } > g$$

to stay on the track

$$F_{N3} + mg = m\frac{v_3^2}{r}$$

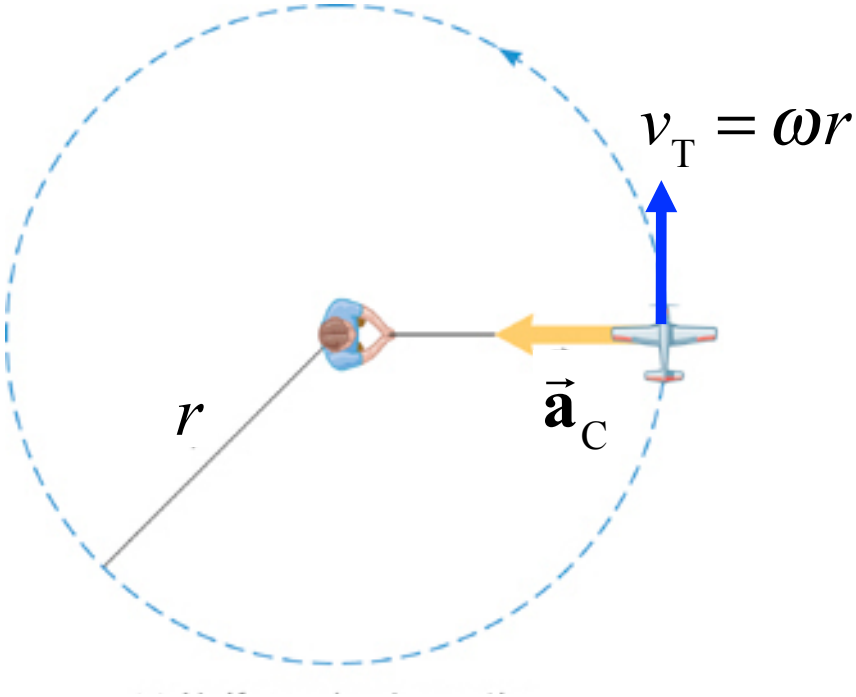


$$F_{N4} = m\frac{v_4^2}{r}$$

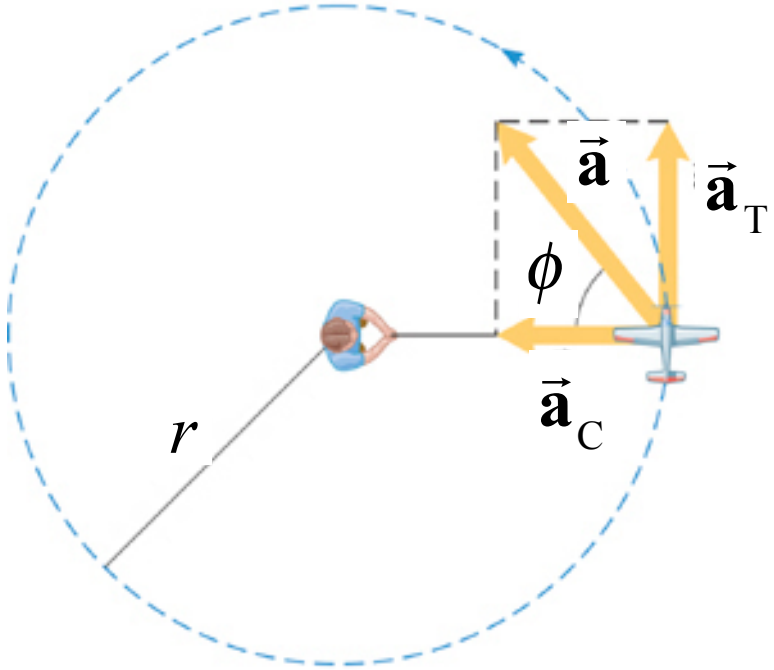
$$F_{N2} = m\frac{v_2^2}{r}$$

$$F_{N1} - mg = m\frac{v_1^2}{r}$$

8.3 Centripetal Acceleration with Tangential Acceleration



Uniform circular motion



Non-uniform circular motion

ω in rad/s constant

$$a_C = \frac{v_T^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

Changing $\omega = \omega_0 + \alpha t$

$$a_T = \alpha r$$

$$a_{total} = \sqrt{a_C^2 + \alpha^2 r^2}$$

Chapter 9

Gravitation and Circular Orbits

Newton's Law of Universal Gravitation

Every particle in the universe exerts an attractive force on every other particle.

A particle is a piece of matter, small enough in size to be regarded as a mathematical point.

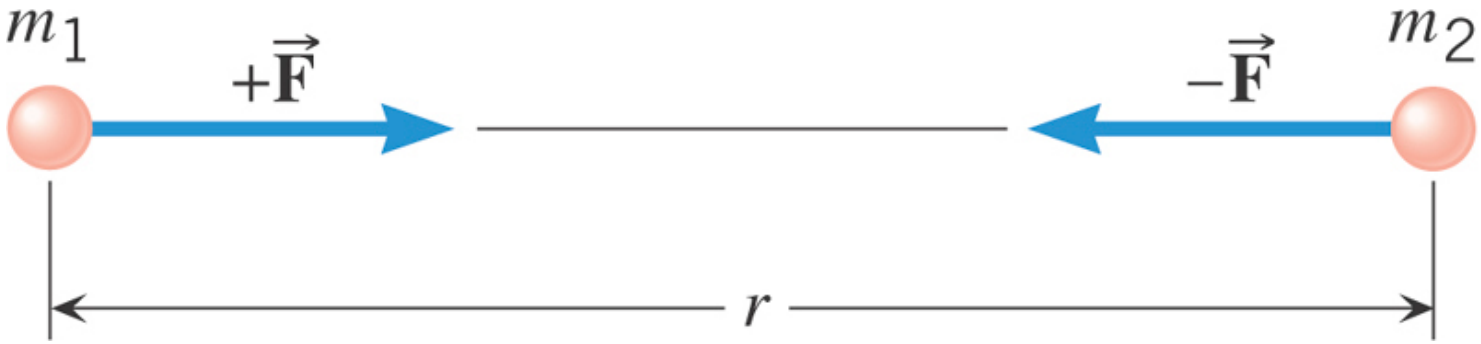
The force that each exerts on the other is directed along the line joining the particles.

9.1 The Gravitational Force

For two particles that have masses m_1 and m_2 and are separated by a distance r, the force has a magnitude given by

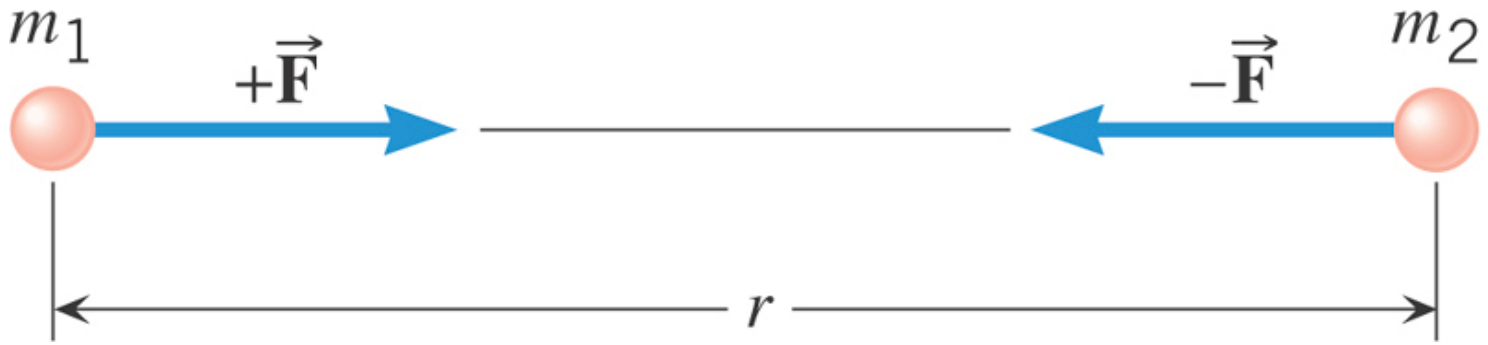
$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$



the same magnitude of force acts on each mass, no matter what the values of the masses.

9.1 The Gravitational Force

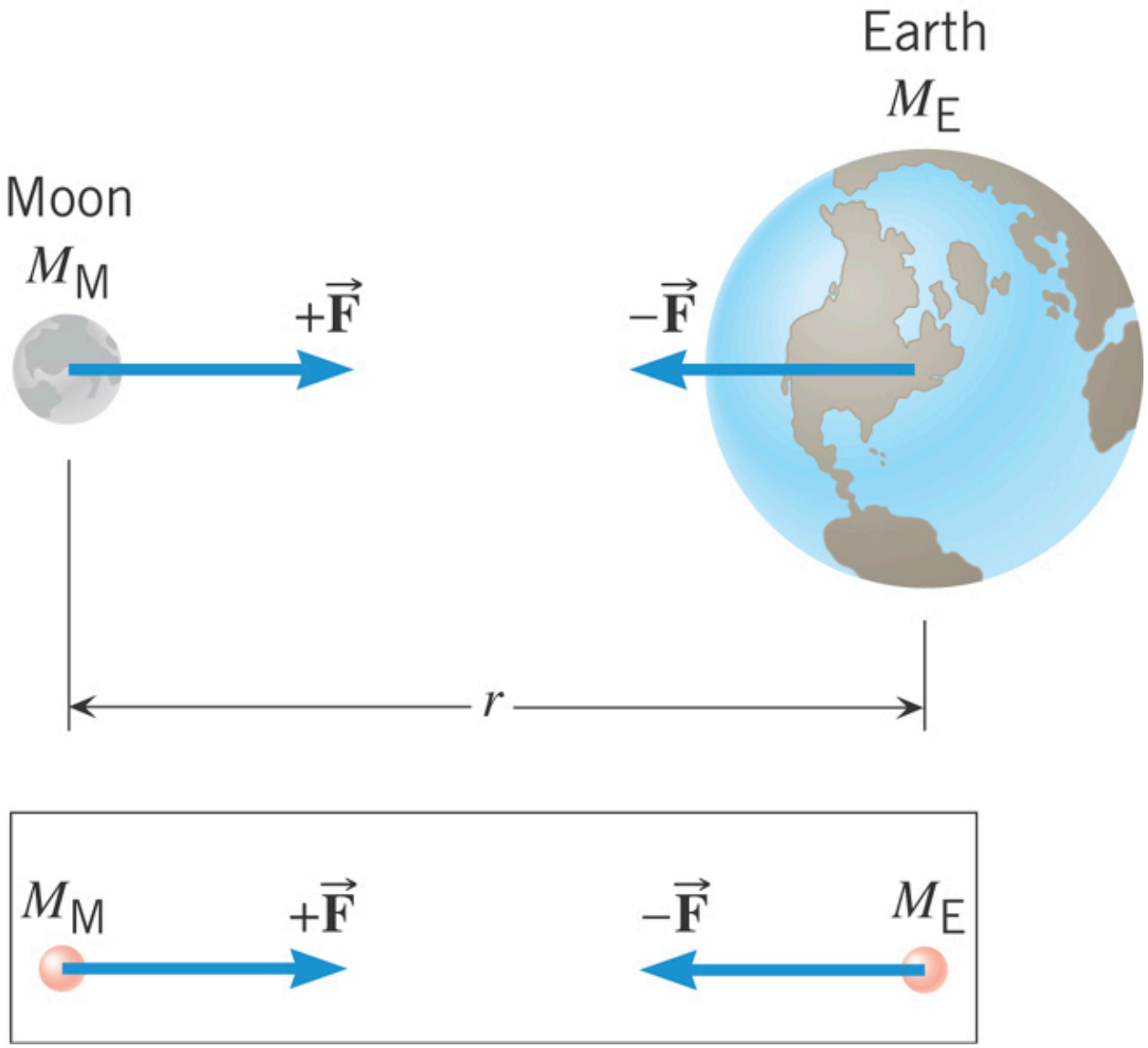


$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(12\ \text{kg})(25\ \text{kg})}{(1.2\ \text{m})^2}$$

$$= 1.4\times10^{-8}\ \text{N}$$

9.1 The Gravitational Force



Reminder: Definition of Weight

The weight of an object on or above the earth is the gravitational force that the earth exerts on the object. The weight always acts downwards, toward the center of the earth.

On or above another astronomical body, the weight is the gravitational force exerted on the object by that body.

SI Unit of Weight: newton (N)

9.1 The Gravitational Force

Relation Between Mass and Weight

WEIGHT of mass *m*, is a force vector

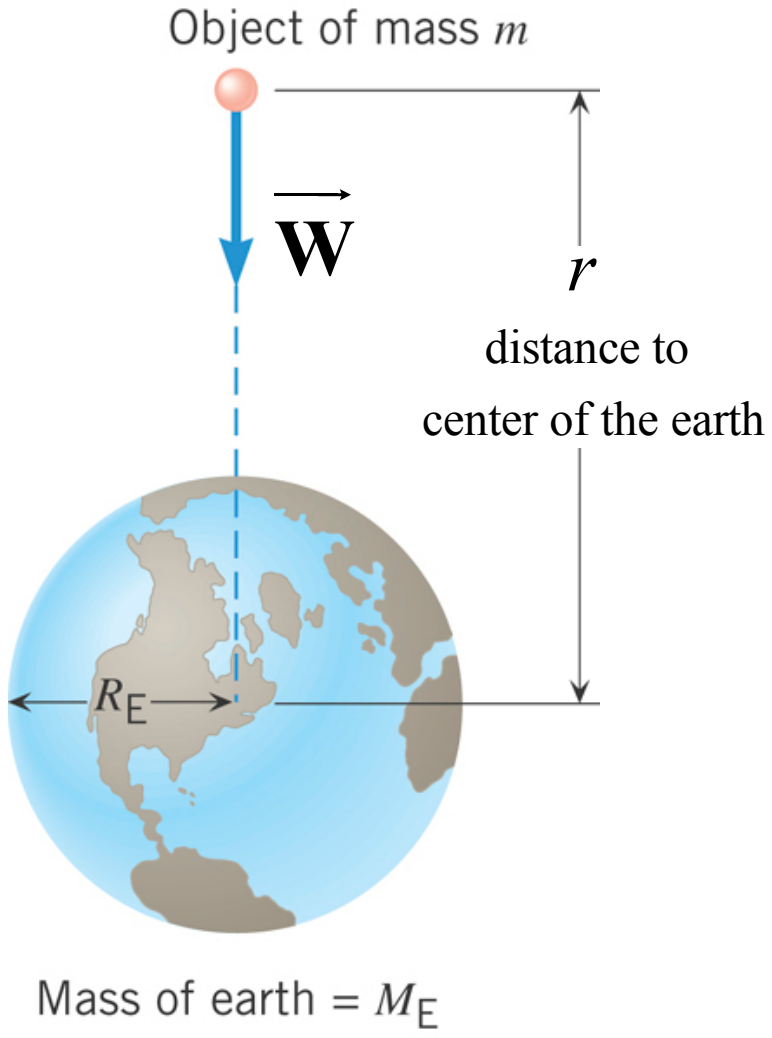
$$\vec{\mathbf{W}} = G\frac{mM_{\mathrm{E}}}{r^2}, \text{ downward}$$

WEIGHT(magnitude) of mass *m*

$$W = mg, \qquad g = G\frac{M_{\mathrm{E}}}{r^2}$$

Your WEIGHT

Your "weight" is the force that gravity applies to your body.
But, what you feel is the normal force of the floor pushing up on your feet



Near the earth's surface

$$\boxed{r = R_{\mathrm{E}} = 6.38\times10^{6}\ \mathrm{m}}$$

Radius of the earth

$$g = G\frac{M_{\mathrm{E}}}{R_{\mathrm{E}}^{2}}$$

$$= \left(6.67\times10^{-11}\ \mathrm{N\cdot m^{2}/kg^{2}}\right)\frac{\left(5.97\times10^{24}\ \mathrm{kg}\right)}{\left(6.37\times10^{6}\ \mathrm{m}\right)^{2}}$$

$$= 9.81\ \mathrm{m/s^{2}}$$

This is why acceleration due to gravity is this value on the earth.

Your WEIGHT on the earth

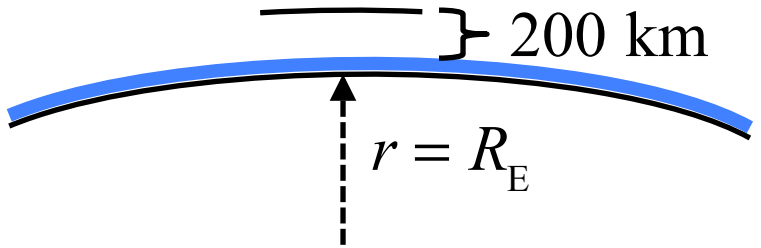
$$W = mg$$

for example: $m = 80.0\ \mathrm{kg}$,

$$W = mg = 784\ \mathrm{N}$$

Near the earth's surface

In orbit at altitude = 200 km



$g = 9.81\ \text{m/s}^2$ At radius of the earth

$g' = 9.20\ \text{m/s}^2$

At 200 km above the earth

$$r' = R_E + 200\ \text{km} = 6.37\times10^6 + 0.20\times10^6\ \text{m}$$

$$= 6.57\times10^6\ \text{m}$$

$$g' = \frac{GM_E}{r'^2} = 9.21\ \text{m/s}^2$$

In low-earth orbit,
your weight is almost the same as on earth. NOT ZERO!

Clicker Question 9.1

A person weighs 500 N on the earth. Consider this person on planet P where the acceleration due to gravity is, $g_P = 4.9\ m/s^2$. Chose the answer that is *false* or answer e).

a) On the earth, the mass of the person is 51 kg.

b) Everywhere on the earth, the person has a mass of 51 kg.

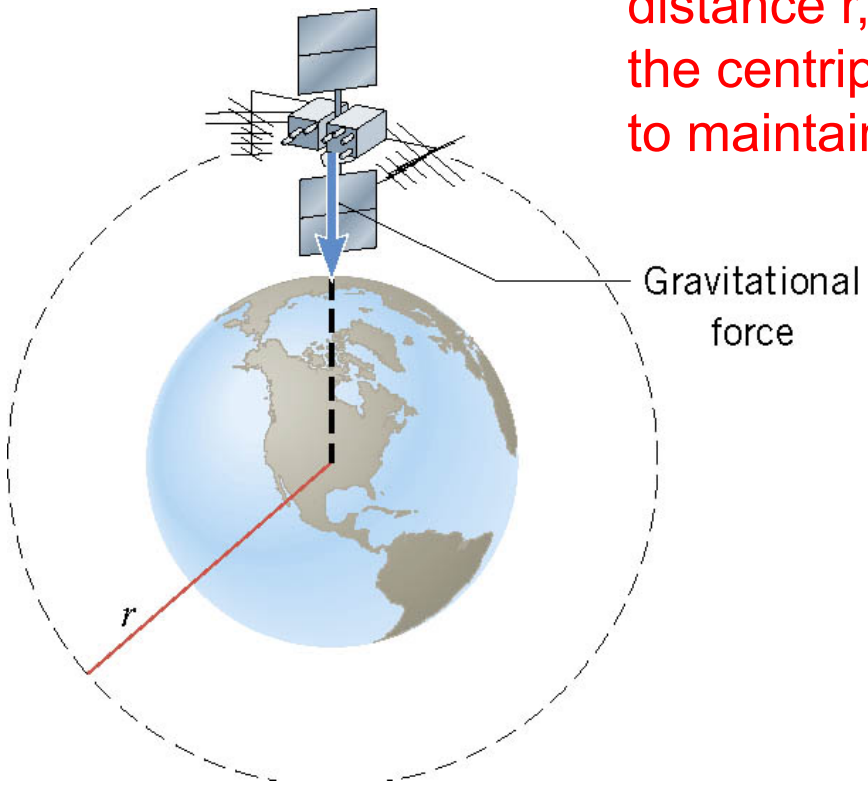
c) On the planet P, the mass of the person is 51 kg.

d) On the planet P, the weight of the person is 250 N.

e) All of the above are true.

9.2 *Satellites in Circular Orbits*

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



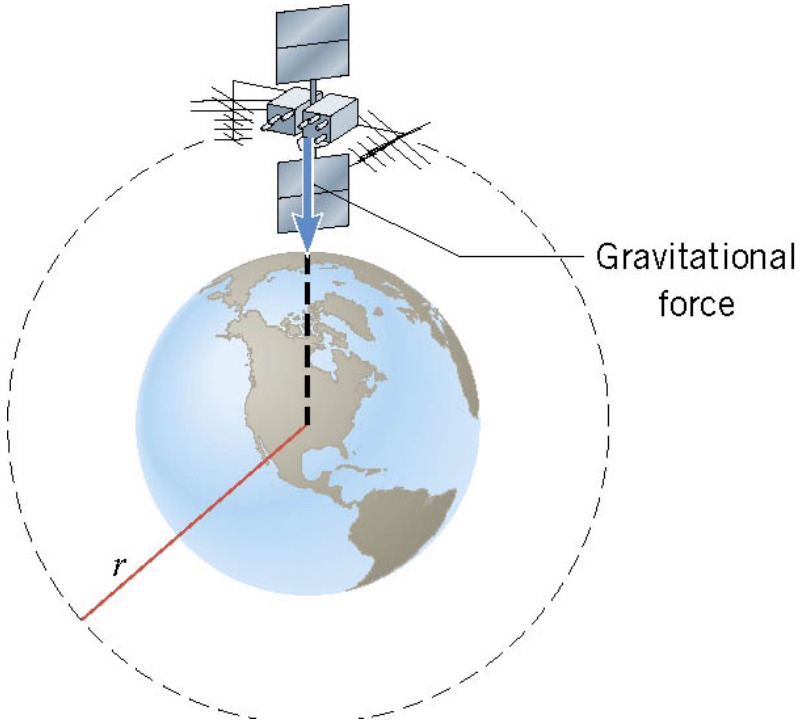
Gravitational force at the distance r, is the source of the centripetal force necessary to maintain the circular orbit.

9.2 *Satellites in Circular Orbits*

Gravitational force at the distance r

Centripetal force

$$F_C = G\frac{mM_E}{r^2} = m\frac{v^2}{r}$$



$$v = \sqrt{\frac{GM_E}{r}}$$

Speed to keep satellite in the orbit with radius r.

9.2 Satellites in Circular Orbits

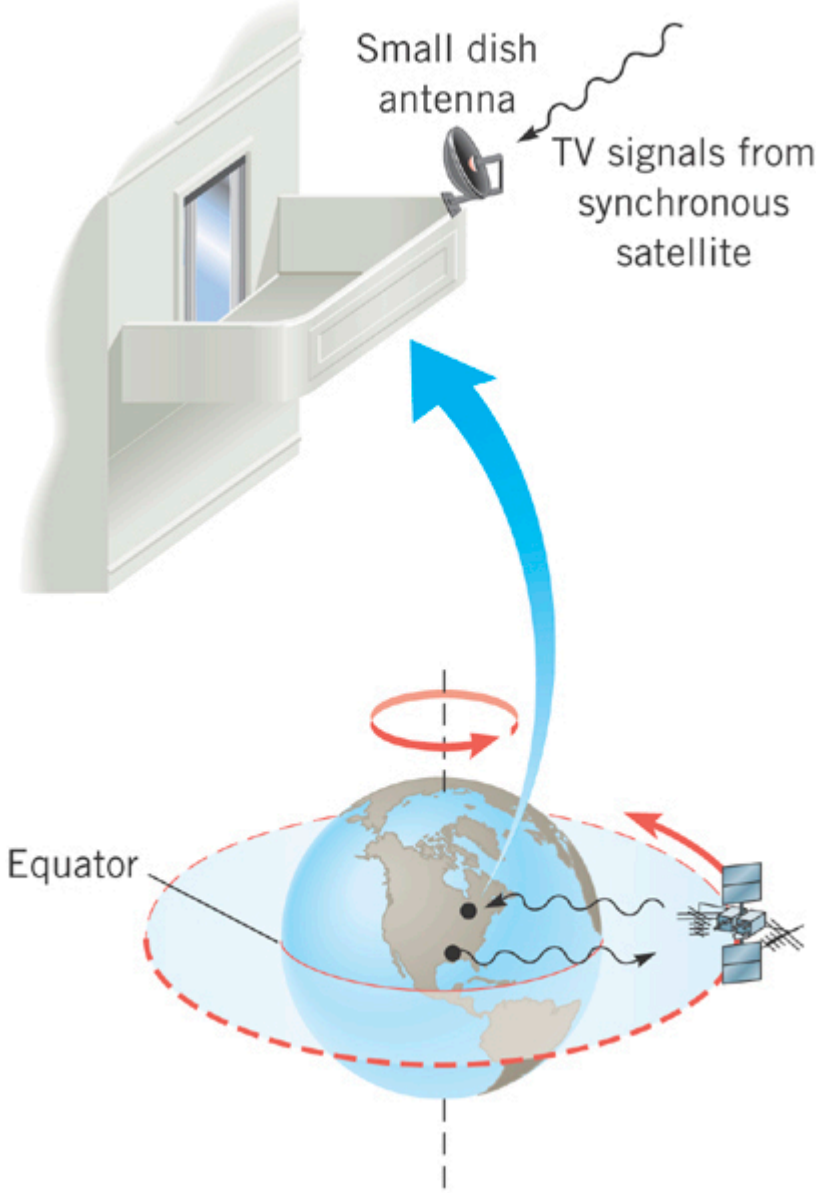
There is a radius where the speed will make the satellite go around the earth in exactly 24 hours. This keeps the satellite at a fixed point in the sky.

$$\omega = \frac{2\pi}{(3600\,\text{s/hr})(24\,\text{hr})}$$
$$= 72.7\times10^{-6}\,\text{rad/s}$$

$$\omega r_s = v_T = \sqrt{\frac{GM_E}{r_s}}$$

$$r_s^3 = \frac{GM_E}{\omega^2}$$
$$= \frac{\left(6.67\times10^{-11}\,\text{Nm}^2/\text{kg}^2\right)\left(5.97\times10^{24}\,\text{kg}\right)}{(72.7\times10^{-6}\,\text{rad/s})^2}$$

$$r_s = 42{,}200\,\text{km (synchronous orbit)}$$



9.2 Apparent Weightlessness

Can you feel gravity? We previously determined that you can't.

1) Hanging from a 100 m high diving board
– your arms feel stretched by the bending of the board.

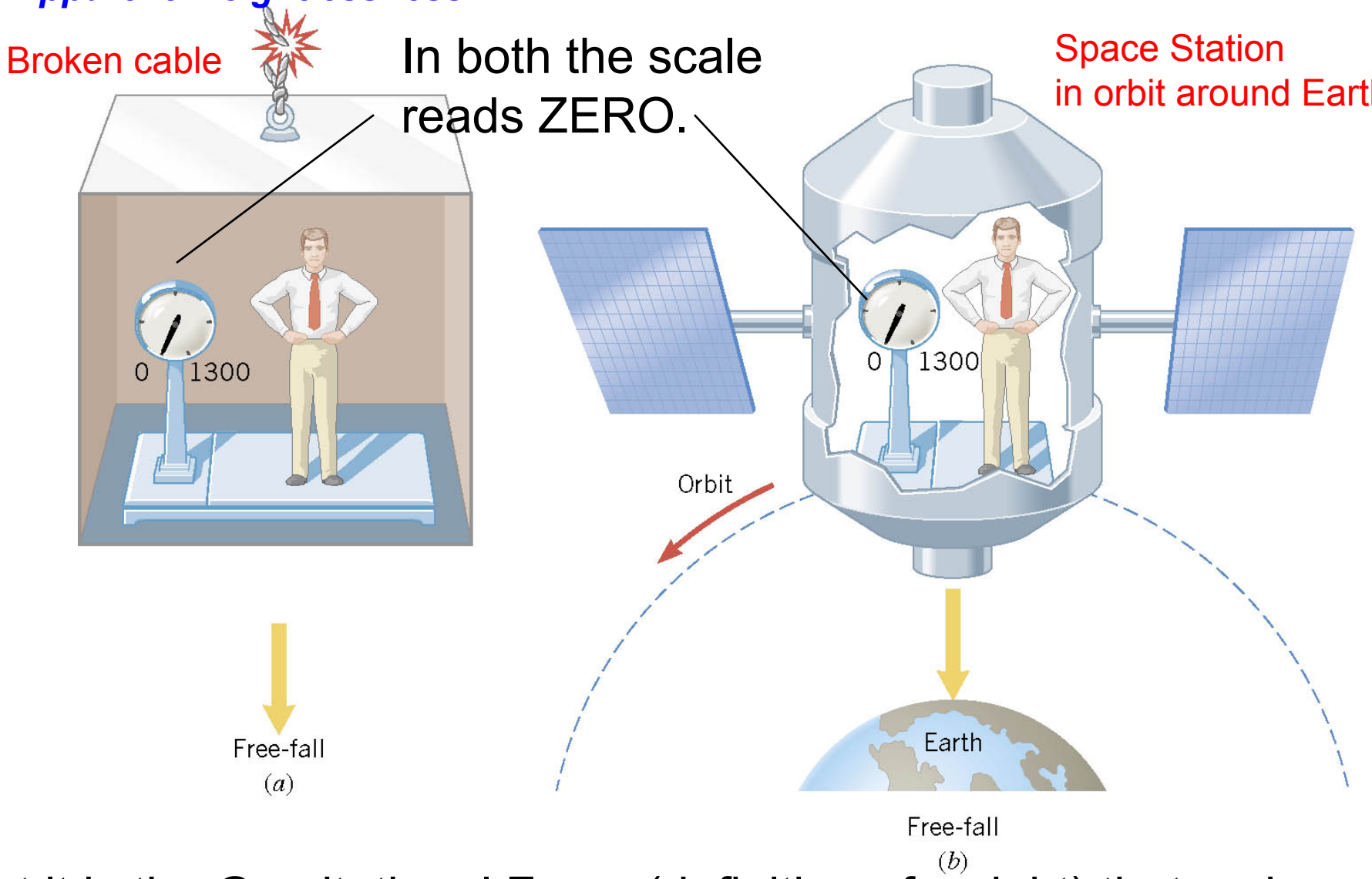
2) Standing on a bed – your legs feel compressed by the springs in the mattress.

The bent diving board or the compressed springs provide the force to balance the gravitational force on your whole body.

When you let go of the diving board and before you hit the ground the ONLY force on you is gravity. It makes you accelerate downward, but it does not stretch or compress your body.

In free fall one cannot feel the force of gravity!

9.2 *Apparent Weightlessness*



But it is the Gravitational Force (definition of weight) that makes both the elevator and the body free-fall with the same acceleration. FEELING weightless and BEING weightless are VERY different.