Practical Lab 2
Rotational Motion
Moment of inertia

Objectives

- to familiarize yourself with the concept of moment of inertia, I, which plays the same role in the description of the rotation of a rigid body as mass plays in the description of linear motion
- calculate the linear acceleration of a falling object

APPARATUS

See Figure 3a.

THEORY

If we apply a single unbalanced force, F, to an object, the object will undergo a linear acceleration, a, which is determined by the unbalanced force acting on the object and the mass of the object. The mass is a measure of an object's inertia, or its resistance to being accelerated. Newton's Second Law expresses this relationship:

$$F = ma$$

If we consider rotational motion, we find that a single unbalanced torque

$$\tau = (Force)(lever\ arm)^{\#}$$

produces an angular acceleration, α, which depends not only on the mass of the object but on how that mass is distributed. The equation which is analogous to F = ma for an object that is rotationally accelerating is

$$\tau = I\,\alpha. \tag{1}$$

where the Greek letter tau (τ) represents the torque in Newton-meters, α is the angular acceleration in radians/sec^2 and I is the **moment of inertia** in kg*m^2. The moment of inertia is a measure of the way the mass is distributed on the object and determines its resistance to angular acceleration.

Every rigid object has a definite moment of inertia about any particular axis of rotation. Here are a couple of examples of the expression for I for two special objects:

In this lab the lever arm will be the radius at which the force is applied (the radius of the axle). This is due to the fact that the forces will be applied tangentially, i.e., perpendicular to the radius. The general form of this relationship is $\tau = force * lever\ \ arm * \sin\theta$, where θ is the angle between the force and the lever arm. However, in this experiment θ is 90° and sin(90°) = 1.

One point mass m on a weightless rod of radius r ($\boldsymbol{I = mr^2}$):

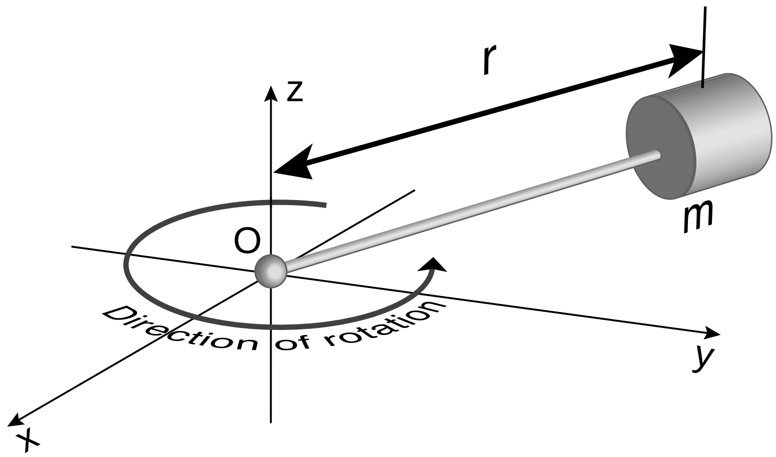

Figure 1

Two point masses on a weightless rod ($I = m_1 r_1^2 + m_2 r_2^2$):

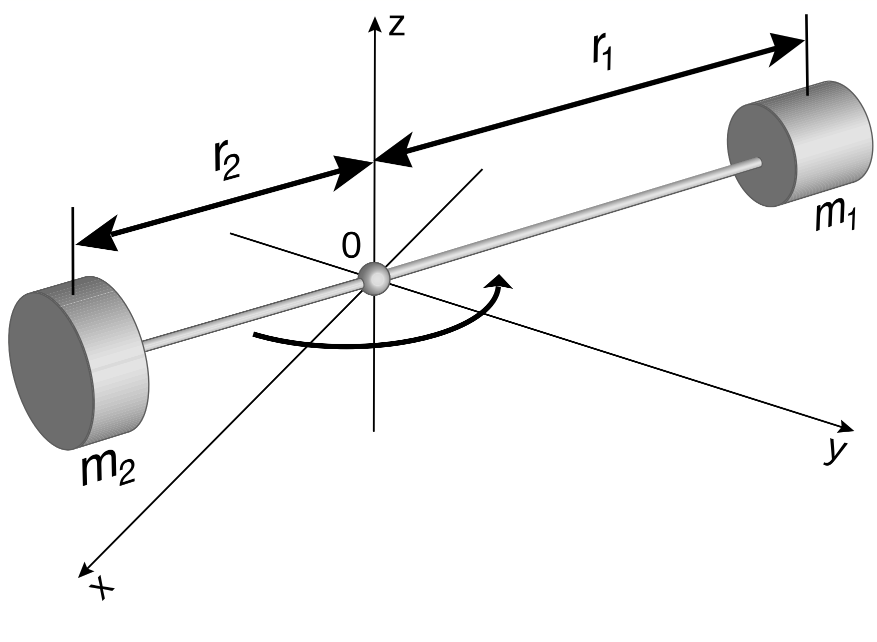

Figure 2

To illustrate we will calculate the moment of inertia for a mass of 2 kg at the end of a massless rod that is 2 m in length (object #1 above):

$$I = mr^2 = (2\text{ kg})\,(2\text{ m})^2 = 8\ \text{kg}\cdot\text{m}^2$$

If a force of 5 N were applied to the mass <u>perpendicular</u> to the rod (to make the lever arm equal to r) the torque is given by:

$$\tau = Fr = (5\text{ N})\,(2\text{ m}) = 10\ \text{N}\cdot\text{m}$$

By equation (1) we can now calculate the angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{10\ \text{N}\cdot\text{m}}{8\ \text{kg}\cdot\text{m}^2} = 1.25\ \frac{\text{rad}}{\text{sec}^2}$$

NOTE: The moment of inertia of a complicated object is found by adding up the moments of each individual piece (Figure#2 above is the sum of two Figure #1 components).

Experimental Apparatus

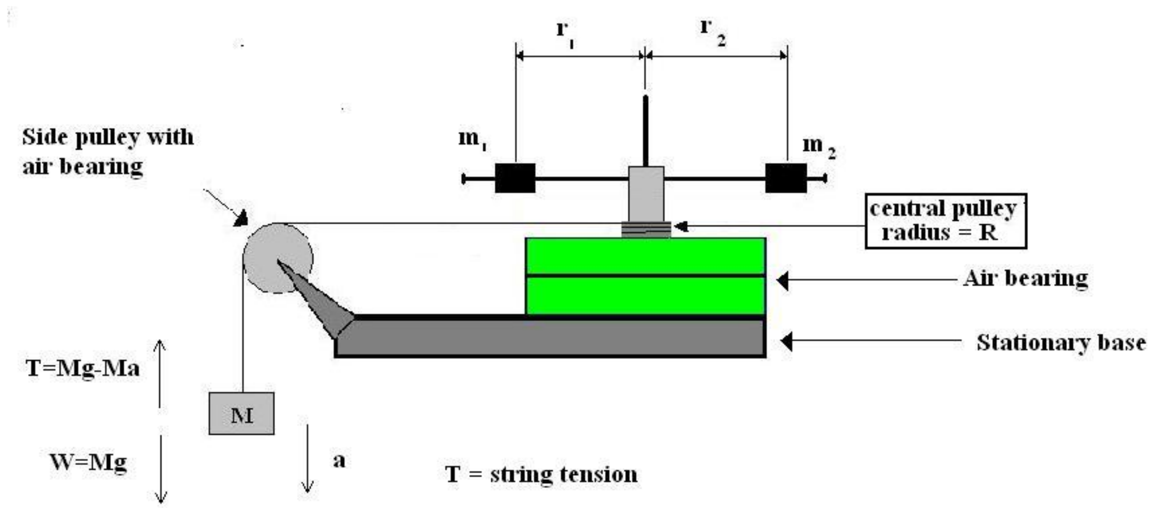

Figure 3a: Moment of Inertia Apparatus

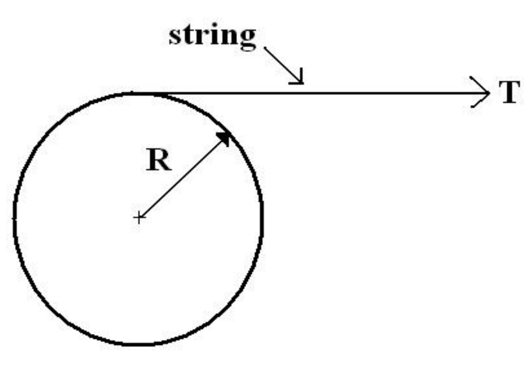

Figure 3b: Central Pulley (axle)

In our case, the rigid body consists of two cylinders, which are placed on a metallic rod at varying radii from the axis of rotation. The cylinders and rod are supported by a rotating platform attached to a central pulley and nearly frictionless air bearings. A side view of the apparatus is shown in Figure 3a and a top view of the central pulley is shown in Figure 3b.

In this experiment, we will keep the distribution of mass on the rotating body constant and therefore, its moment of inertia will remain constant as well. We will place the two cylindrical masses at specified radius such that $r = r_1 = r_2$. We will then use our measurements of fall time and initial height to calculate the linear acceleration of a falling mass. We will take multiple measurements of the fall time and use the average value in our calculations.

To set up your rigid body, wrap the string around the central pulley (axle) and run it over the side pulley to a known weight as shown in Figure 3a.

Consider the following steps:

If we release the weight from rest, the tension in the string will exert a torque on the rigid body causing it to rotate with a constant angular acceleration α. The angular acceleration of the rigid body is related to the linear acceleration of the falling mass by:

$$\alpha = \frac{Linear\_acceleration}{radius\_of\_axle} = \frac{a}{R}$$

or

$$a = R\alpha \tag{2}$$

From Figure 3a and Newton's Second Law, the tension in the string is:

$$T = Mg - Ma \tag{3}$$

The tension in the string causes a net torque on the rigid body. Since torque = (Lever arm) (Force), the net torque on the rigid body is given by:

$$\tau = R \times T \tag{4}$$

The moment of inertia of the rigid body is then found from equation 1 ($\tau = I\alpha$).

PROCEDURE

The apparatus is delicate. Do not rotate the rotating table or side pulley unless air is flowing through the bearings.

1. Place the cylinders, m_1 and m_2 on the horizontal rod such that the axes of the cylinders are along the horizontal rod as shown in Figure 4 below. ***You will be assigned one of these two mass placements to use when you do the practical exam.*** Make sure the thumbscrew on each cylinder is tightened. The center of mass of each cylinder must be the same distance (r) from the axis of rotation (i.e. $r_1 = r_2$ in Figure 3a).

Figure 4: The masses are placed with a small r, all the way in on the left; and masses are placed with a large r, all the way out on the right.

2. With the air supply on, attach the hanging mass (M) to one end of a string and wind the other end around the central pulley. The string should also pass over the side pulley such that the hanging mass is just below the side pulley. Hold the hanging mass stationary and measure its elevation ($y=h$) using the floor as your reference level. Record this elevation in your spreadsheet and assign an appropriate uncertainty to this measurement.
3. Release the hanging mass and simultaneously start the desktop timer. When the mass hits the floor, stop the timer and record the time in your Excel spreadsheet.
4. Rewind the string around the central pulley and repeat step 3.
5. Repeat steps 3 and 4 until you have measure the falling time five times.
6. Have Excel calculate the mean of your five measurements and the standard deviation of this mean value, s_m. You will use the standard deviation of your mean value of your falling time as the uncertainty in the fall time. The Excel equations for the mean value and standard deviation of this mean value are: "AVERAGE(CELL1:CELL)" and "=STDEV(CELL1:CELL5)/SQRT(5)".

7. The position, y of an object released from rest a distance h above the floor is found using: $y = h - \frac{1}{2}at^2$. The final position of the mass is $y = 0$, so the acceleration is found using:

$$a = \frac{2h}{t^2}$$

Use this equation, your measured starting height and mean falling time, have Excel calculate the acceleration of the falling mass In addition, have Excel calculate the uncertainty. The uncertainty of in the acceleration is found using:

$$\delta a = a\left(2\frac{\delta t}{t} + \frac{\delta y}{y}\right).$$

Checklist

Your lab report should include the following four items:

1) Your spreadsheet
2) The formula view of the spreadsheet
3) Answers to questions

Uncertainties

To test the compatibility of two measurements, $d_1 \pm \delta d_1$ and $d_2 \pm \delta d_2$, find the difference $\Delta = |d_1 - d_2|$ and calculate its uncertainty, $\delta\Delta = \delta d_1 + \delta d_2$. If $|\Delta| < \delta\Delta$, the two measurements are compatible.

Your assigned setting is to place the masses as shown below: given at practical lab

QUESTIONS

1) What is the acceleration of the falling mass and its uncertainty?

2) Using equation (3), $T = Mg - Ma$ what is the tension in the string while the mass is falling?

3) Assume the location of the two masses, m_1 and m_2 were moved to a different location (either closer to the rotation axis or further away from the rotation axis). Qualitatively, how would this affect the acceleration of the falling mass and the tension in the string?

Assume the radial position of the masses m_1 and m_2 were varied and the moment of inertia of the entire system was measured for each radial position (just like you did in the Moment of Inertial experiment). This data were then used to construct the I vs. r^2 plot and to calculate the fit parameters shown below (you will be given an actual graph when you do the practical lab).

Your graph to analyze will be here

Recall, if the two masses are now each placed a distance r from the axis of rotation then equation describing the moment of inertia of the entire apparatus, including the masses m_1 and m_2 is:

$$I = (m_1 + m_2)r^2 + I_0$$

It is helpful to compare this equation to the form of an equation for a straight line when answering the last questions:

$$y = mx + b$$

Use the information on this graph to answer the following questions.

4) What is the slope of the best fit line and its uncertainty?

5) What is the mass of m_1 and its uncertainty? You may assume the masses m_1 and m_2 are identical.

6) The mass of m_1 is measured with an electronic scale and found to be (to be given when you do the practical lab) grams. Is the mass you obtained from your graph consistent with this measurement?

Copy of the spreadsheet for this practical lab:

Practical Lab 2

Measure grey fields | Calculate yellow fields

Initial height of M y=		±	(insert units)

Trial	*t*
	sec
1	
2	
3	
4	
5	
MEAN	0
s_m	

Linear acceleration

a =		(insert units)
δa =		(insert units)