Chapter 5

Work and Energy

5.1 Work Done by a Constant Force

The concept of forces acting on a mass (one object) is intimately related to the concept of ENERGY production or storage.

- A mass accelerated to a non-zero speed carries energy (mechanical)
- A mass raised up carries energy (gravitational)
- The mass of an atom in a molecule carries energy (chemical)
- The mass of a molecule in a hot gas carries energy (thermal)
- The mass of the nucleus of an atom carries energy (nuclear)

(The energy carried by radiation will be discussed in PHY232)

The concept of energy relates to forces acting on moving masses.

WORK

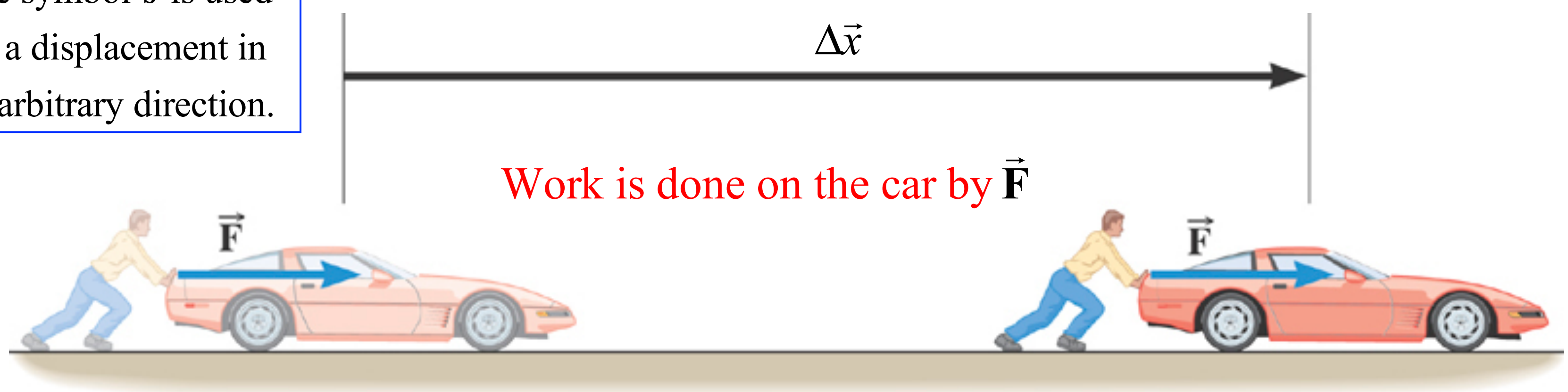
Sorry, but there is no other way to understand the concept of energy.

5.1 Work Done by a Constant Force

Work is *done on* a moving object (a mass) *by* the force component acting on the object, that is parallel to the displacement of the object.

Only acceptable definition.

The symbol $\vec{\mathbf{s}}$ is used for a displacement in an arbitrary direction.



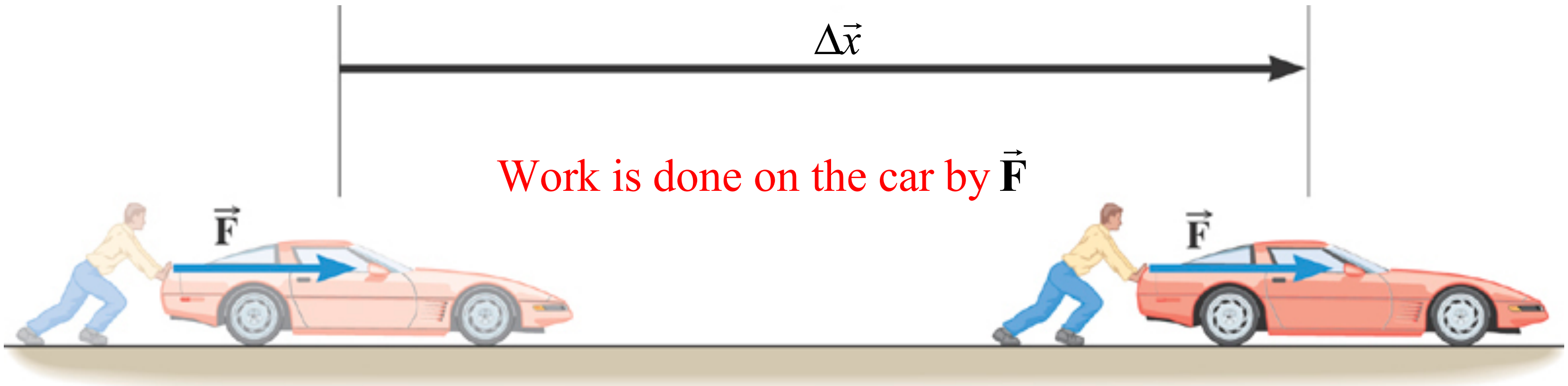
The case shown is the simplest: the directions of $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{x}}$ are the same.

F and Δx are the magnitudes of these vectors.

5.1 Work Done by a Constant Force

Only acceptable definition.

Work is *done on* a moving object (a mass) *by* the force component acting on the object, that is parallel to the displacement of the object.



Sorry about using the symbol W again. Hard to avoid it.

$$\boxed{W = F\Delta x}$$

$$\boxed{1\ \mathrm{N}\cdot\mathrm{m} = 1\ \text{joule}\ (\mathrm{J})}$$

Work is a scalar

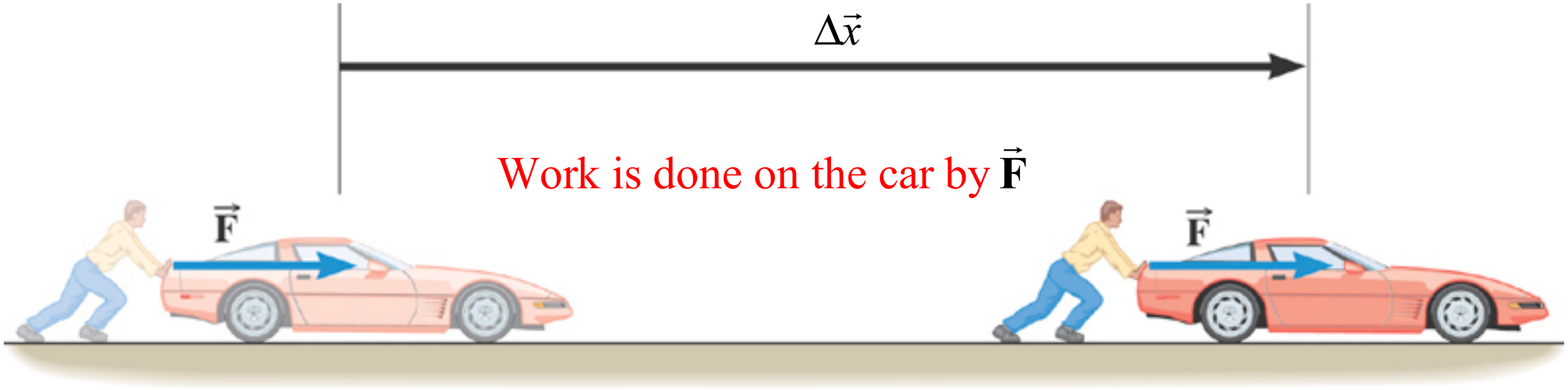
(Positive or Negative)

The nature (or source) of the force is a DIFFERENT issue, covered later.

Other forces may be doing work on the object at the same time.

The net amount of work done on the object is the result of the net force on it.

5.1 Work Done by a Constant Force



With only one force acting on the car (m_{Car}), the car must accelerate, and over the displacement s, the speed of the car will increase.

Newton's 2nd law: acceleration of the car, $a = F/m_{Car}$

Starting with velocity v_0, find the final speed.

$$v^2 = v_0^2 + 2a\Delta x$$

$$\underline{v = \sqrt{v_0^2 + 2a\Delta x}}$$

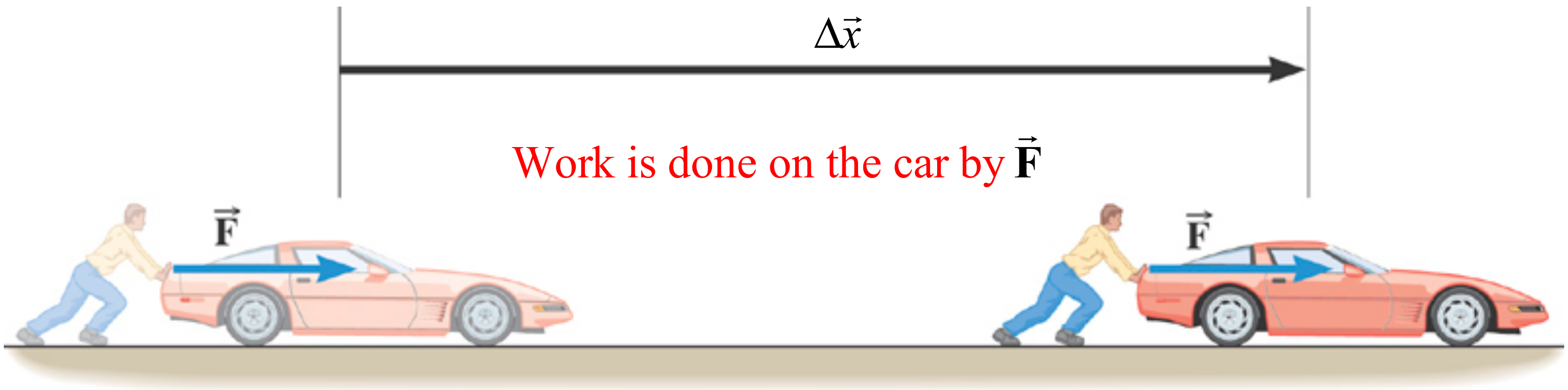
The work done on the car by the force:

$$\boxed{W = F\Delta x} \qquad \boxed{1\ \mathrm{N}\cdot\mathrm{m} = 1\ \text{joule}\ (\mathrm{J})}$$

has increased the speed of the car.

5.1 Work Done by a Constant Force

Other forces may be doing work on the object at the same time.



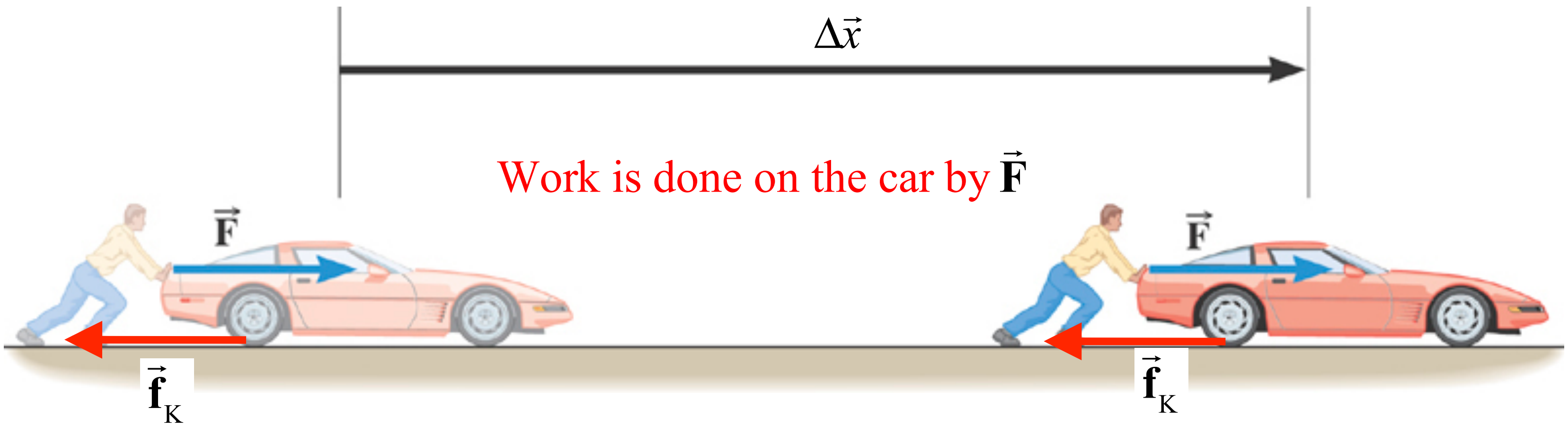
Example:

This time the car is not accelerating, but maintaining a constant speed, v_0.

Constant speed and direction: net force $\sum \mathbf{F} = 0$.

There must be at least one other force acting on the car !

$\vec{\mathbf{f}}_K$ and $\Delta\vec{\mathbf{x}}$ point in opposite directions, work is negative!

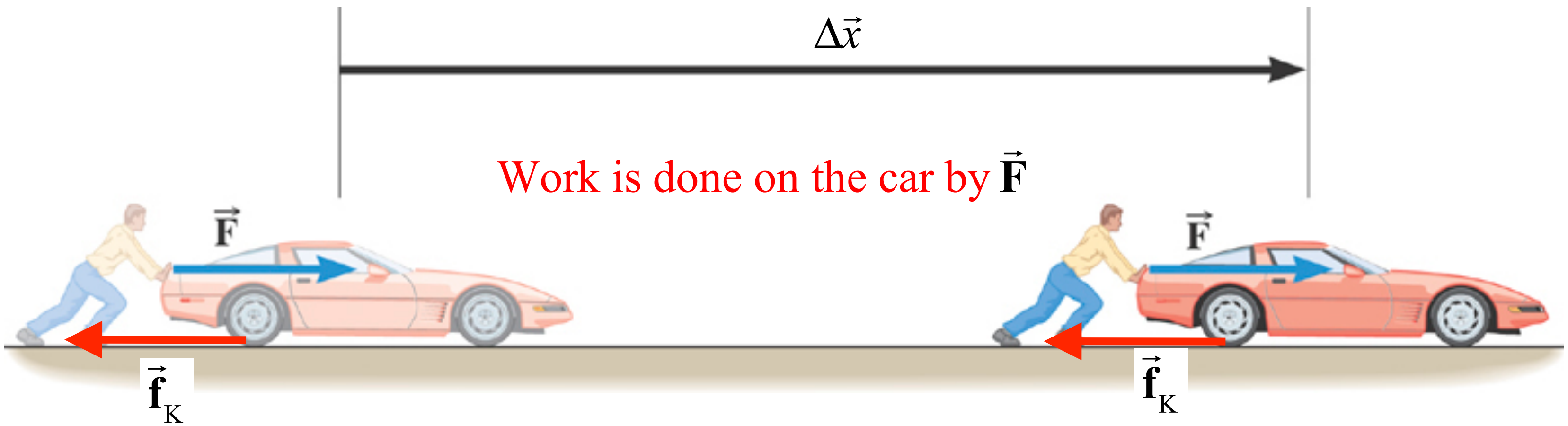


Also acting on the car is a kinetic friction force, $\vec{\mathbf{f}}_K = -\vec{\mathbf{F}}$.

Net force on car must be ZERO, because the car does not accelerate !

5.1 Work Done by a Constant Force

$\vec{\mathbf{f}}_K$ and $\Delta\vec{x}$ point in opposite directions, work is negative!



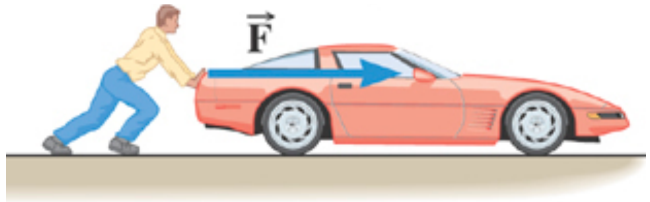
Also acting on the car is a kinetic friction force, $\vec{\mathbf{f}}_K = -\vec{\mathbf{F}}$.

Net force on car must be ZERO, because the car does not accelerate !

$$W = F\Delta x$$
$$W_f = -f_K \Delta x = -F\Delta x$$

The work done on the car by $\vec{\mathbf{F}}$ was countered by the work done by the kinetic friction force, $\vec{\mathbf{f}}_K$.

5.1 *Work Done by a Constant Force*



Car's emergency brake was not released. What happens?

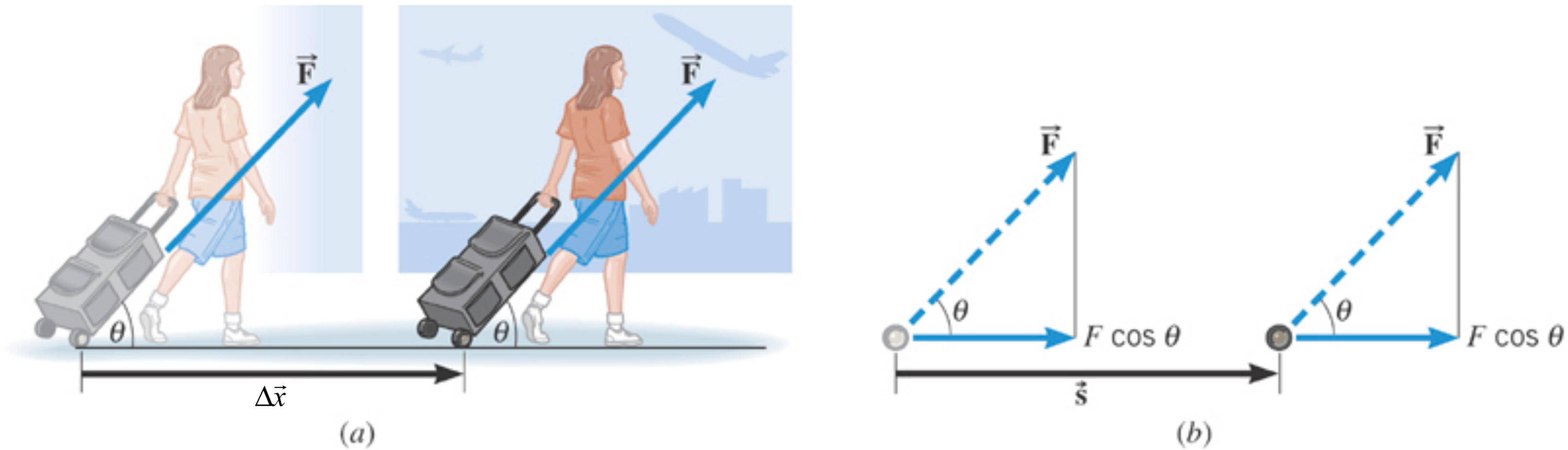
The car does not move. No work done on the car.

Work by force $\vec{\mathbf{F}}$ is zero. What about the poor person?

The person's muscles are pumping away but the attempt to do work on the car, has failed. What happens to the person we will discuss later.

What must concern us here is: if the car does not move the work done on the car by the force $\vec{\mathbf{F}}$ is ZERO.

5.1 Work Done by a Constant Force



If the force and the displacement are not in the same direction, work is done by only the component of the force in the direction of the displacement.

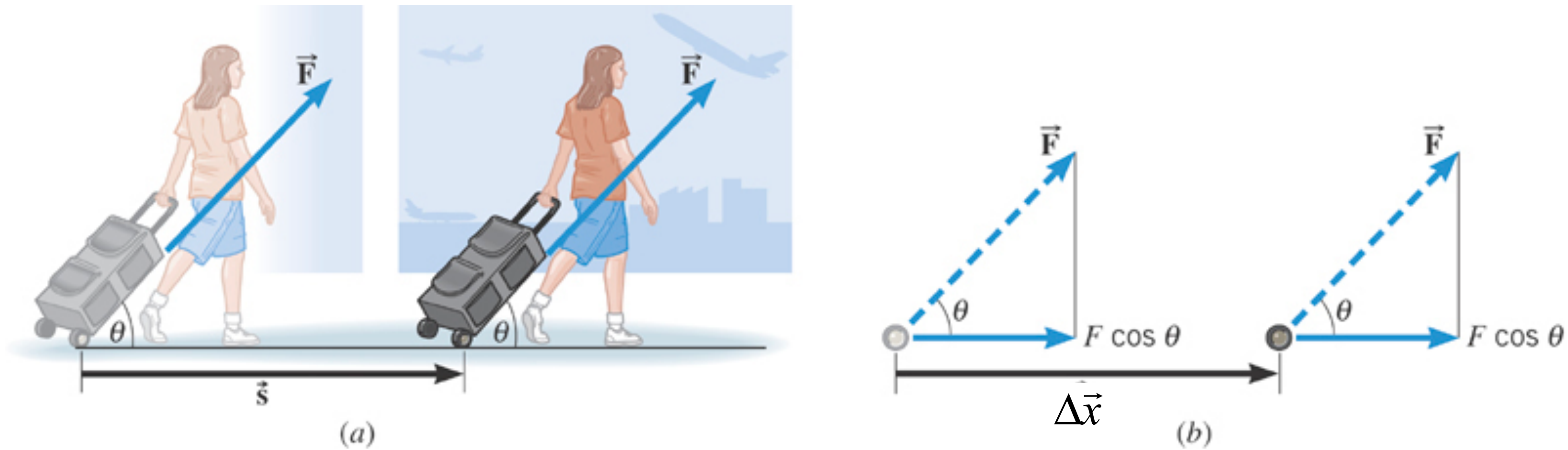
$$\boxed{W = \left(F\cos\theta\right)\Delta x}$$

F and Δx are <u>magnitudes</u>

$\cos 0^\circ = 1$ $\quad$ $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{x}}$ in the same direction. $\quad W = F\Delta x$

$\cos 90^\circ = 0$ $\quad$ $\vec{\mathbf{F}}$ perpendicular to $\Delta\vec{\mathbf{x}}$. $\quad W = 0$

$\cos 180^\circ = -1$ $\quad$ $\vec{\mathbf{F}}$ in the opposite direction to $\Delta\vec{\mathbf{x}}$. $\quad W = -F\Delta x$



Example: Pulling a Suitcase-on-Wheels

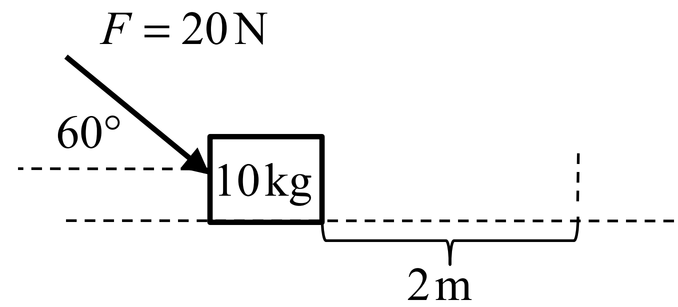
Find the work done if the force is 45.0-N, the angle is 50.0 degrees, and the displacement is 75.0 m.

$$W = (F\cos\theta)\Delta x = [(45.0\text{ N})\cos 50.0^\circ](75.0\text{ m})$$

$$= 2170\text{ J}$$

Clicker Question 5.1

A 10 kg is pushed with a 20 N force with an angle of 60° to the horizontal for a distance of 2.0m.



What work was done by this force?

a) $0\,\mathrm{J}$

b) $10\,\mathrm{J}$

c) $20\,\mathrm{J}$

d) $40\,\mathrm{J}$

e) $200\,\mathrm{J}$

5.1 Work Done by a Constant Force

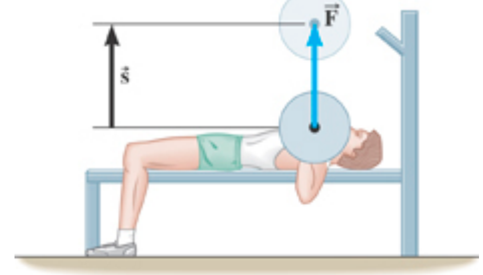
The bar bell (mass m) is moved slowly at a constant speed $\Rightarrow\ F = mg$.
The work done by the gravitational force will be discussed later.

(a)

Raising the bar bell, the displacement is up, and the force is up.

$$W = \left(F\cos 0^\circ\right)\Delta x = F\Delta x$$

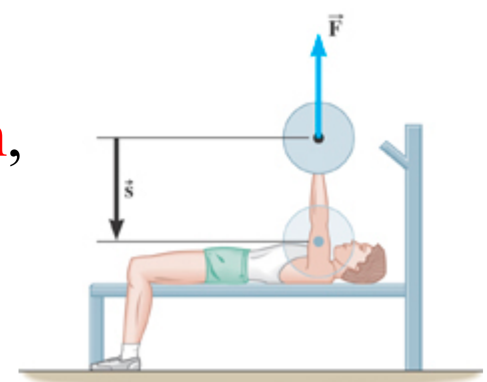
these are magnitudes!



(b)

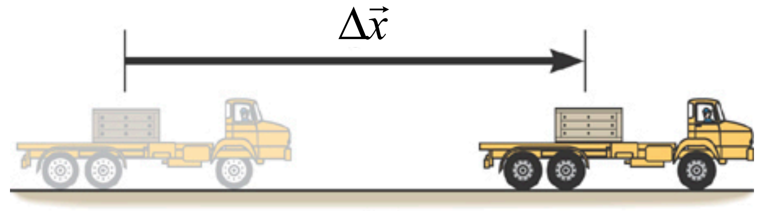
Lowering the bar bell, the displacement is down, and the force is (STILL) up.

$$W = \left(F\cos 180^\circ\right)\Delta x = -F\Delta x$$



(c)

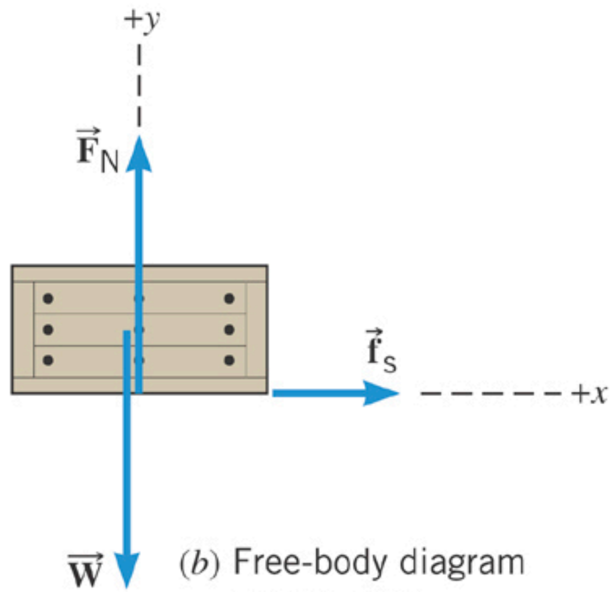
Example: Accelerating a Crate



(*a*)

The truck is accelerating at a rate of +1.50 m/s². The mass of the crate is 120-kg and it does not slip. The magnitude of the displacement is 65 m.

What is the total work done on the crate by all of the forces acting on it?



(*b*) Free-body diagram for the crate

$$\text{(normal force)}\ W = \left(F_N \cos 90^\circ\right)\Delta x = 0$$

$$\text{(gravity force)}\ W = \left(F_G \cos 90^\circ\right)\Delta x = 0$$

$$\text{(friction force)}\ W = \left(f_S \cos 0^\circ\right)\Delta x = f_S \Delta x$$

$$= (180\ \text{N})(65\ \text{m}) = 12\ \text{kJ}$$

$$\boxed{\begin{aligned} f_S &= ma = (120\ \text{kg})(1.50\ \text{m/s}^2) \\ &= 180\,\text{N} \end{aligned}}$$

$$\boxed{1\ \text{N}\cdot\text{m} \ = \ 1\ \text{joule}\ (\text{J})}$$

Clicker Question 5.2

A 10 kg mass is dropped a distance of 2m.

What is the work done on the mass by gravity?

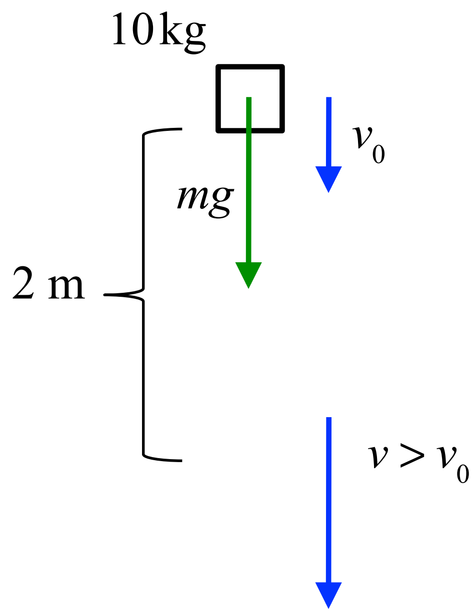
a) 0 J

b) 10 J

c) 20 J

d) 40 J

e) 200 J



Clicker Question 5.3

A 10 kg mass at the end of a string is lowered 2m at a constant speed.

What is the work done on the mass by the tension in the string?

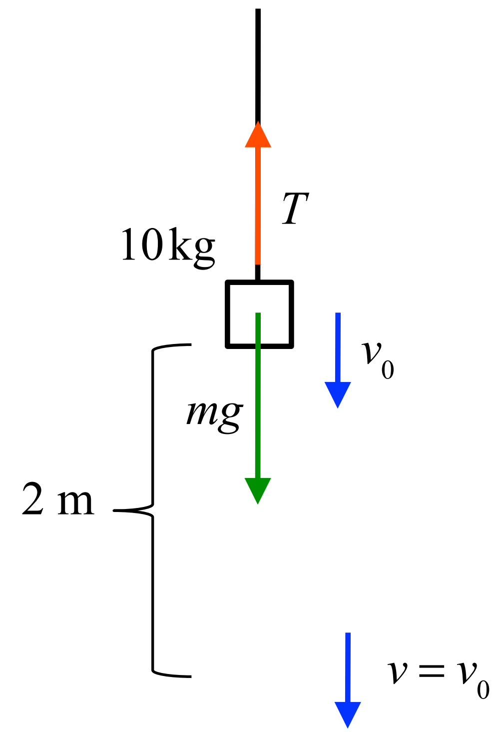
a) $0\,\text{J}$

b) $400\,\text{J}$

c) $200\,\text{J}$

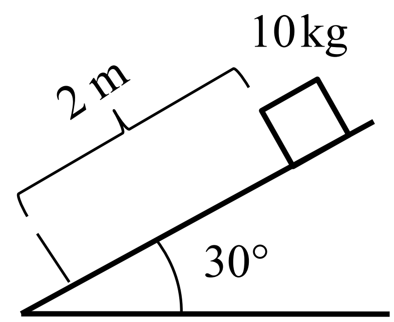
d) $-400\,\text{J}$

e) $-200\,\text{J}$



Clicker Question 5.4

A 10 kg mass slides down a inclined plane with an angle of 30° to the horizontal at a *constant speed* for a distance of 2.0m. What kinetic frictional force acts up the ramp?



a) 0 N

b) 490 N

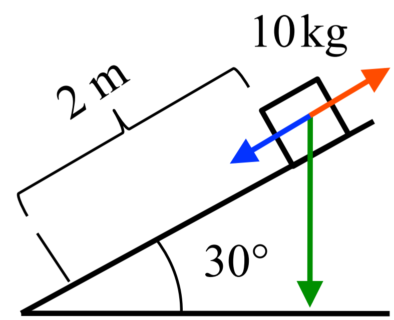
c) 980 N

d) 49 N

e) 98 N

Clicker Question 5.5

A 10 kg mass slides down a inclined plane with an angle of 30° to the horizontal at a *constant speed* for a distance of 2.0m.
What kinetic frictional force acts up the ramp?



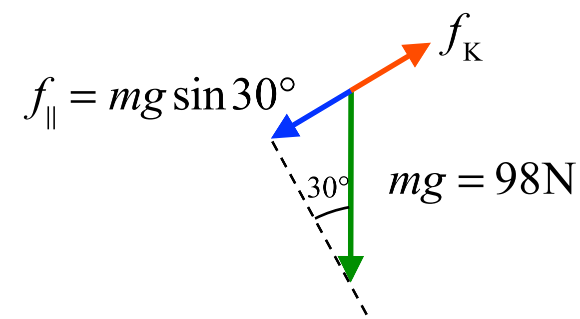
a) $0\,\mathrm{N}$

b) $490\,\mathrm{N}$

c) $980\,\mathrm{N}$

d) $49\,\mathrm{N}$

e) $98\,\mathrm{N}$



The kinetic frictional force does what work on the mass?

a) $0\,\mathrm{J}$

b) $-49\,\mathrm{J}$

c) $-98\,\mathrm{J}$

d) $-490\,\mathrm{J}$

e) $-980\,\mathrm{J}$

5.2 Work on a Spring & Work by a Spring

HOOKE'S LAW

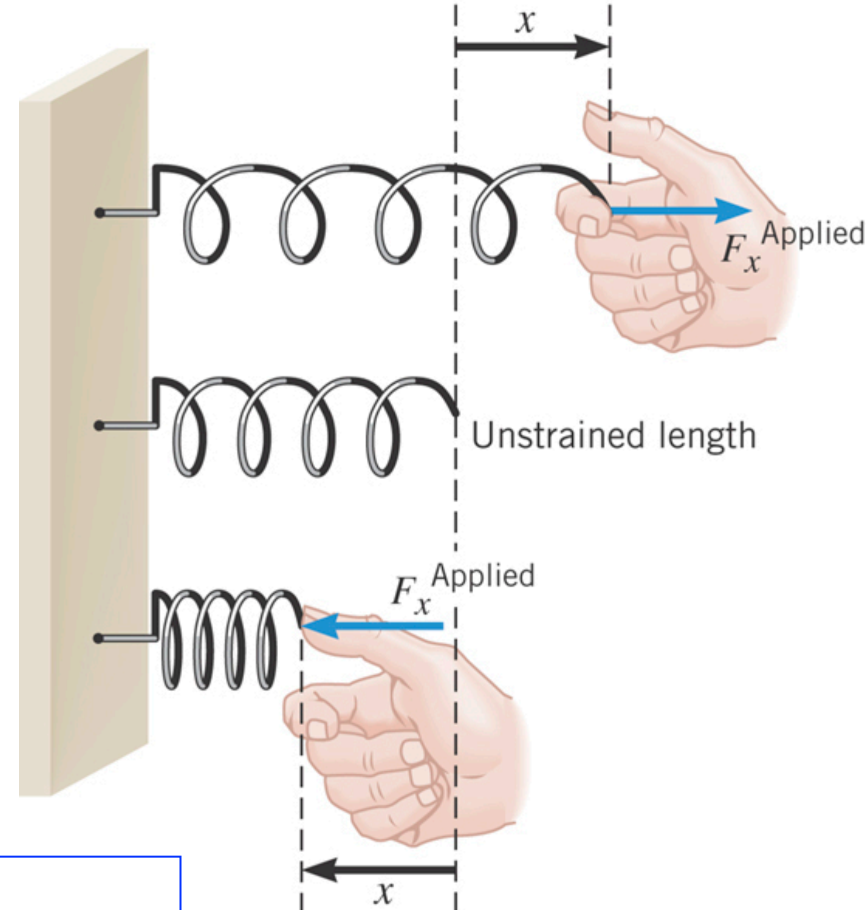
Force Required to Distort an Ideal Spring

The force applied to an ideal spring is proportional to the displacement of its end.

$$F_x^{\text{Applied}} = kx$$

spring constant

Units: N/m



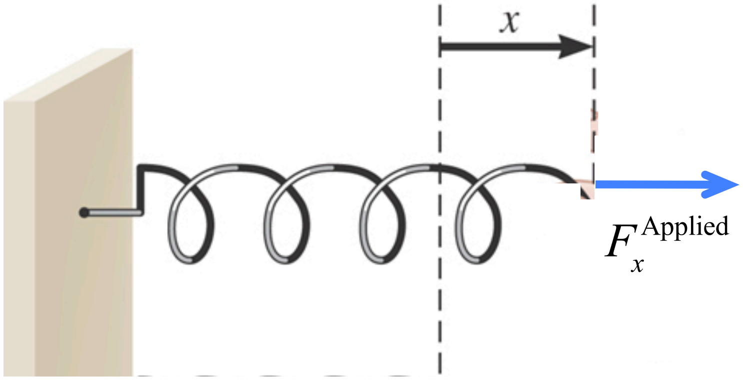
This is a scalar equation

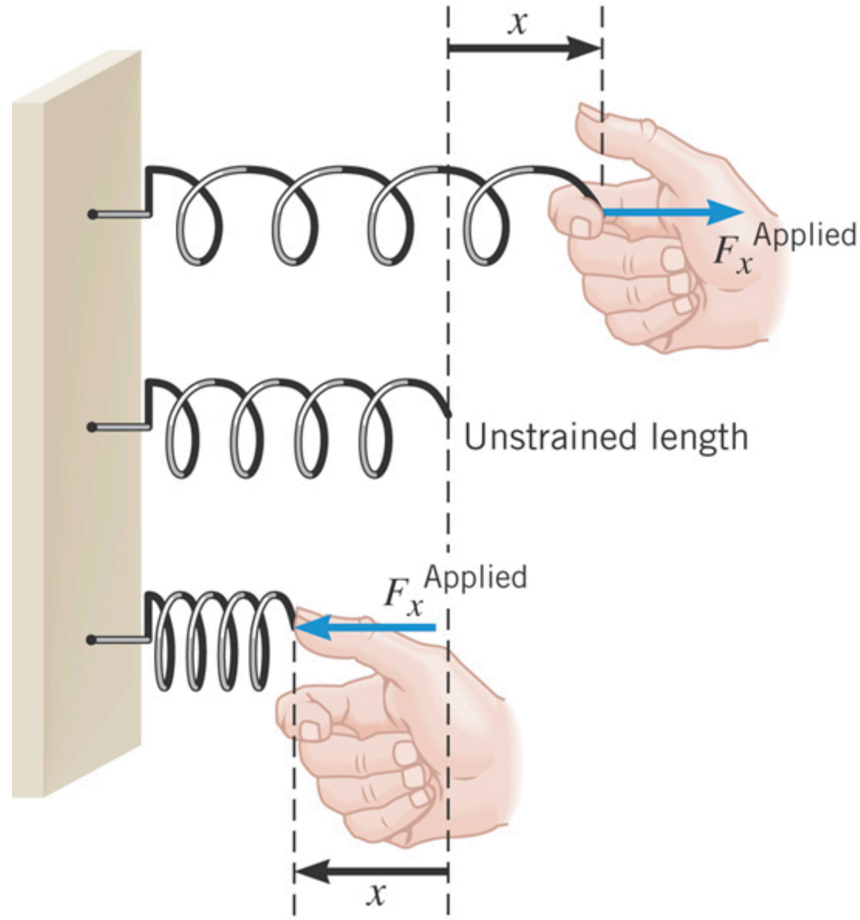
F_x^{Applied} is magnitude of applied force.

x is the magnitude of the spring displacement

k is the spring constant (strength of the spring)

5.2 Work on a Spring & Work by a Spring





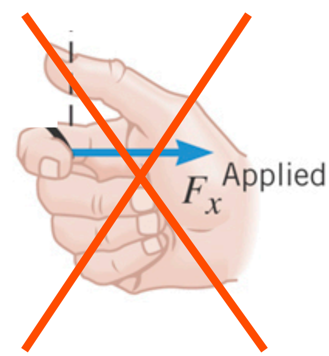
F_x^{Applied} is applied to the spring.

This force can come from anywhere.

The wall generates a force on the spring.

F_x^{Applied} acts ON the SPRING

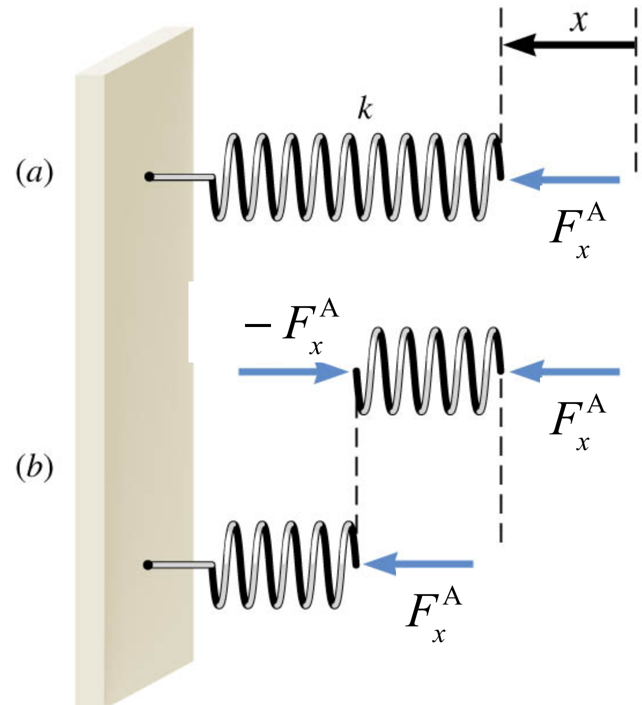
NOT on the HAND



5.2 Work on a Spring & Work by a Spring

Conceptual Example: Is ½ a spring stronger or weaker?
A 10-coil spring has a spring constant *k*. The spring is cut in half, so there are two 5-coil springs. What is the spring constant of each of the smaller springs?

$$\boxed{\text{Original Spring: } F_x^{\mathrm{A}} = kx; \quad k = \frac{F_x^{\mathrm{A}}}{x}}$$



Compression of each piece $x' = x/2$.

Apply the same force as before!

<u>Spring constant of each piece</u>

$$k' = \frac{F_x^{\mathrm{A}}}{x'} = \frac{F_x^{\mathrm{A}}}{x/2}$$

$$= 2\left(\frac{F_x^{\mathrm{A}}}{x}\right) = 2k \text{ (twice as strong)}$$

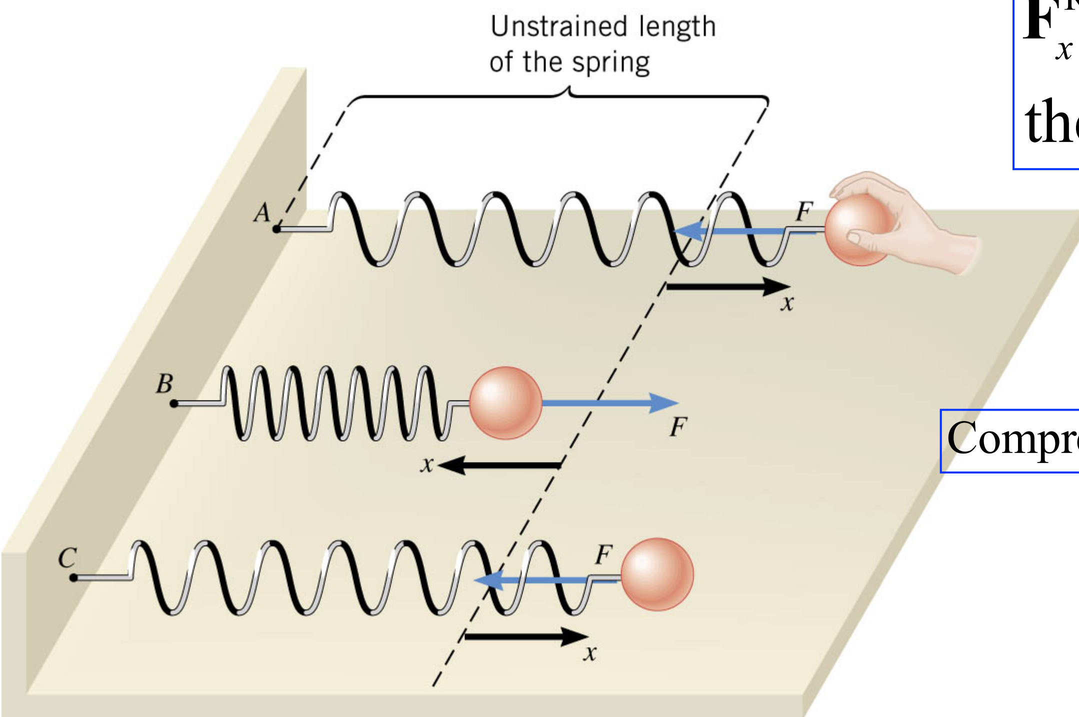
5.2 Work on a Spring & Work by a Spring

HOOKE'S LAW
Restoring Force Generated by a Distorted Ideal Spring

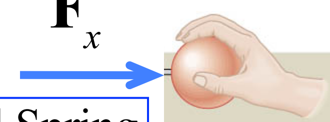
The restoring force generated by an ideal spring is proportional to the displacement of its end:

$$F_x^R = -kx$$

F_x^R : restoring force generated by the stretched or compressed spring.

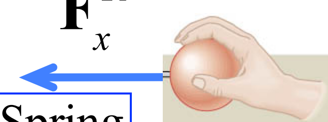


F_x^R



Compressed Spring

F_x^R



Stretched Spring

Restoring forces act on ball/hand.

Conceptual Example 2 Are Shorter Springs Stiffer?

A 10-coil spring has a spring constant *k*. If the spring is cut in half, so there are two 5-coil springs, what is the spring constant of each of the smaller springs?

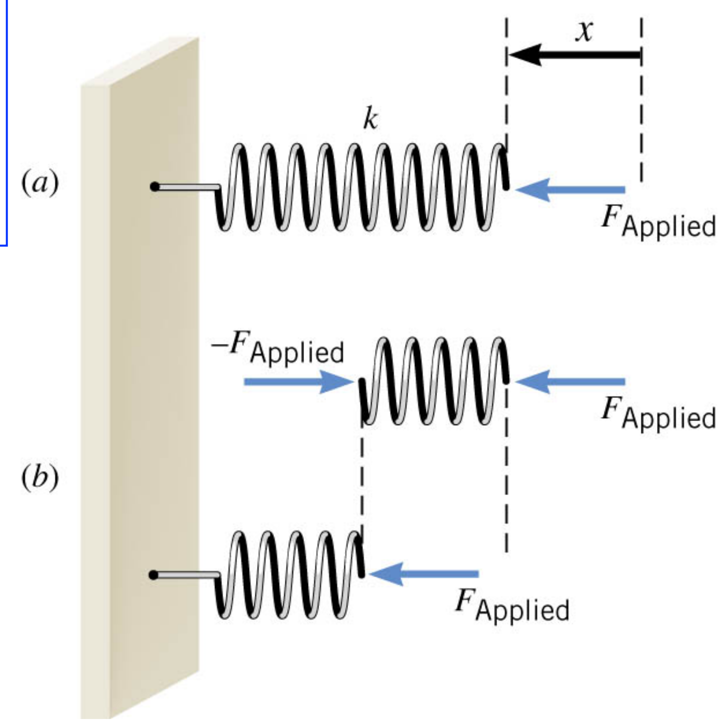
$$F_A = kx; \quad k = \frac{F_A}{x}$$

Each piece $x' = x/2$. Same force applied.

New spring constant of each piece

$$k' = \frac{F_A}{x'} = \frac{F_A}{x/2}$$

$$= 2\left(\frac{F_A}{x}\right) = 2k \text{ (twice as strong)}$$



5.2 Work on a Spring & Work by a Spring

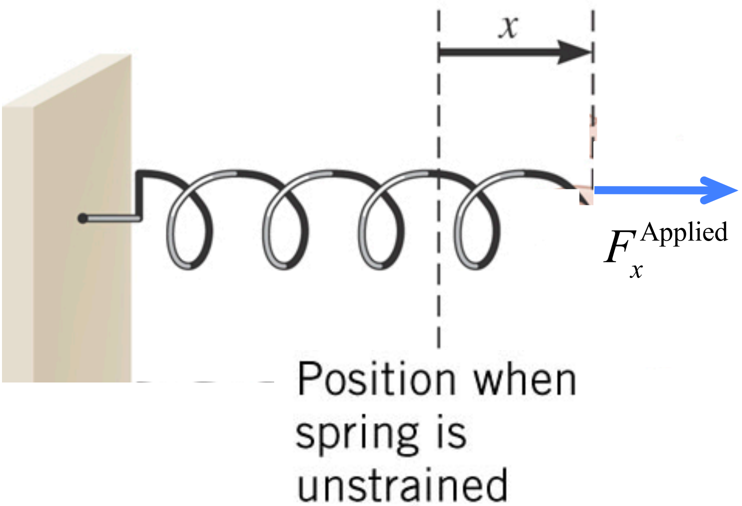
Work done by applied force stretching (or compressing) a spring.
Force is changing while stretching – so use the average force.

$\bar{F}$ is the magnitude of the <u>average force</u> while stretching, $\frac{1}{2}(kx+0)$

Δx is the magnitude of the displacement, (x)

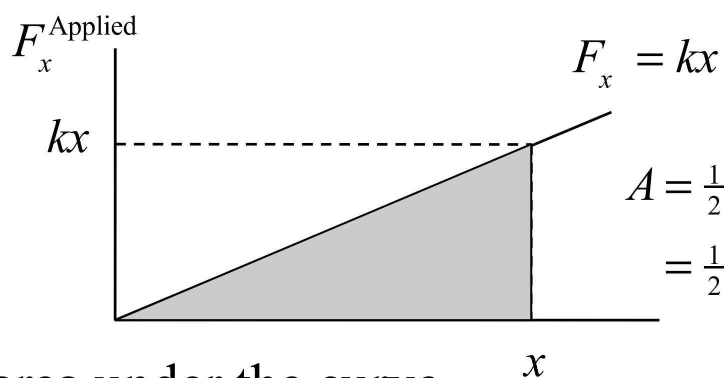
θ is the angle between the force and displacement vectors, $(0°)$

W is the work done on the spring by the applied force



$$W = \left(\bar{F}\cos\theta\right)\Delta x$$

$$= \tfrac{1}{2}\left(kx\right)\cos(0^\circ)(x) = \tfrac{1}{2}kx^2 \quad \text{(positive)}$$



$$A = \tfrac{1}{2}bh$$

$$= \tfrac{1}{2}(x)(kx) = \tfrac{1}{2}kx^2$$

work is the area under the curve

5.2 Work on a Spring & Work by a Spring

Restoring force of a stretched spring can do work on a mass.

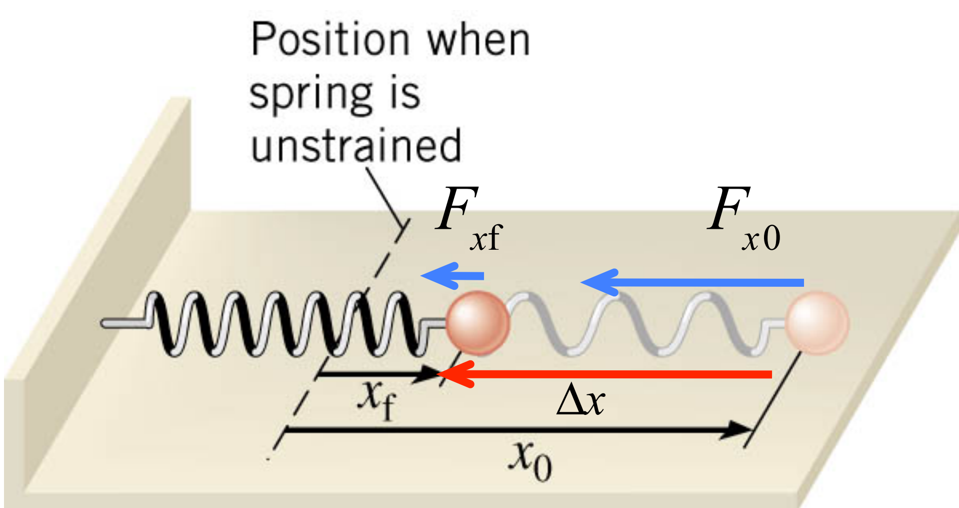
$\bar{F}$ is the magnitude of the average force, $\frac{1}{2}(kx_0 + kx_f)$

Δx is the magnitude of the displacement, $(x_0 - x_f)$

θ is the angle between the force and displacement vectors, $(0°)$

$$W_{\text{elastic}} = \left(\bar{F}\cos\theta\right)\Delta x$$

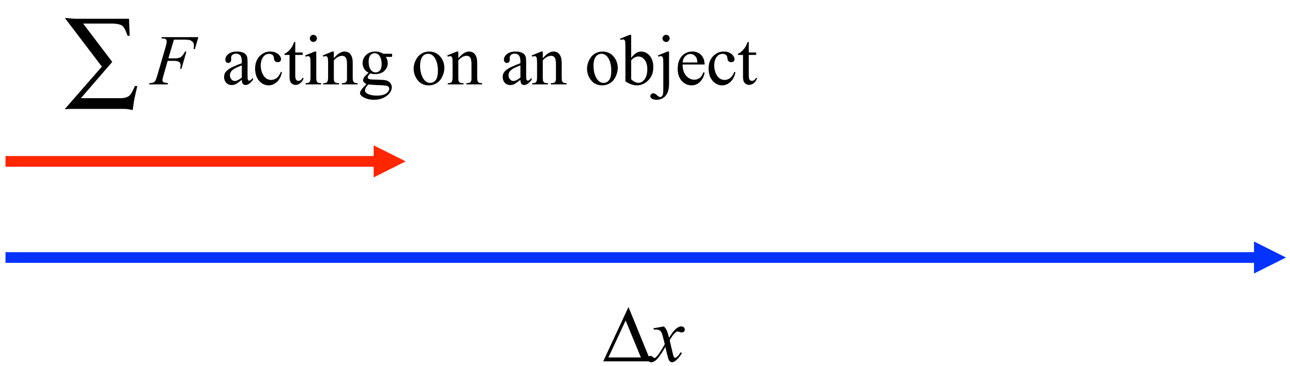
$$= \tfrac{1}{2}\left(kx_f + kx_0\right)\cos(0^\circ)(x_0 - x_f) = \tfrac{1}{2}kx_0^2 - kx_f^2 \quad \text{(positive)}$$



5.3 *The Work-Energy Theorem and Kinetic Energy*

Consider a constant net external force acting on an object.

The object is displaced a distance Δx, in the same direction as the net force.



The work is simply $W = \left(\sum F\right)\Delta x = (ma)\Delta x$

5.3 The Work-Energy Theorem and Kinetic Energy

We have often used this 1D motion equation using v_x for final velocity:

$$v_x^2 = v_{0x}^2 + 2a\Delta x$$

Multiply equation by $\frac{1}{2}m$ (why?)

$$\tfrac{1}{2}mv_x^2 = \tfrac{1}{2}mv_{0x}^2 + ma\Delta x \quad \text{but } F_{\text{Net}} = ma$$

$$\tfrac{1}{2}mv_x^2 = \tfrac{1}{2}mv_{0x}^2 + F_{\text{Net}}\Delta x \quad \text{but net work, } W = F_{\text{Net}}\Delta x$$

DEFINE KINETIC ENERGY of an object with mass m speed v:

$$\boxed{\mathrm{K} = \tfrac{1}{2}mv^2}$$

Now it says, Kinetic Energy of a mass changes due to Work:

$$\boxed{K = K_0 + W}$$

or

$$\boxed{K - K_0 = W}$$

Work–Energy Theorem