Chapter 3

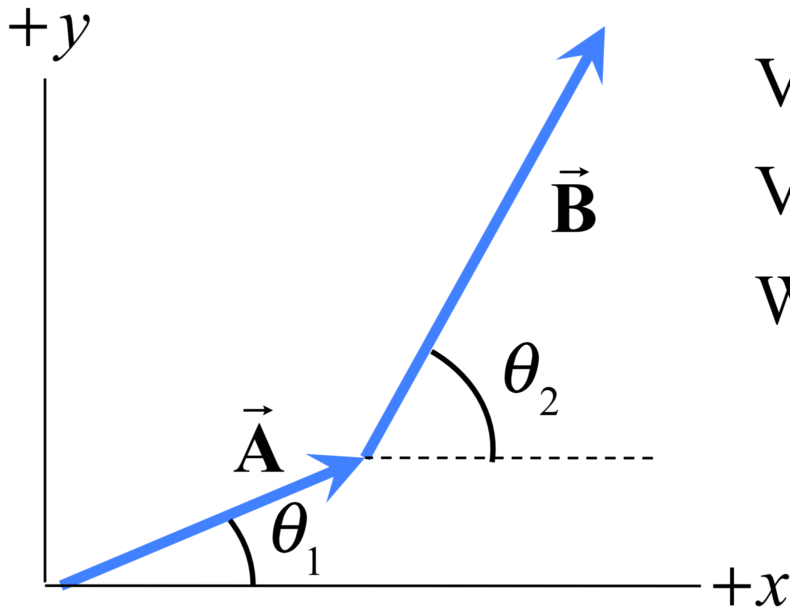
Kinematics in Two Dimensions

continued

Office Hrs for Prof. Bromberg are cancelled 1/23/13
11:00-12:00 See R. Hallstein – BPS 1253 (knock loudly)
12:00-1:00   See J. Huston -- BPS 3230/3239

3.2 Addition of Vectors by Means of Components

Summary of adding two vectors together



Vector $\vec{\mathbf{A}}$ has magnitude A and angle θ_1

Vector $\vec{\mathbf{B}}$ has magnitude B and angle θ_2

What is the vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$?

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \qquad \vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}$$

1) Determine components of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$: A_x, A_y and B_x, B_y

2) Add x-components to find $C_x = A_x + B_x$

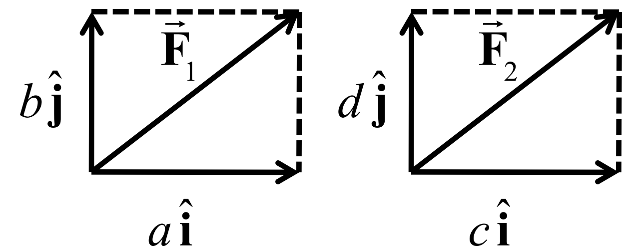
3) Add y-components to find $C_y = A_y + B_y$

4) Determine the magnitude and angle of vector $\vec{\mathbf{C}}$

magnitude $C = \sqrt{C_x^2 + C_y^2}$; $\quad \theta = \tan^{-1}(C_y / C_x)$

Clicker Question 3.3

$\vec{\mathbf{F}}_1 = a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}}$, and $\vec{\mathbf{F}}_2 = c\,\hat{\mathbf{i}} + d\,\hat{\mathbf{j}}$.



What is the vector $\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$?

A) $(a+b+c+d)(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

B) $(a+c)\hat{\mathbf{i}}+(b+d)\hat{\mathbf{j}}$

C) $(a+b)\hat{\mathbf{i}}+(c+d)\hat{\mathbf{j}}$

D) $(a-b)\hat{\mathbf{i}}+(c-d)\hat{\mathbf{j}}$

E) $(a+b)\hat{\mathbf{j}}+(c+d)\hat{\mathbf{i}}$

Clicker Question 3.3

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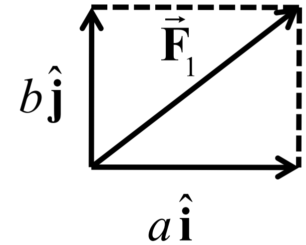

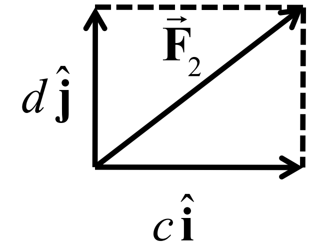

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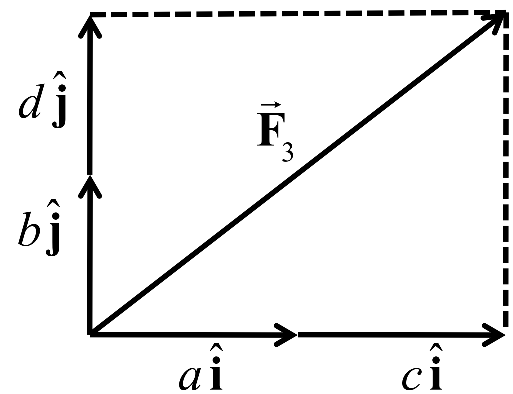

A) $(a+b+c+d)(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

B) $(a+c)\hat{\mathbf{i}}+(b+d)\hat{\mathbf{j}}$

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D) $(a-b)\hat{\mathbf{i}}+(c-d)\hat{\mathbf{j}}$

E) $(a+b)\hat{\mathbf{j}}+(c+d)\hat{\mathbf{i}}$

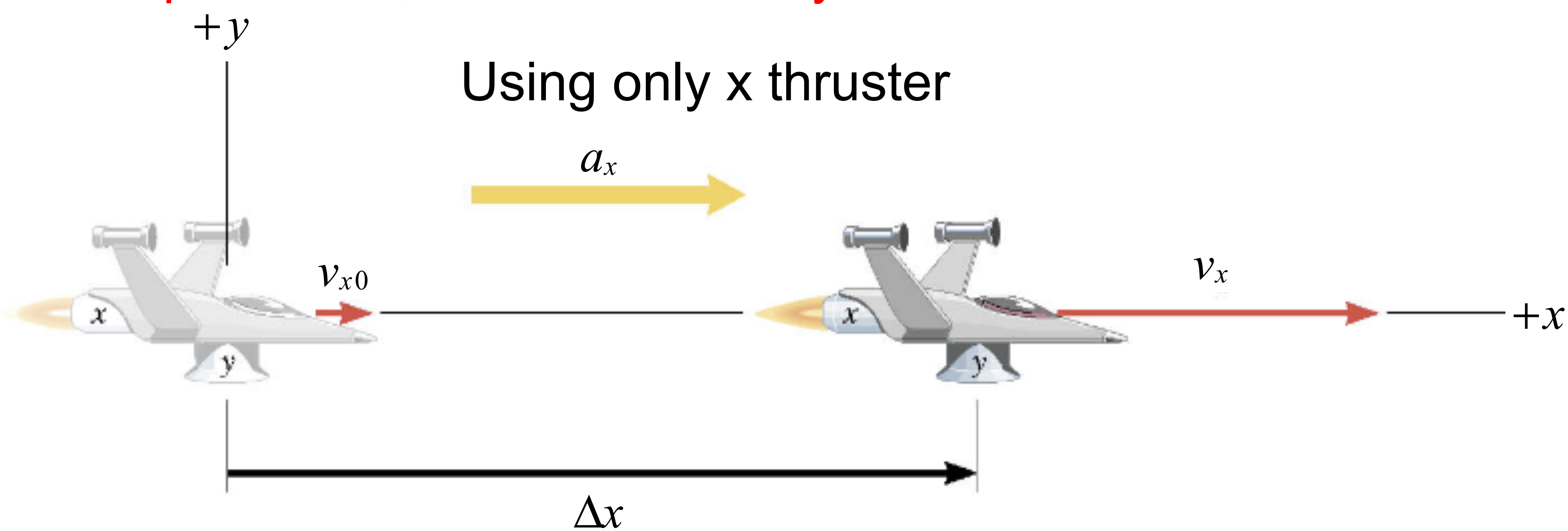
$$\begin{array}{rcccc} \vec{\mathbf{F}}_1 & = a\,\hat{\mathbf{i}} & + & b\,\hat{\mathbf{j}} \\ +\vec{\mathbf{F}}_2 & = c\,\hat{\mathbf{i}} & + & d\,\hat{\mathbf{j}} \\ \hline \end{array}$$

$$\vec{\mathbf{F}}_3 = (a+c)\,\hat{\mathbf{i}} + (b+d)\,\hat{\mathbf{j}}$$

Homework: What is the vector $\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_1 - \vec{\mathbf{F}}_2$?

3.2 Equations of Kinematics in Two Dimensions

Except for time, motion in x and y directions are INDEPENDENT.



Motion in x direction with constant acceleration.

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right) t$$

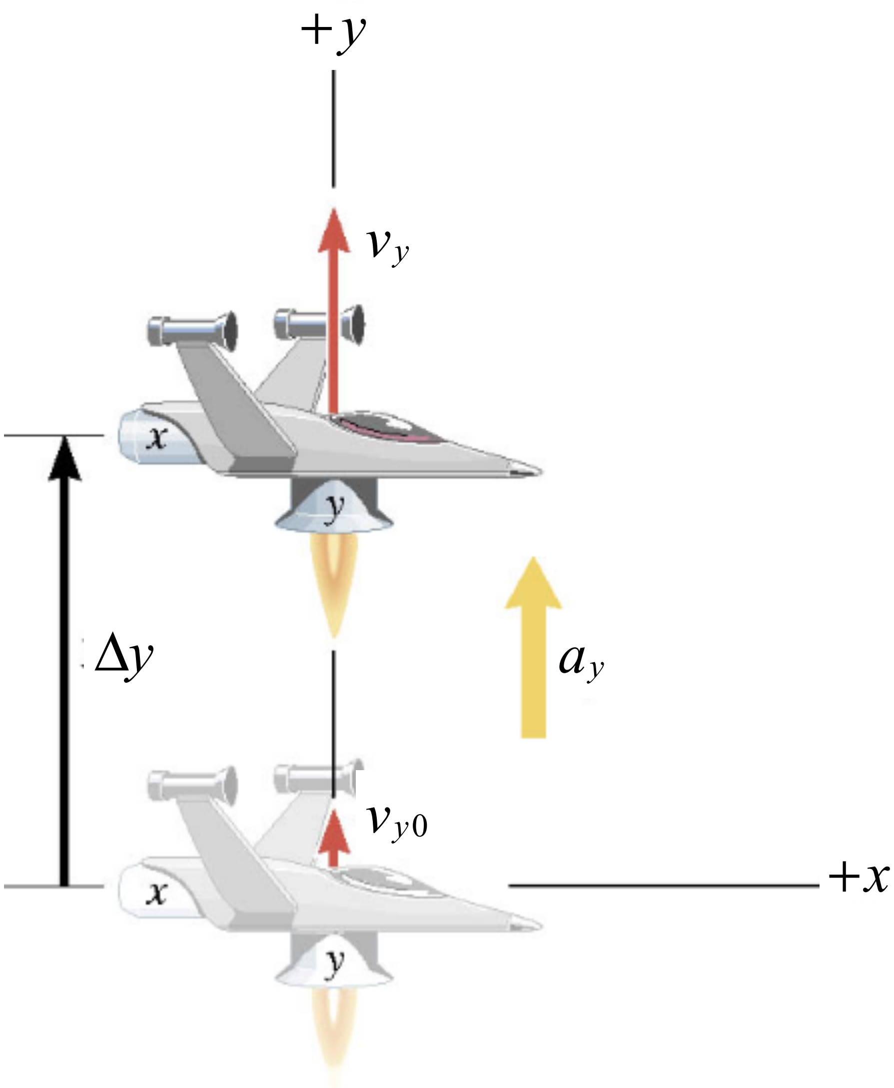
$$\Delta x = v_{x0} t + \tfrac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

3.2 Equations of Kinematics in Two Dimensions

Except for time, motion in x and y directions are INDEPENDENT.

Using only y thruster



Constant acceleration motion in y direction.

$$v_y = v_{y0} + a_y t$$

$$\Delta y = v_{y0} t + \tfrac{1}{2} a_y t^2$$

$$\Delta y = \tfrac{1}{2}\left(v_{y0} + v_y\right) t$$

$$v_y^2 = v_{y0}^2 + 2 a_y \Delta y$$

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

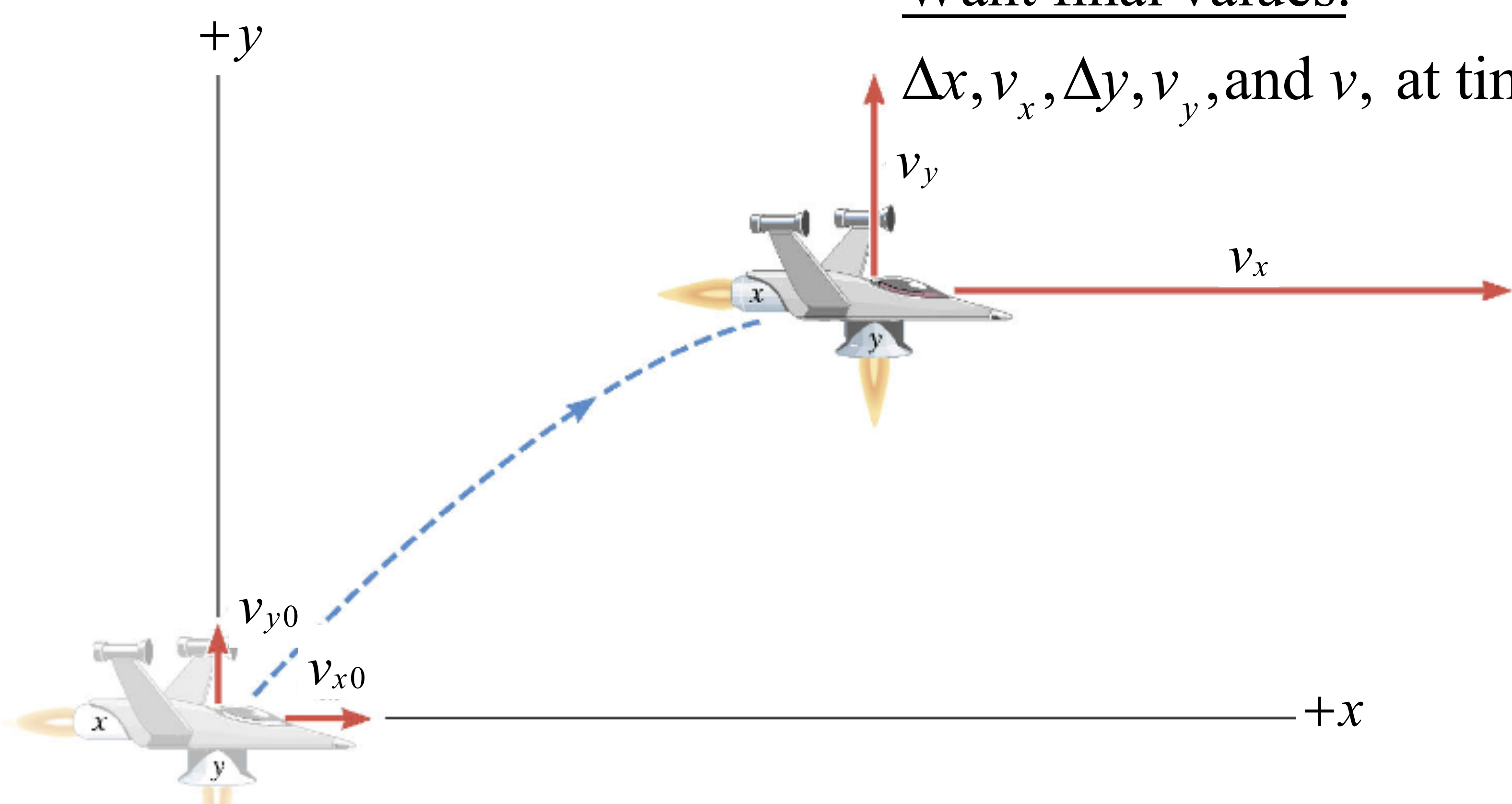
3. Write down the values that are given for any of the five kinematic variables associated with each direction.

Example: A Moving Spacecraft

In the x direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the y direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) Δx and v_x, (b) Δy and v_y, and (c) the final velocity of the spacecraft at a time 7.0 s later.

Want final values:

$$\Delta x, v_x, \Delta y, v_y, \text{and } v, \text{ at time } t = 7\text{s}.$$



3.2 Equations of Kinematics in Two Dimensions

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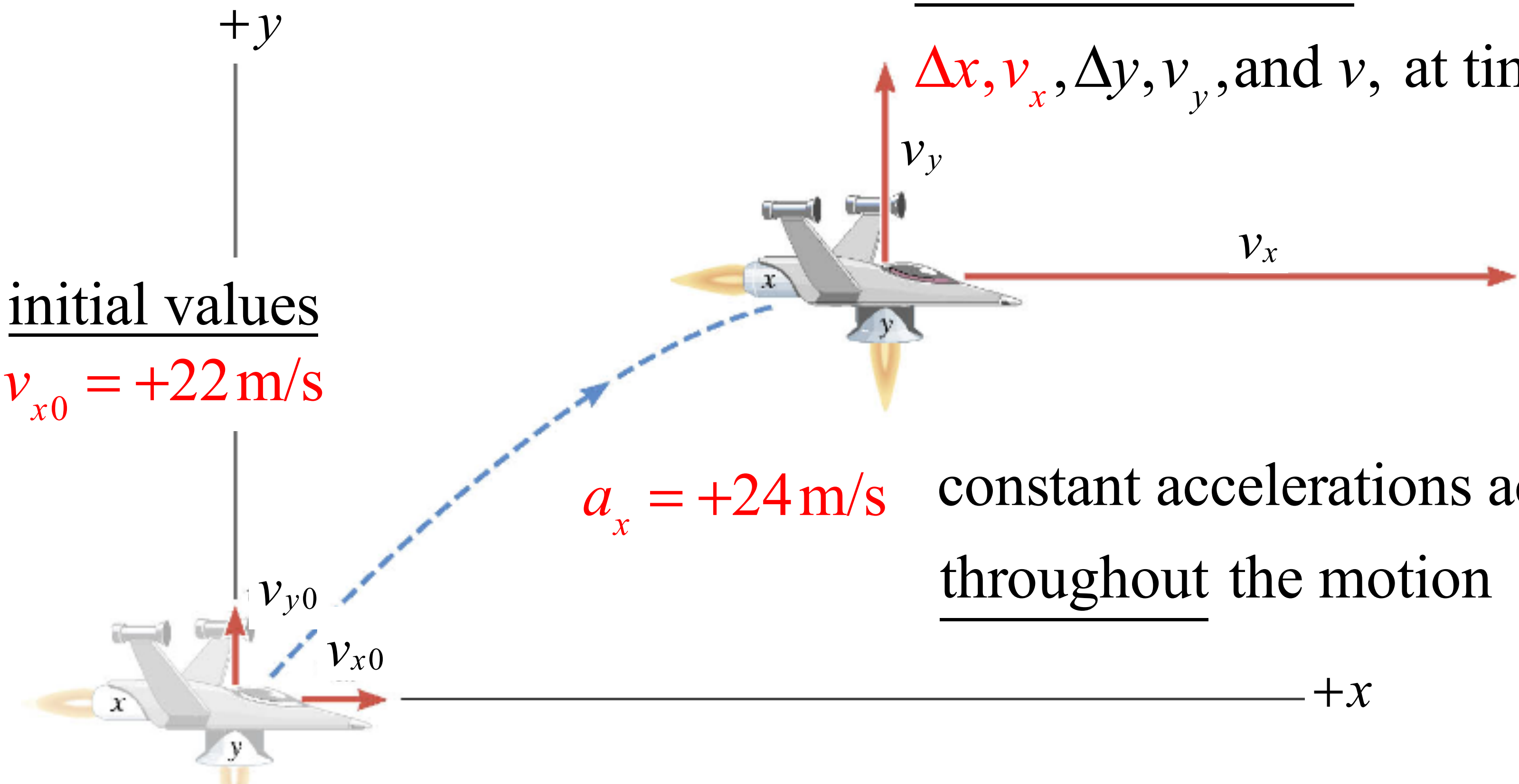

initial values

$v_{x0} = +22\,\text{m/s}$

$a_x = +24\,\text{m/s}$

constant accelerations act throughout the motion

3.2 *Equations of Kinematics in Two Dimensions*

Example: A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s². In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s². Find (a) Δx and v_x, (b) Δy and v_y, and (c) the final velocity of the spacecraft at a time 7.0 s later.

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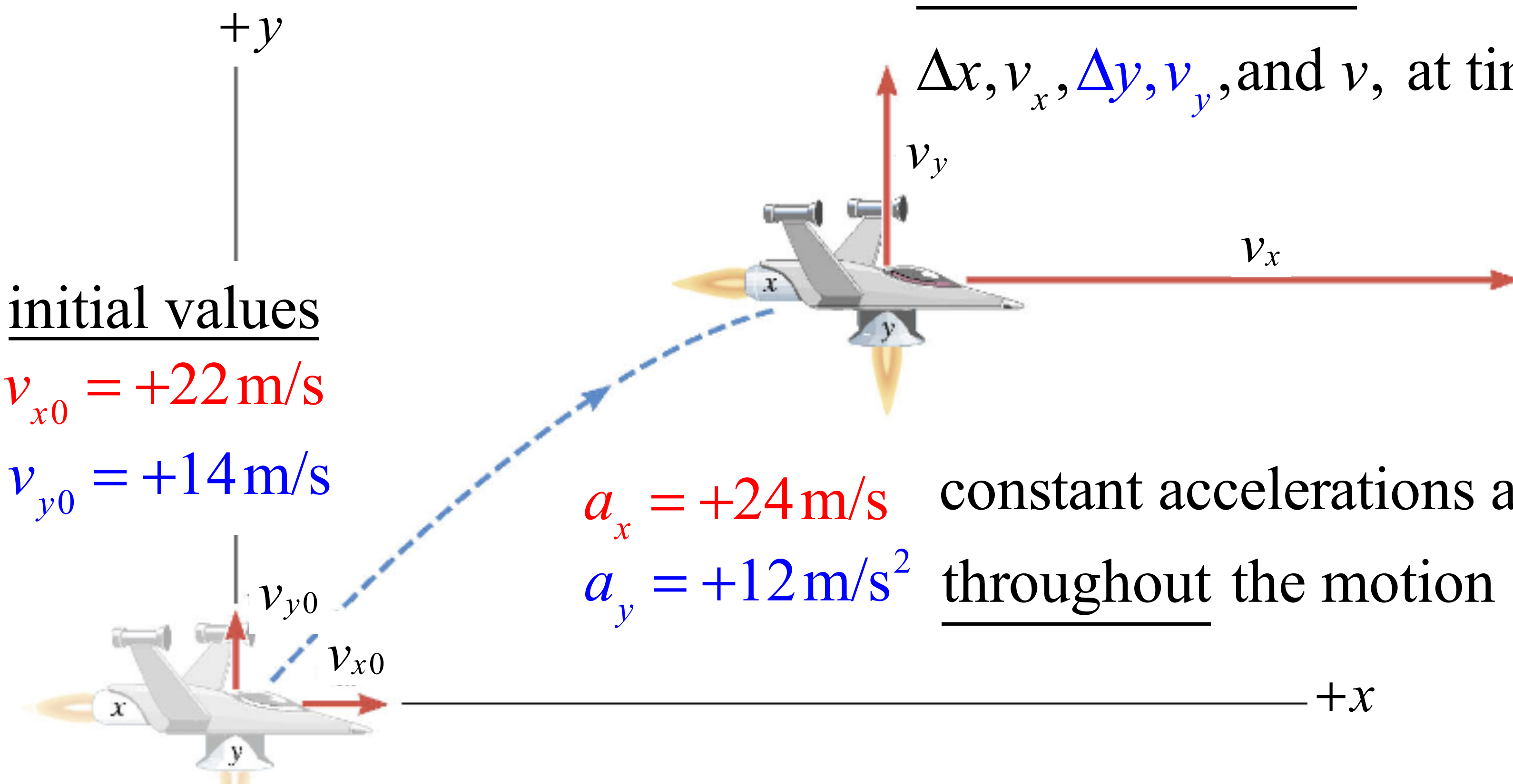

initial values

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$a_x = +24\,\text{m/s}$

$a_y = +12\,\text{m/s}^2$

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3.2 Equations of Kinematics in Two Dimensions

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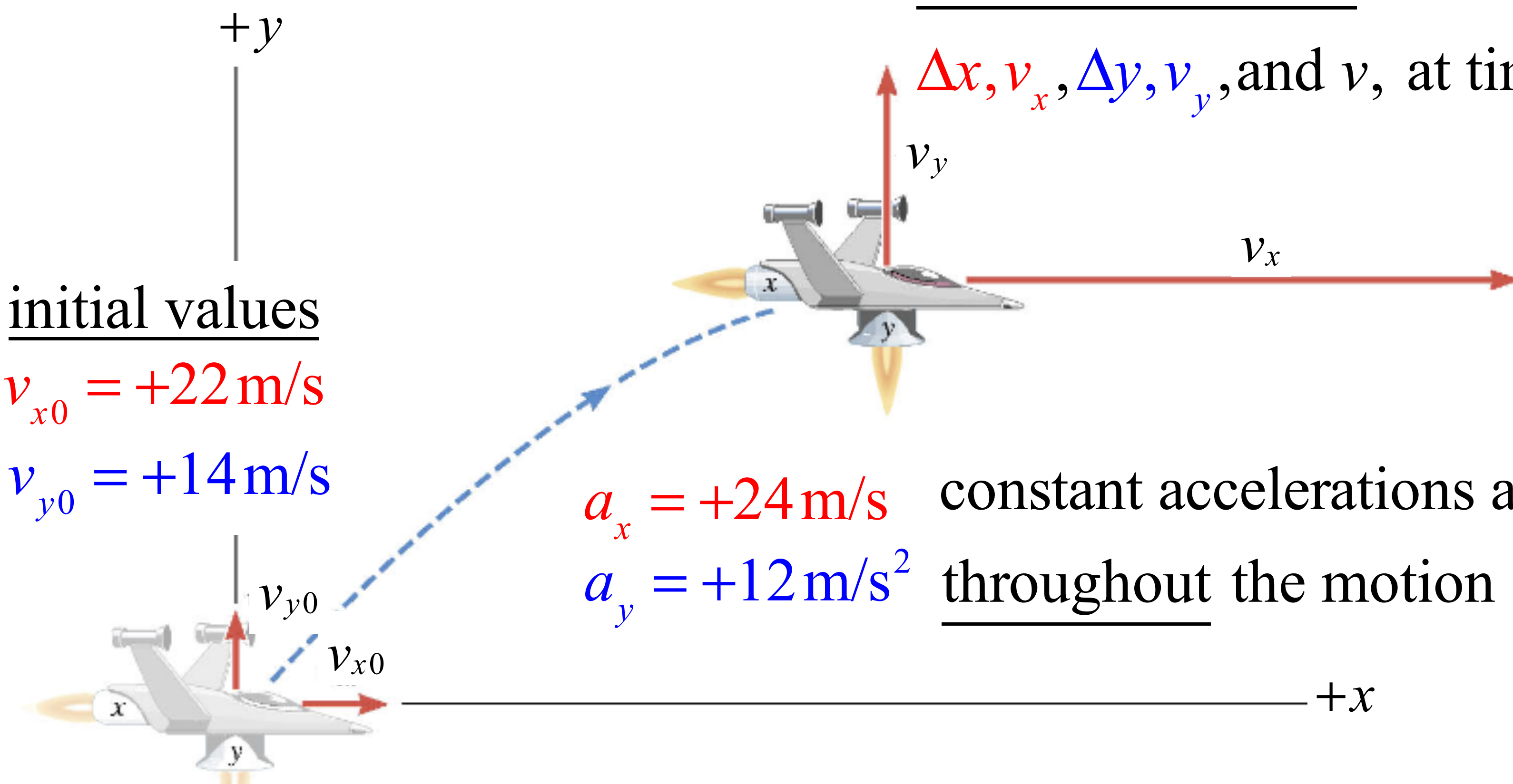
$\Delta x, v_x, \Delta y, v_y$, and v, at time $t = 7$s.



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$a_y = +12\,\text{m/s}^2$

constant accelerations act throughout the motion

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables associated with each direction.

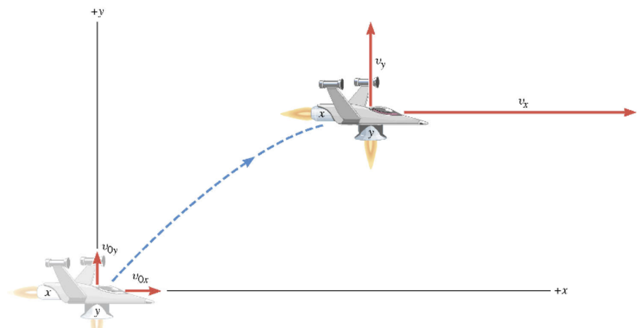
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y*. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

3.2 *Equations of Kinematics in Two Dimensions*

Example: A Moving Spacecraft:



x direction motion

Δx	a_x	v_x	v_{x0}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{x0}t + \tfrac{1}{2}a_x t^2$$

x displacement

$$= \left(22\,\mathrm{m/s}\right)\left(7.0\ \mathrm{s}\right) + \tfrac{1}{2}\left(24\,\mathrm{m/s^2}\right)\left(7.0\ \mathrm{s}\right)^2 = +740\ \mathrm{m}$$

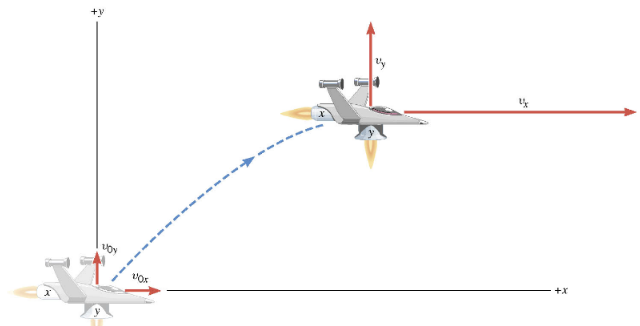
$$v_x = v_{x0} + a_x t$$

x component of velocity

$$= \left(22\,\mathrm{m/s}\right) + \left(24\,\mathrm{m/s^2}\right)\left(7.0\ \mathrm{s}\right) = +190\,\mathrm{m/s}$$

3.2 Equations of Kinematics in Two Dimensions

Example: A Moving Spacecraft:



y direction motion

Δy	a_y	v_y	v_{y0}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

y displacement

$$= (14\,\text{m/s})(7.0\text{ s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\text{ s})^2 = +390\text{ m}$$

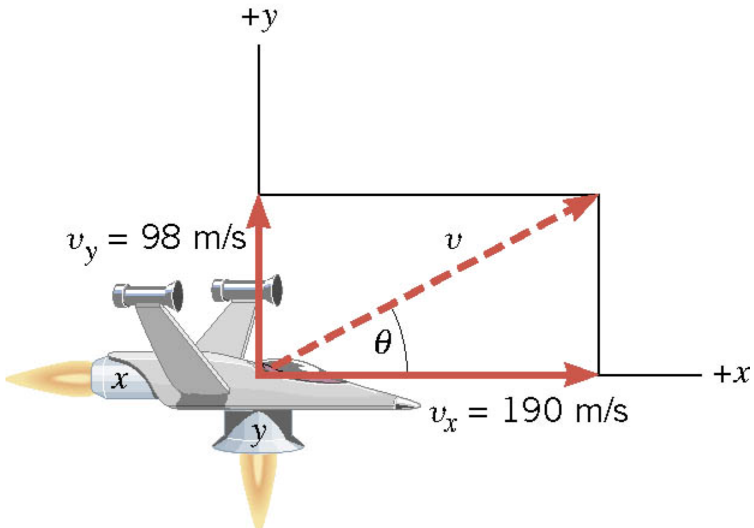
$$v_y = v_{y0} + a_y t$$

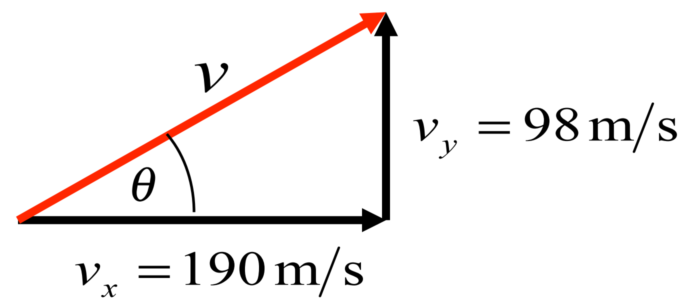
y component of velocity

$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\text{ s}) = +98\,\text{m/s}$$

3.2 Equations of Kinematics in Two Dimensions

Can also find final speed and direction (angle) at $t = 7\text{s}$.





$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(190\,\text{m/s})^2 + (98\,\text{m/s})^2} = 210\,\text{m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

3.3 Projectile Motion

Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at 9.81m/s^2.

Great simplification for projectiles !

$\underline{y \text{ direction}}$

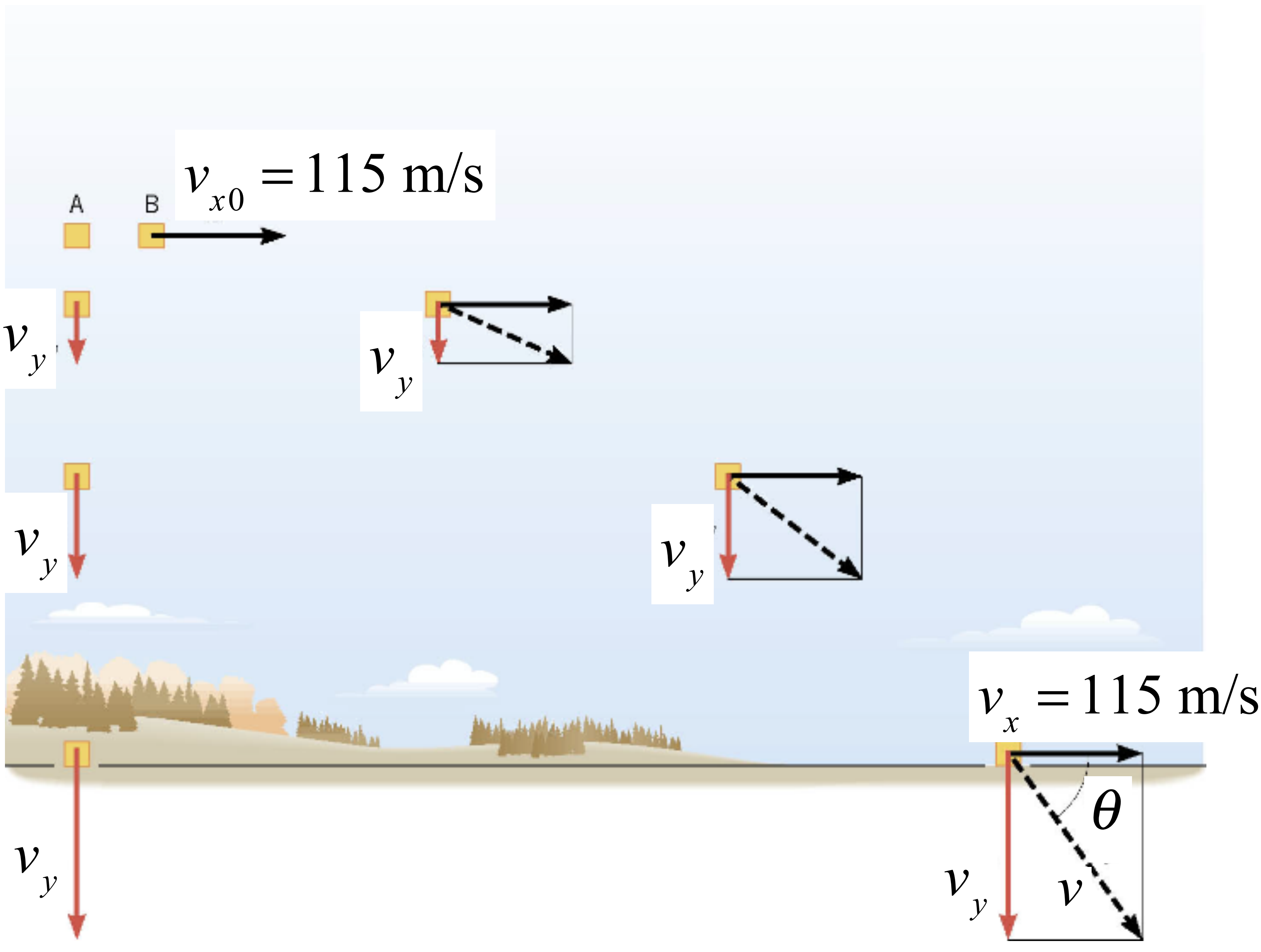
up is positive so

$$a_y = -9.81\,\mathrm{m/s^2}$$

$\underline{x \text{ direction}}$

$$a_x = 0$$

$$v_x = v_{x0} = \text{ constant}$$



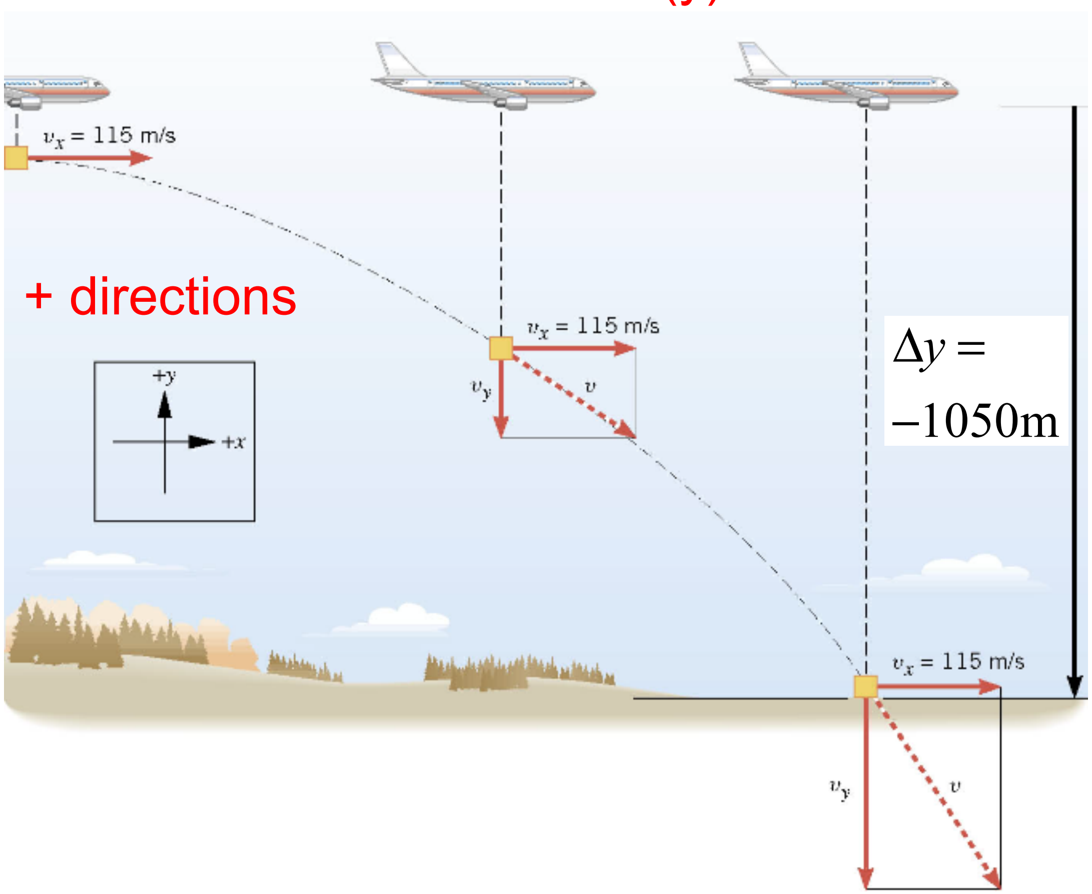
A
B
$v_{x0} = 115$ m/s
v_y
v_y
v_y
v_y
v_y
$v_x = 115$ m/s
θ
v_y
v

3.3 Projectile Motion

Example: A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.

Time to hit the ground depends ONLY on vertical (y) motion



$$v_{y0} = 0$$

$$a_y = -9.81\ \text{m/s}^2$$

$$\Delta y = -1050\ \text{m}$$

Displacement in y is in the negative direction

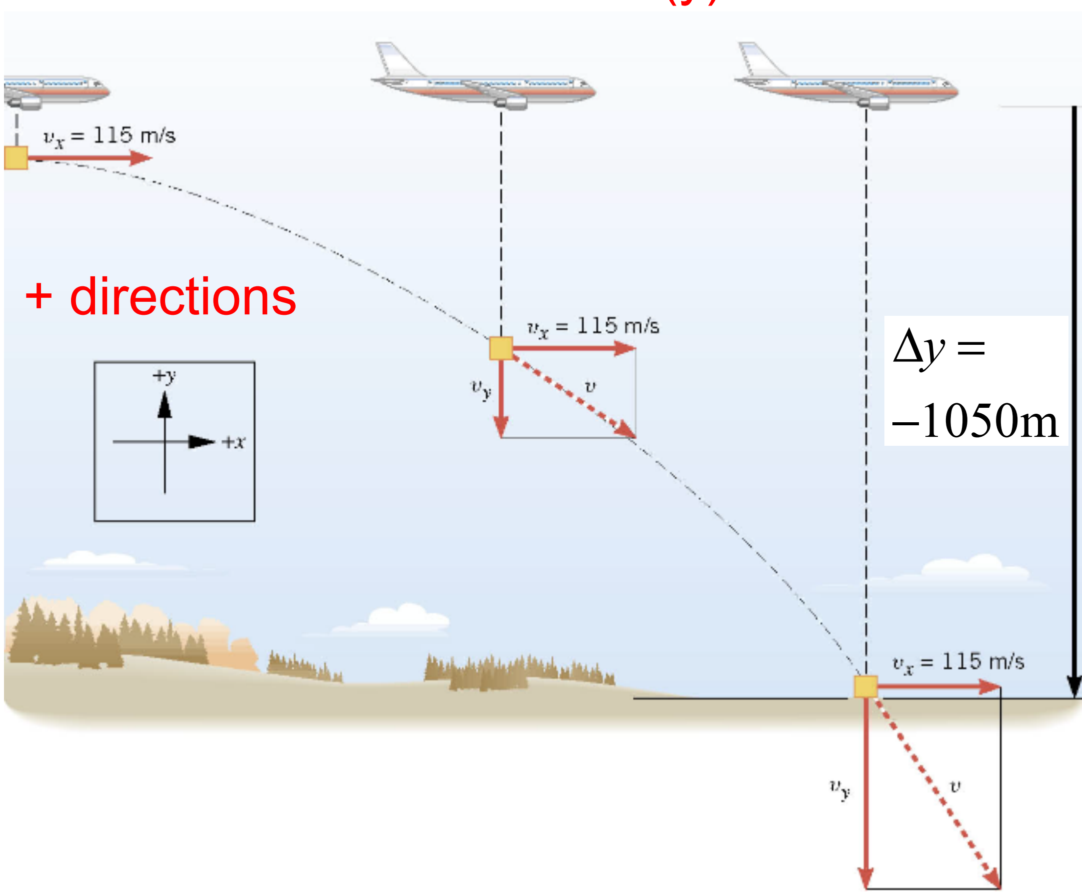
Why not positive?

Example: A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.

Time to hit the ground depends ONLY on vertical (y) motion

Put y = 0 at the TOP



$$y_0 = 0 \text{ m}$$

$$\Delta y = y - y_0$$

$$= (-1050 - 0)$$

$$= -1050$$

$$y = -1050 \text{ m}$$

Final y is 1050m BELOW y = 0

3.3 Projectile Motion

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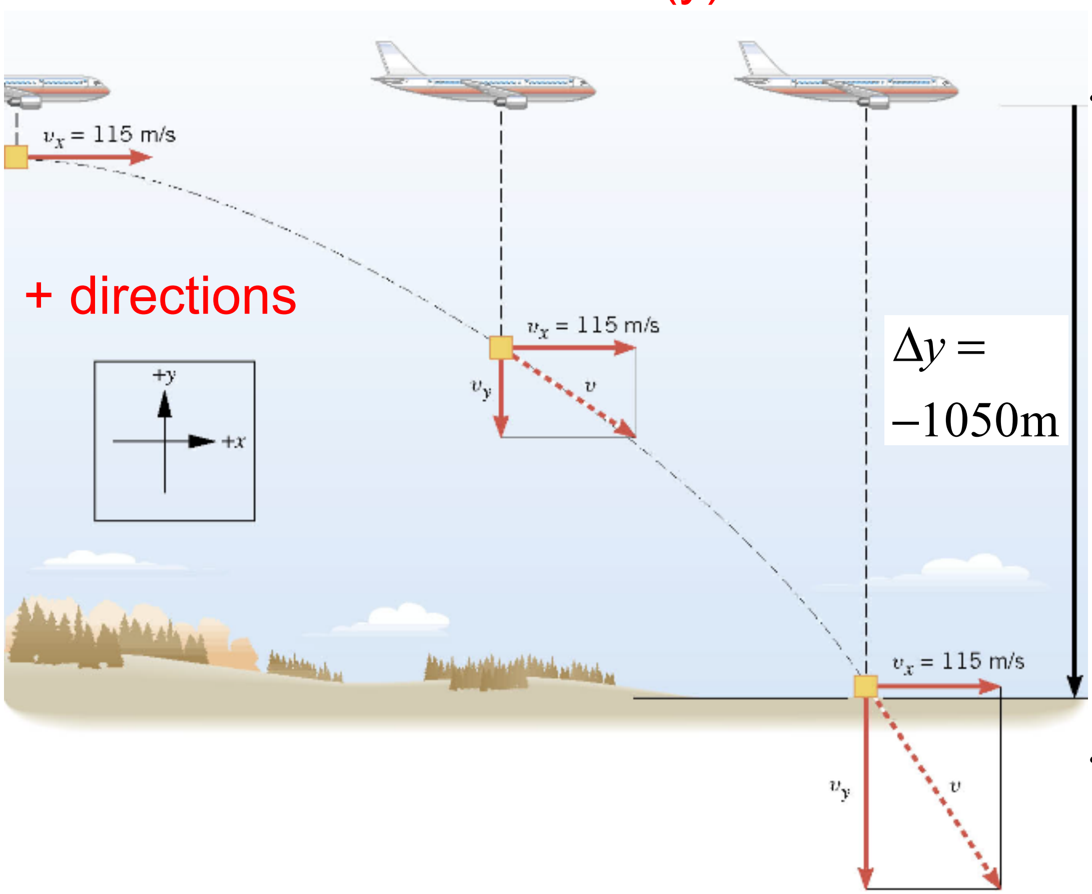

$$y_0 = +1050 \text{ m}$$

$$\Delta y = y - y_0$$
$$= (0 - 1050)$$
$$= -1050$$

$$y = 0 \text{ m}$$

Put y = 0 at the BOTTOM

3.3 Projectile Motion

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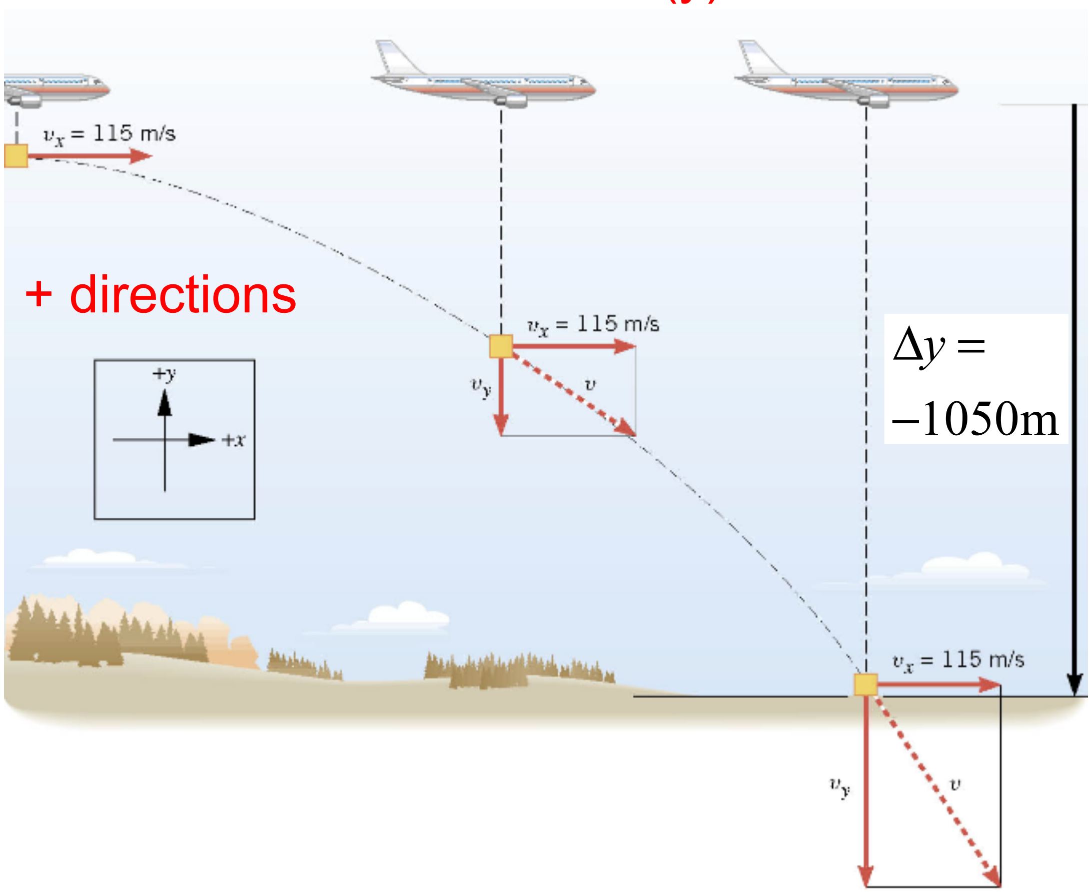


$$v_{y0} = 0$$

$$a_y = -9.81 \text{ m/s}^2$$

$$\Delta y = -1050 \text{ m}$$

3.3 Projectile Motion

Δy	a_y	v_y	v_{y0}	t
–1050 m	–9.81 m/s^2		0 m/s	?

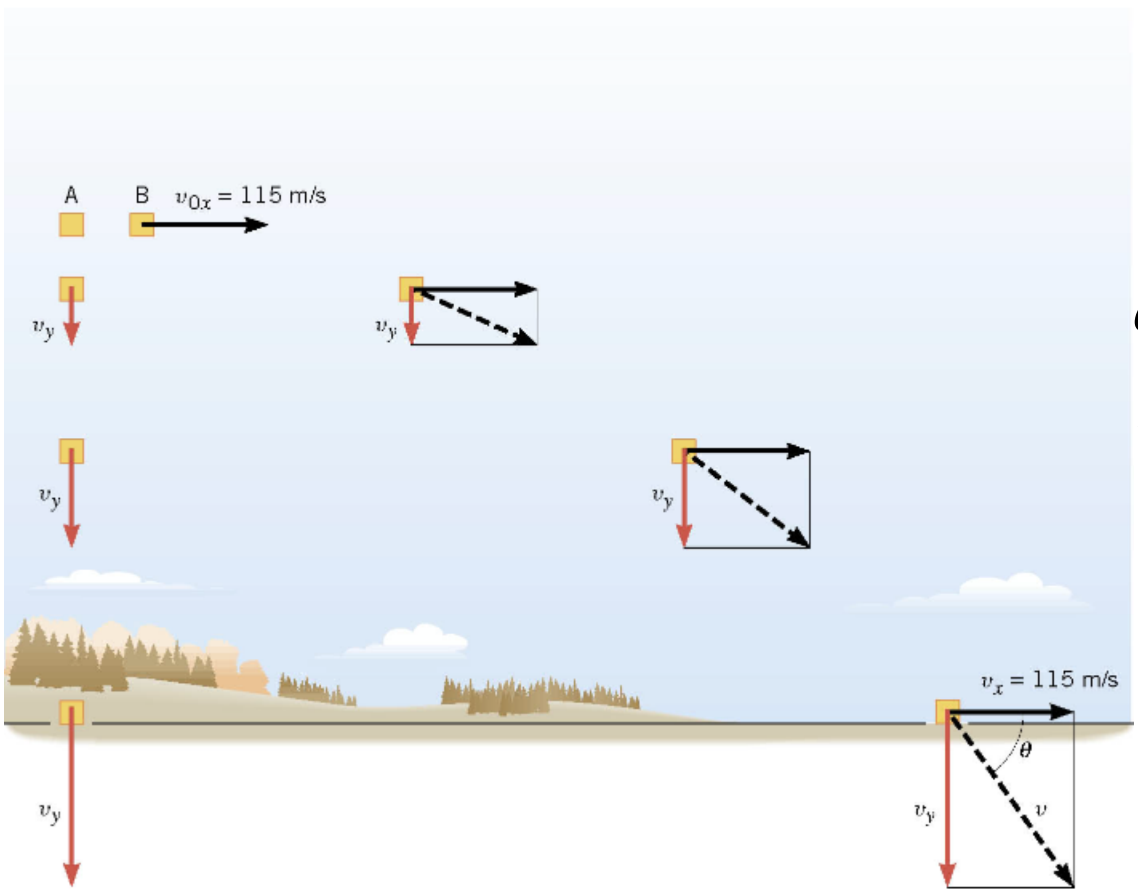
$$\boxed{\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2} \Longrightarrow \Delta y = \tfrac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2\left(-1050\text{ m}\right)}{-9.81\text{m}/\text{s}^2}}$$

$$= 14.6\text{ s}$$

Example: The Velocity of the Care Package

What are the magnitude and direction of the final velocity of the care package?



$$t = 14.6 \text{ s}$$

BECAUSE x-component of acceleration is zero

$$a_x = 0; \; v_{x0} = +115 \text{ m/s}$$

$$v_x = v_{x0} + a_x t$$

$$= +115 \text{ m/s}$$

x-component of velcoity does not change

3.3 Projectile Motion

Δy	a_y	v_y	v_{y0}	t
–1050 m	–9.81 m/s^2	?	0 m/s	14.6 s

$$v_y = v_{y0} + a_y t = 0 + \left(-9.81\,\mathrm{m/s^2}\right)\left(14.6\ \mathrm{s}\right)$$

$$= -143\,\mathrm{m/s}$$ y-component of final velocity.

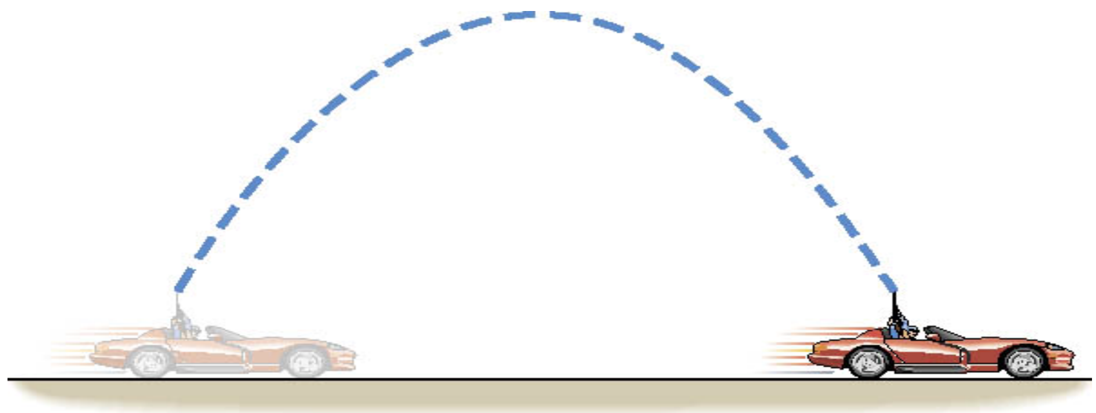
Now ready to get final speed and direction

$$v_x = v_{x0} = +115\,\mathrm{m/s} \qquad v = \sqrt{v_x^2 + v_y^2} = 184\ \mathrm{m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-143}{+115}\right) = -51^\circ$$

Conceptual Example: I Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land – behind you, ahead of you, or in the barrel of the rifle?



Ballistic Cart Demonstration

Clicker Question 3.4

A cannon is on a flat-car train moving at constant velocity. Which direction should the cannon be AIMED so that the cannon-ball lands right on the cannon? Ignore air friction.

A) You have to “lead” the train
B) If the train is moving fast, it can’t be done
C) Far ahead if the the train is moving really fast
D) At exactly 45 degrees for all train speeds
E) Straight upward

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3.3 *Projectile Motion*

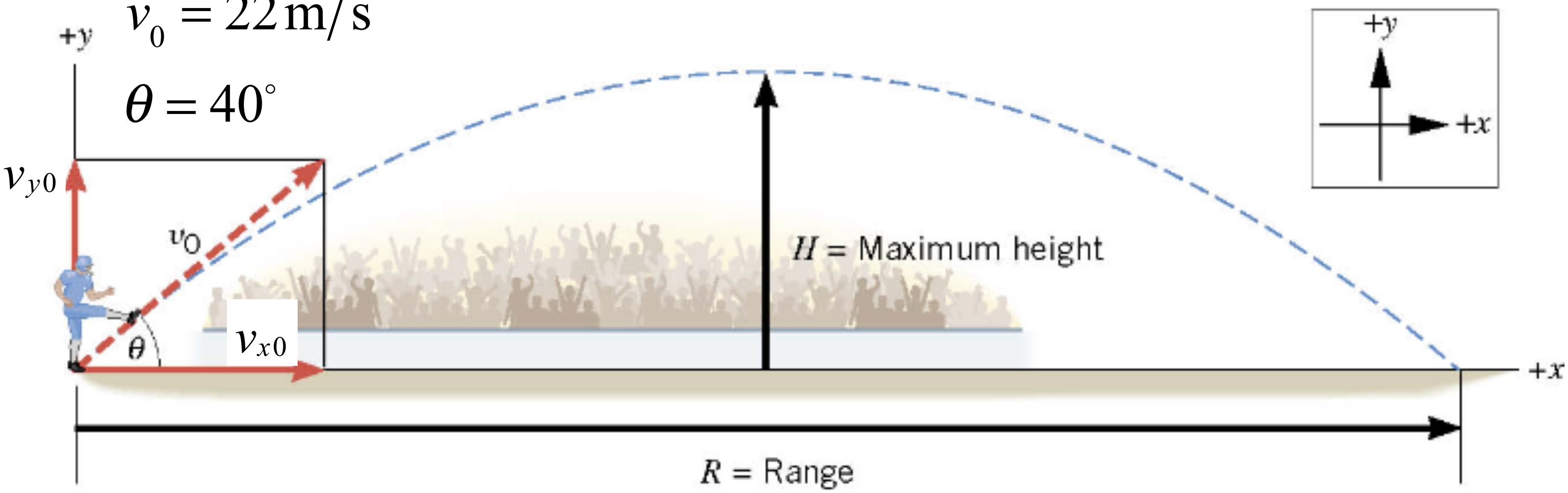
Example: The Height of a Kickoff

A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.

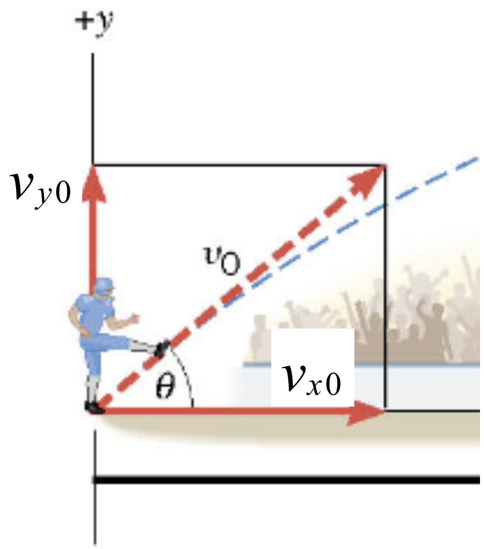
maximum height and "hang time" depend only on the y-component of initial velocity

$v_0 = 22\,\mathrm{m/s}$

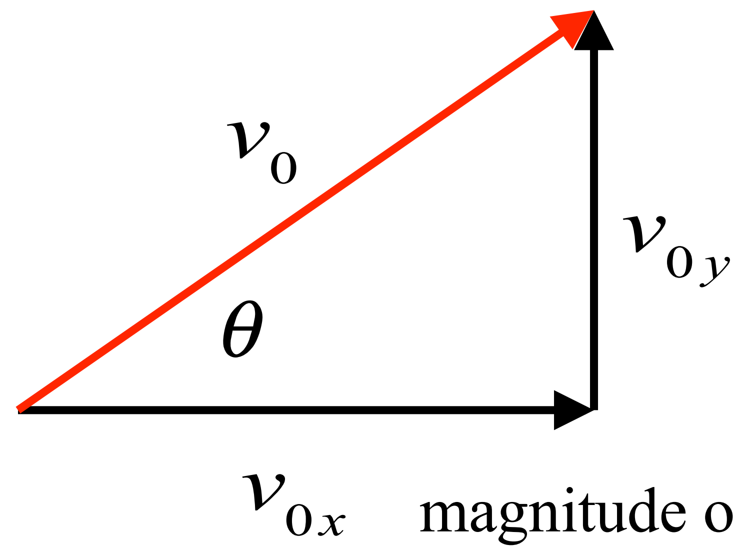
$\theta = 40^{\circ}$



3.3 Projectile Motion



$$v_0 = 22\,\mathrm{m/s}$$
$$\theta = 40^\circ$$

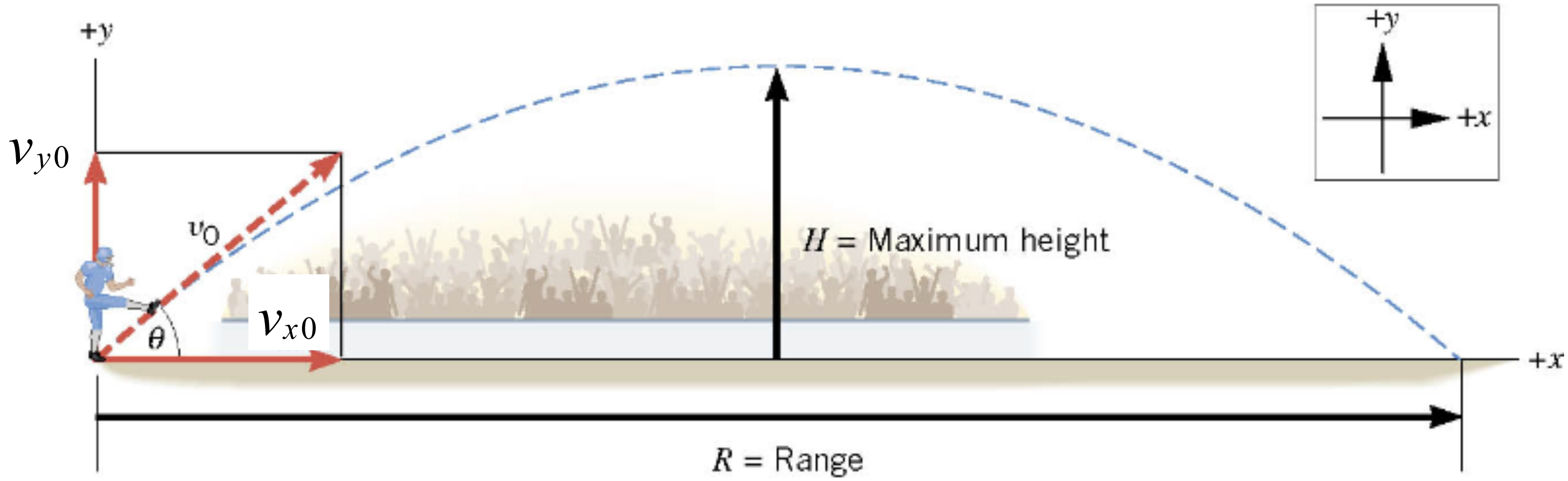


magnitude of the component vectors

$$v_{y0} = v_0 \sin\theta = \left(22\,\mathrm{m/s}\right)\sin 40^\circ = 14\,\mathrm{m/s}$$

$$v_{x0} = v_0 \cos\theta = \left(22\,\mathrm{m/s}\right)\cos 40^\circ = 17\,\mathrm{m/s}$$

3.3 Projectile Motion



Δy	a_y	v_y	v_{y0}	t
?	$-9.80\ \mathrm{m/s^2}$	0	14 m/s	

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y \quad \Rightarrow \quad \Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

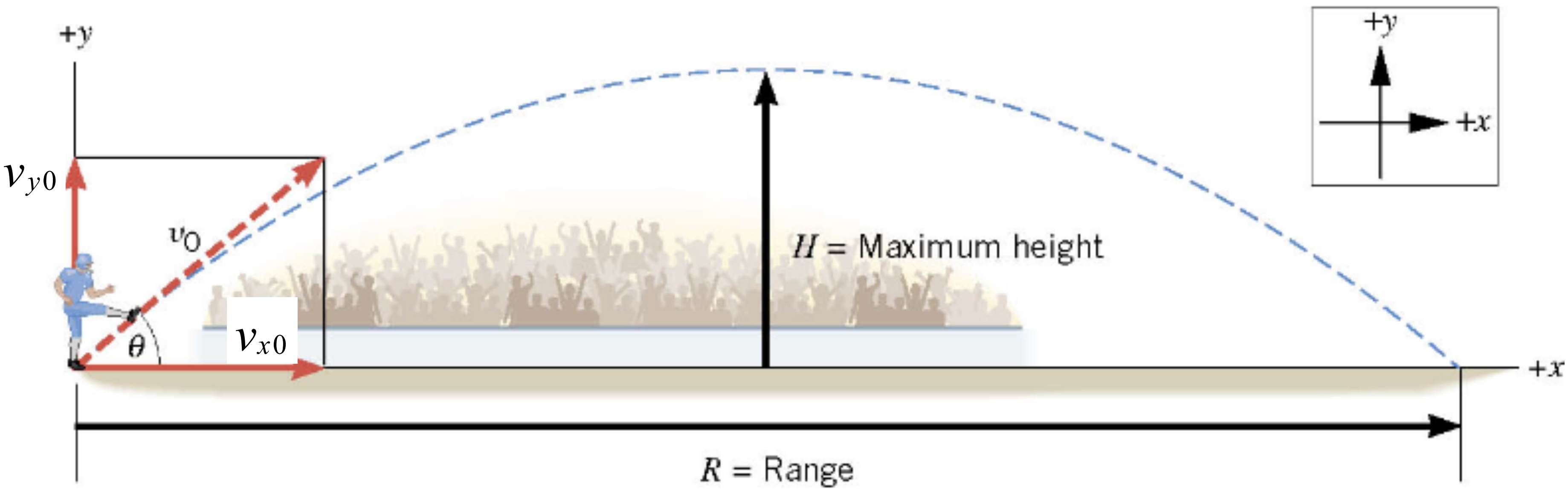
maximum height

$$H = \Delta y = \frac{0 - \left(14\,\mathrm{m/s}\right)^2}{2\left(-9.8\,\mathrm{m/s^2}\right)} = +10\ \mathrm{m}$$

3.3 Projectile Motion

Example: The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



Δy	a_y	v_y	v_{y0}	t
0	–9.80 m/s^2		14 m/s	?

3.3 Projectile Motion

Δy	a_y	v_y	v_{y0}	t
0	–9.81 m/s²		14 m/s	?

$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

$$0 = \left(14\,\mathrm{m/s}\right)t + \tfrac{1}{2}\left(-9.81\,\mathrm{m/s^2}\right)t^2$$

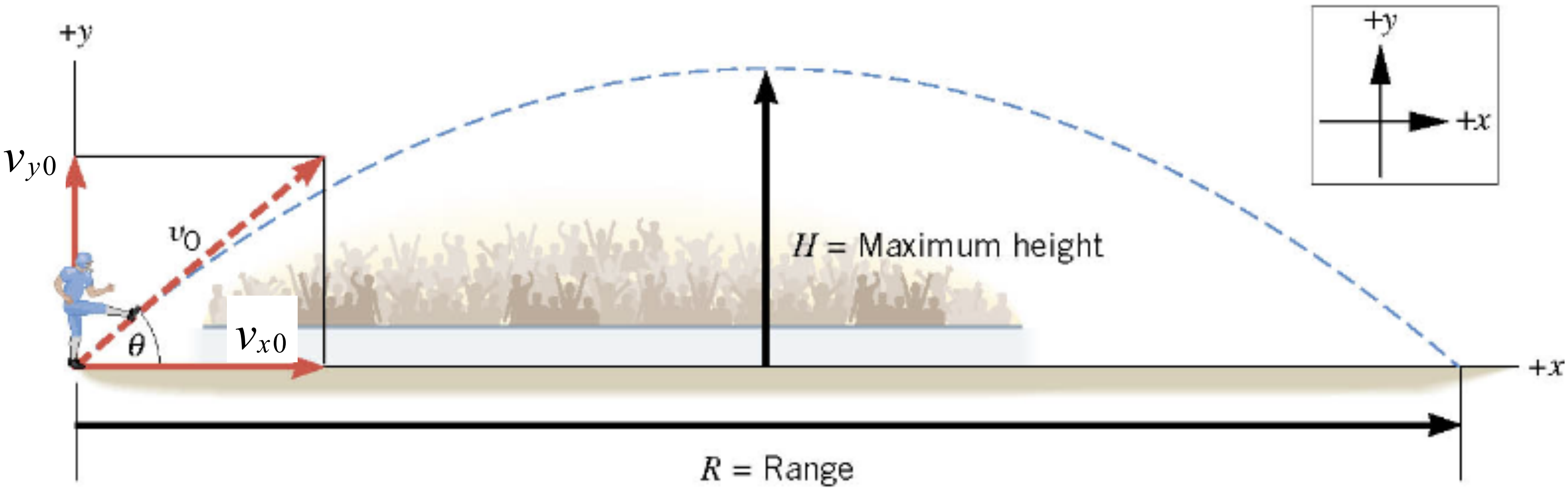
$$0 = 2\left(14\,\mathrm{m/s}\right) + \left(-9.81\,\mathrm{m/s^2}\right)t$$

$$t = 2.9\,\mathrm{s}$$

Example: The Range of a Kickoff

Calculate the range R of the projectile.

Range depends on the hang time
and x-component of initial velocity



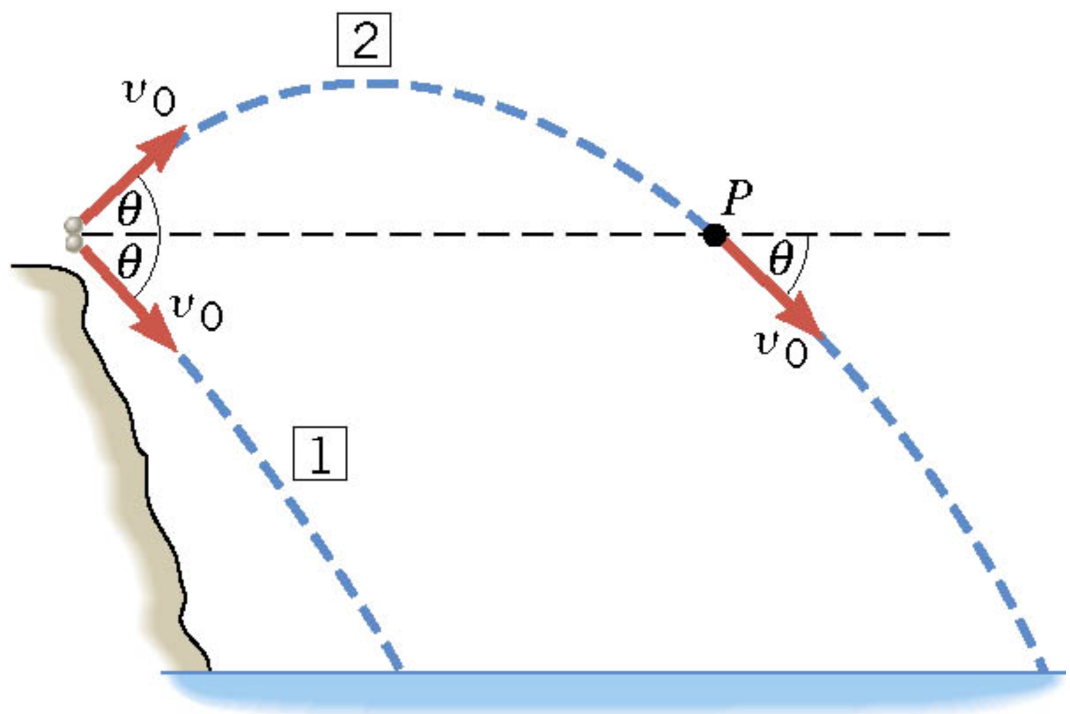
x-direction $$\Delta x = v_{x0}t + \tfrac{1}{2}a_x t^2 = v_{x0}t \qquad a_x = 0\,!$$

$$= (17\,\mathrm{m/s})(2.9\ \mathrm{s}) = +49\ \mathrm{m}$$

3.3 Projectile Motion

Conceptual Example: Two Ways to Throw a Stone

From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle above the horizontal and stone 2 is thrown at the same angle below the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?



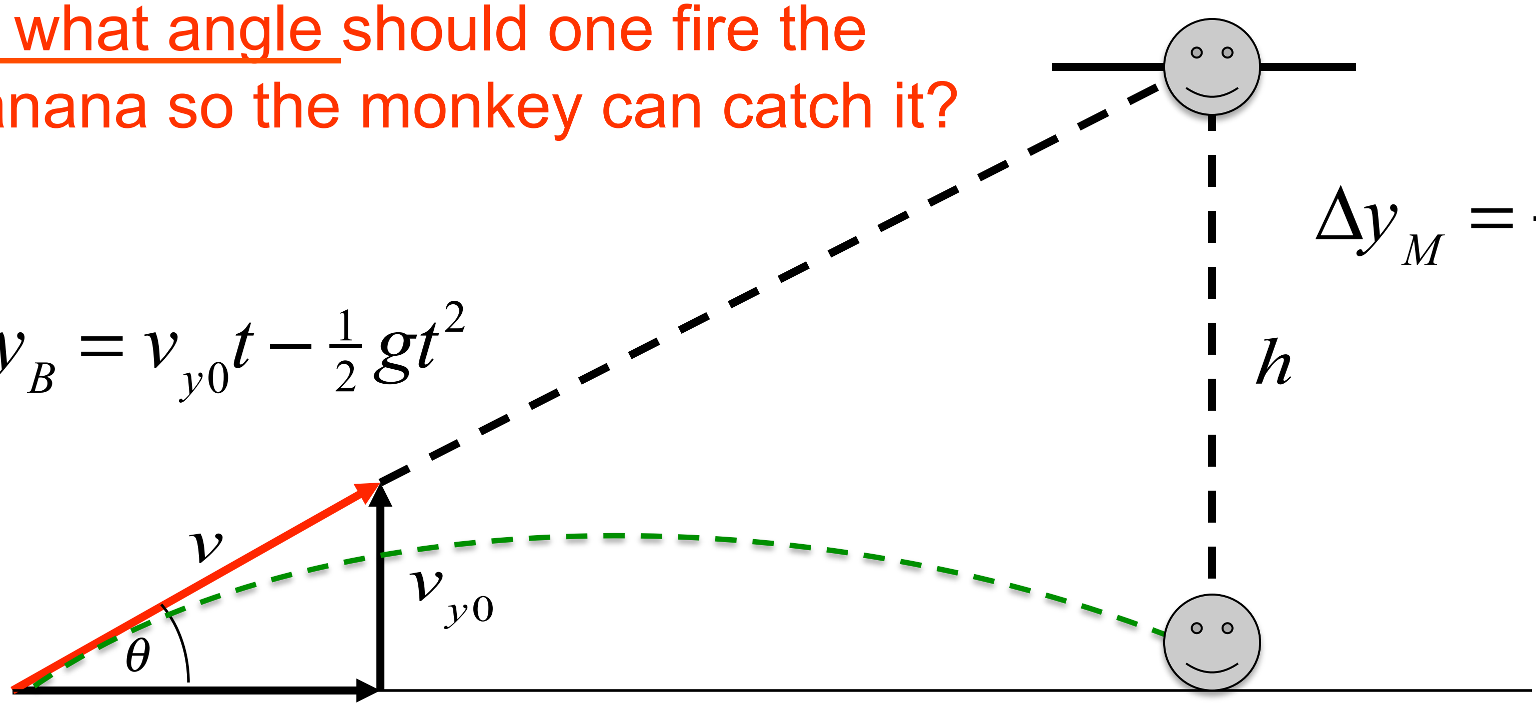
Shoot the Monkey Demonstration

Banana fired and monkey drops at the same time

<u>At what angle</u> should one fire the banana so the monkey can catch it?

$$\Delta y_M = -\tfrac{1}{2} g t^2$$

$$\Delta y_B = v_{y0} t - \tfrac{1}{2} g t^2$$



Shoot at the Monkey !

Hit means: $\Delta y_B - \Delta y_M = h \Rightarrow \quad v_{0y} t = h$

Hit time: $t = \dfrac{\Delta x}{v_{x0}}$ $\qquad \dfrac{v_{y0}}{v_{x0}} \Delta x = h$

$$\boxed{\frac{v_{y0}}{v_{x0}} = \frac{h}{\Delta x} = \tan\theta}$$