Chapter 5

Work and Energy

continued

5.2 Work on a Spring & Work by a Spring

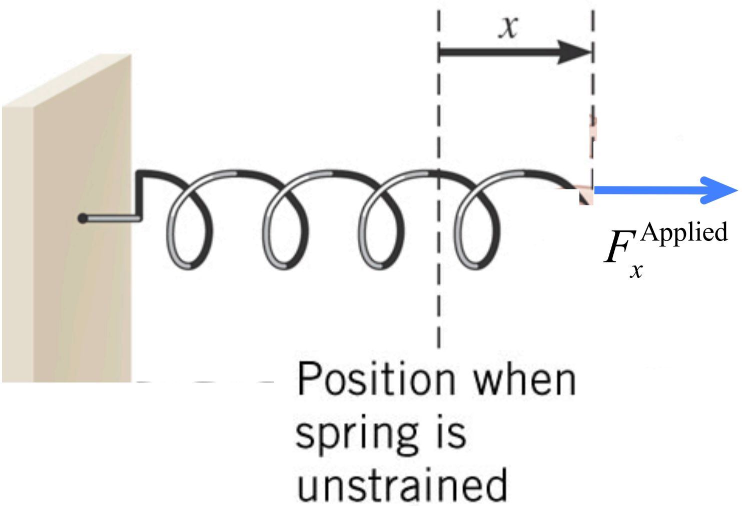
Work done by applied force stretching (or compressing) a spring.
Force is changing while stretching – so use the average force.

$\bar{F}$ is the magnitude of the average force while stretching, $\frac{1}{2}(kx+0)$

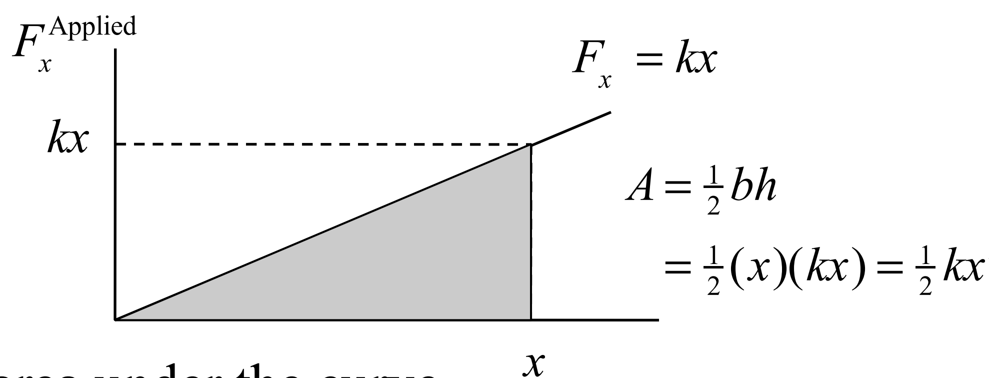
Δx is the magnitude of the displacement, (x)

θ is the angle between the force and displacement vectors, $(0°)$

W is the work done on the spring by the applied force



$$W = \left(\bar{F}\cos\theta\right)\Delta x$$
$$= \tfrac{1}{2}\left(kx\right)\cos(0^\circ)(x) = \tfrac{1}{2}kx^2 \quad \text{(positive)}$$



work is the area under the curve

5.3 The Work-Energy Theorem and Kinetic Energy

We have often used this 1D motion equation

using v_x for final velocity:

$$v_x^2 = v_{0x}^2 + 2a\Delta x$$

Multiply equation by $\frac{1}{2}m$ (why?)

$$\tfrac{1}{2}mv_x^2 = \tfrac{1}{2}mv_{0x}^2 + ma\Delta x \quad \text{but } F_{\text{Net}} = ma$$

$$\tfrac{1}{2}mv_x^2 = \tfrac{1}{2}mv_{0x}^2 + F_{\text{Net}}\Delta x \quad \text{but net work, } W = F_{\text{Net}}\Delta x$$

DEFINE KINETIC ENERGY of an object with mass m speed v:

$$\text{K} = \tfrac{1}{2}mv^2$$

Now it says, Kinetic Energy of a mass changes due to Work:

$$K = K_0 + W$$

or

$$K - K_0 = W$$

Work–Energy Theorem

5.3 The Work-Energy Theorem and Kinetic Energy

Work and Energy

Work: the effect of a force acting on an object making a displacement.

$$W = (F\cos\theta)\Delta x,$$

where W is the work done, $F, \Delta x$ are the magnitudes of the force and displacement, and θ is the angle between $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{x}}$.

The origin of the force does not affect the calculation of the work done.

Work can be done by: gravity, elastic, friction, explosion, or human forces.

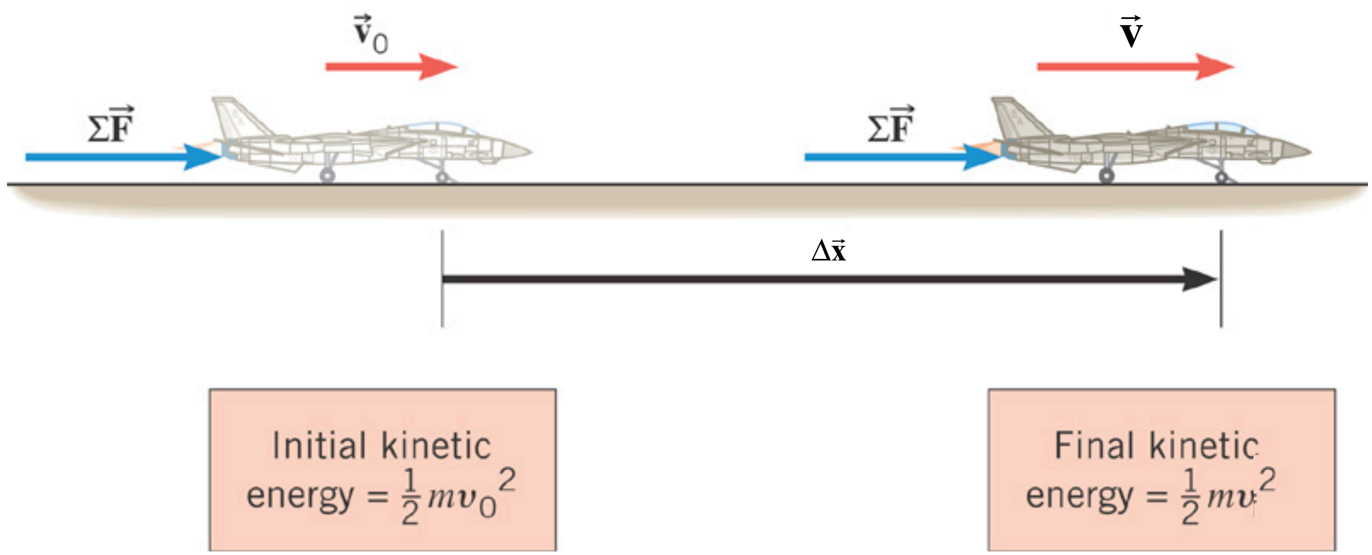
Kinetic energy: property of a mass (m) and the square of its speed (v).

$$\mathrm{K} = \tfrac{1}{2}mv^2$$

Work-Energy Theorem: Work changes the Kinetic Energy of an object.

$$\mathrm{K} = \mathrm{K}_0 + W \quad \text{or} \quad \mathrm{K} - \mathrm{K}_0 = W$$

5.3 The Work-Energy Theorem and Kinetic Energy



THE WORK-ENERGY THEOREM

When a net external force does work on an object, the kinetic energy of the object changes according to

$$W = K - K_0 = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

Clicker Question 5.7

Determine the amount of work done in firing a 2.0-kg projectile to an initial speed of 50 m/s. Neglect any effects due to air resistance

a) 900 J

b) 1600 J

c) 2500 J

d) 4900J

e) zero

Clicker Question 5.7

Determine the amount of work done in firing a 2.0-kg projectile to an initial speed of 50 m/s. Neglect any effects due to air resistance

a) 900 J

b) 1600 J

c) 2500 J

d) 4900J

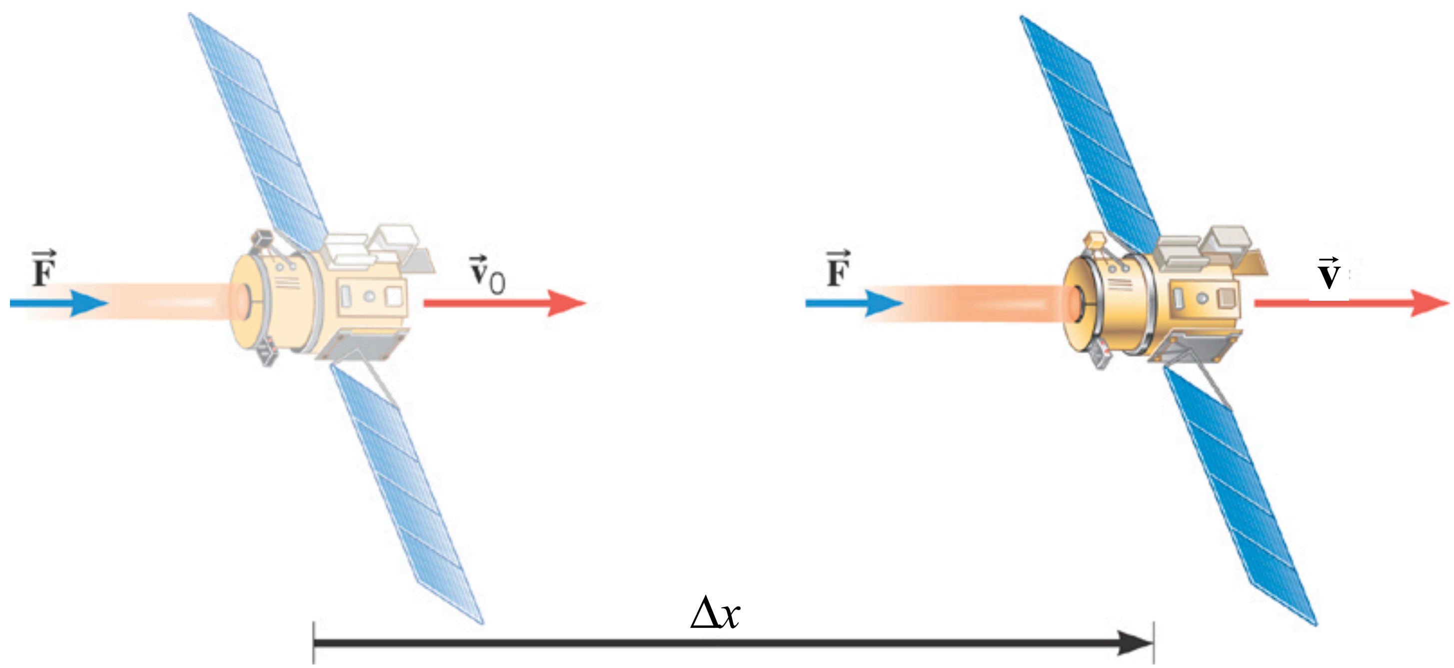
e) zero

Work changes kinetic energy, $K_0 = 0$

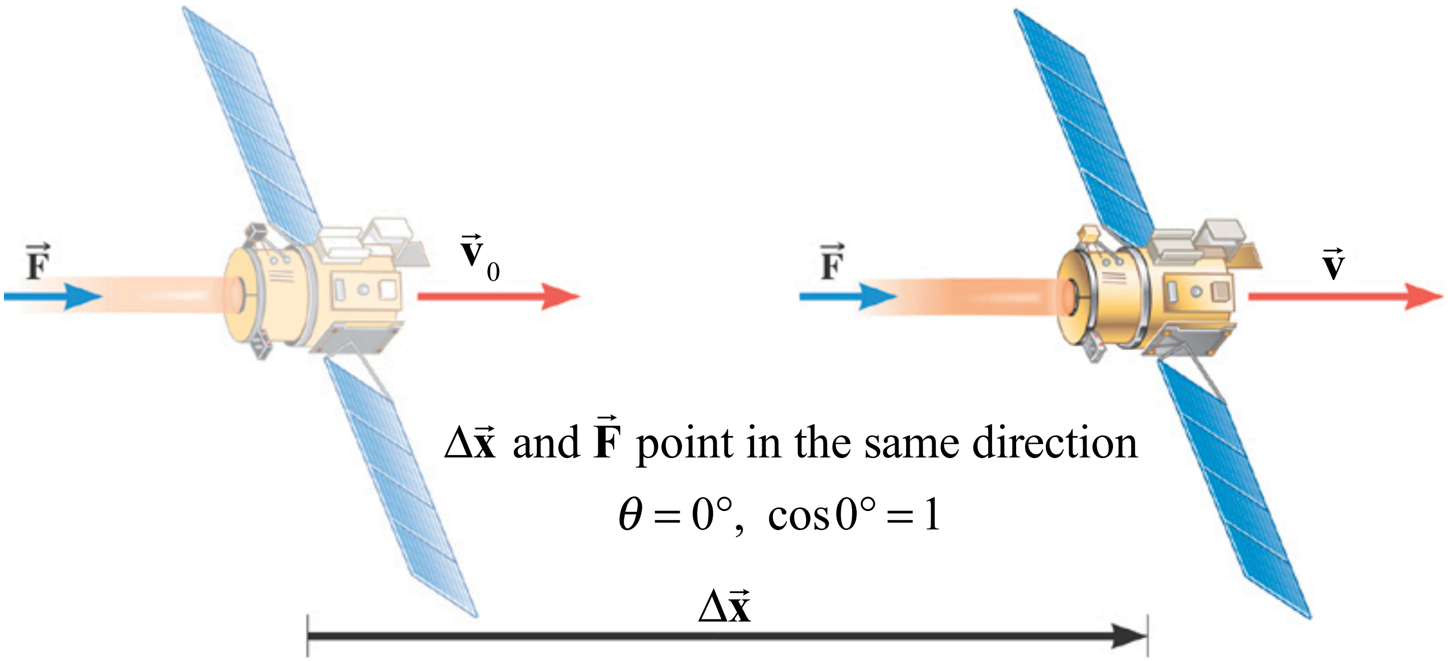
$$W = K - K_0 = \tfrac{1}{2}mv^2 = (0.5)(2.0\text{ kg})(50\text{ m/s})^2 = 2500\text{ J}$$

Example:

The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe through a displacement of 2.42×10^{9}m, what is its final speed?



5.3 The Work-Energy Theorem and Kinetic Energy



$$W = \left[F\cos\theta\right]\Delta x = \left(5.60\times10^{-2}\text{N}\right)\left(2.42\times10^{9}\text{m}\right) = 1.36\times10^{8}\text{J}$$

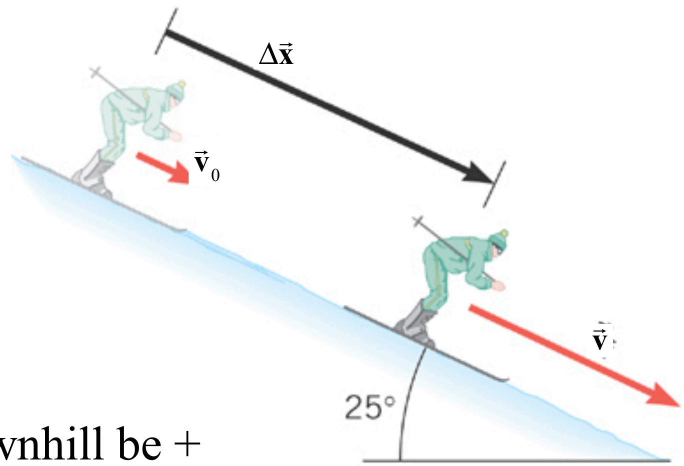
$$W = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$ Solve for final velocity v

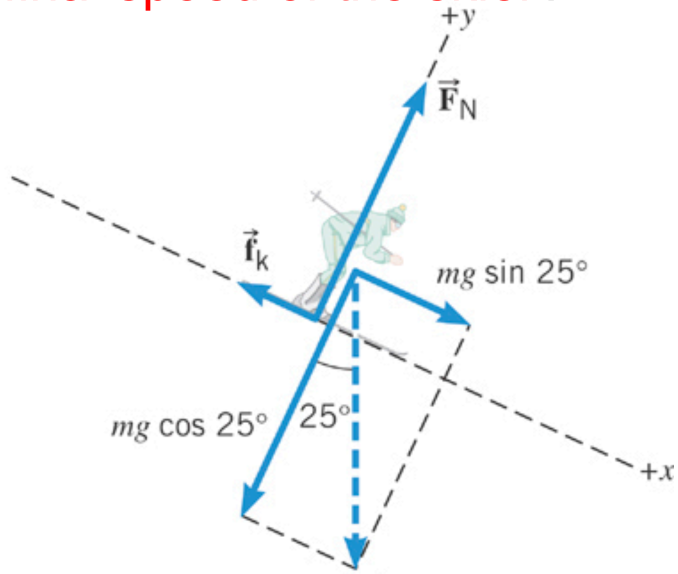
$$v^2 = \frac{2W}{m} + v_0^2 = \frac{2.72\times10^{8}\text{J}}{474\text{ kg}} + (275\text{m/s})^2$$

$$v = 806\text{ m/s}$$

5.3 *The Work-Energy Theorem and Kinetic Energy*

Example - A 58.0 kg skier experiences a kinetic frictional force of 71.0N while traveling down a 25° hill for a distance of 57.0 m. If the skier's initial speed was 3.60 m/s, what is the final speed of the skier?





Let downhill be +

The net force is $F_{Net} = mg\sin 25^\circ - f_k = 170\text{N}$

Decomposition of the downward gravitational force, *mg*.

Work-Energy Theorem:

$$K - K_0 = W \quad \Rightarrow \quad K = K_0 + W$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_0^2 + F_{Net}(\cos 0^\circ)\Delta x$$

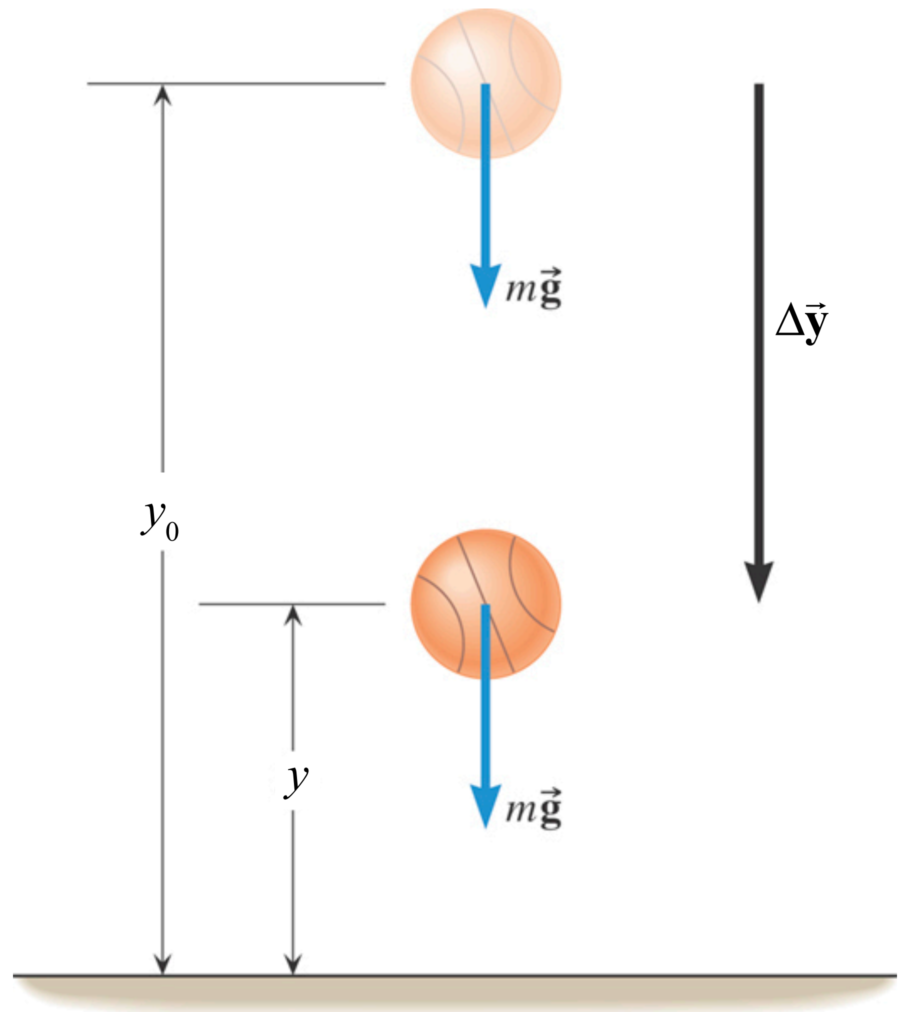
$$v^2 = v_0^2 + \frac{2F_{Net}\Delta x}{m} \Rightarrow v = \sqrt{v_0^2 + \frac{2F_{Net}\Delta x}{m}} = 18.6 \text{ m/s}$$

5.4 Gravitational Potential Energy

This θ is the angle between $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{y}}$.

$$W = \left(F\cos\theta\right)\Delta y$$

$$= mg\Delta y$$



$\Delta y = \text{distance of fall} = (y_0 - y)$

$$W_G = mg\left(y_0 - y\right)$$

Why use $(y_0 - y)$ instead of Δy? $\Rightarrow$ because we already have K and K_0

5.4 Gravitational Potential Energy

When returning to the initial height, y_0, the work done by gravity is zero. There is no displacement, $\Delta y = 0$.

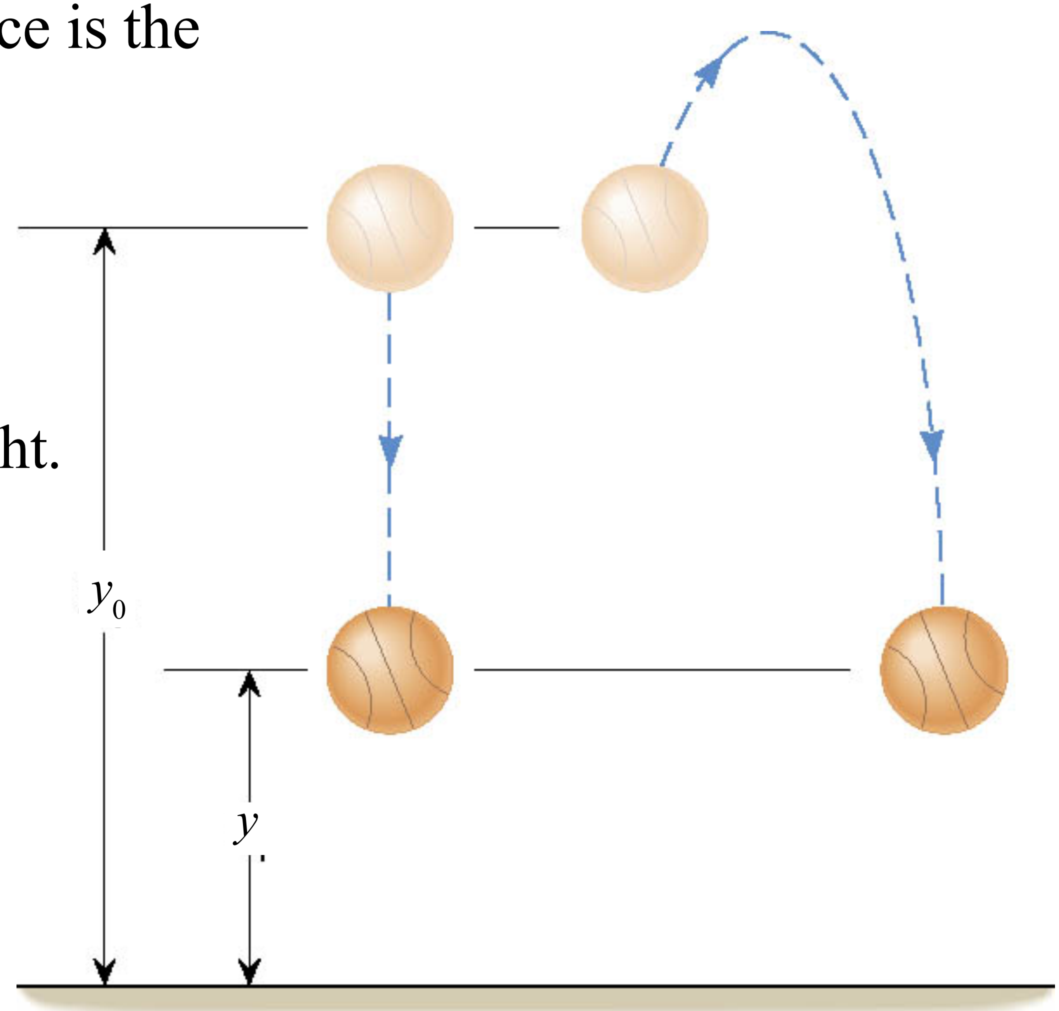
Work by the gravitational force is the same over the two paths

$$W_G = mg\left(y_0 - y\right)$$

Same starting and ending height.

Gravity is unusual!

It is a "conservative force".



DEFINITION OF A CONSERVATIVE FORCE

Version 1 A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.

Version 2 A force is conservative when it does no net work on an object moving around a closed path, starting and finishing at the same point.

Also:

Version 3 A force is conservative when the energy absorbed from a mass by the force can be returned to the mass without loss by that force.

5.4 Conservative Versus Nonconservative Forces

Some Conservative and Nonconservative Forces

Conservative Forces

- Gravitational force
- Elastic spring force
- Electric force

Nonconservative Forces

- Static and kinetic frictional forces
- Air resistance
- Muscular forces
- Explosions
- Jet or rocket forces

5.4 Gravitational Potential Energy

Because gravity is a conservative force, when a mass moves upward against the gravitational force, the kinetic energy of the mass decreases, but when the mass falls to its initial height that kinetic energy returns completely to the mass.

When the kinetic energy decreases, where does it go?

DEFINITION OF GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy U is the energy that an object of mass m has by virtue of its position relative to the surface of the earth. That position is measured by the height y of the object relative to an arbitrary zero level:

$$\boxed{U = mgy} \qquad 1\,\mathrm{N}\cdot\mathrm{m} = 1\ \text{joule}\ (\mathrm{J})$$

(y can be + or –)

Clicker Question 5.8

The potential energy (*mgy*) of a 10 kg mass on the surface of the earth is zero. What is the potential energy of the mass in a 10 m deep hole?

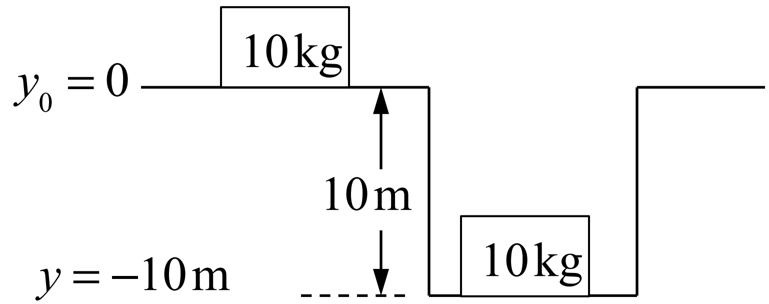
a) 0 J

b) 98 J

c) –98 J

d) 980 J

e) –980 J



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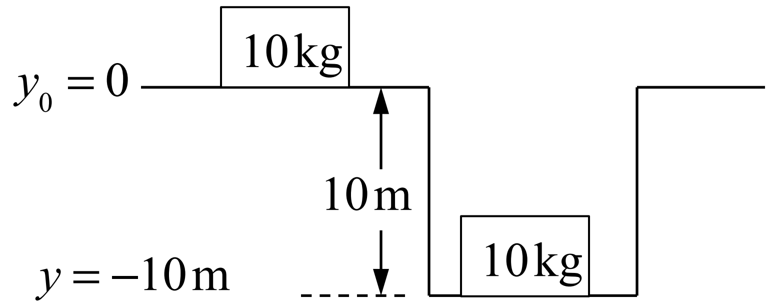


a) 0 J

b) 98 J

c) – 98 J

d) 980 J

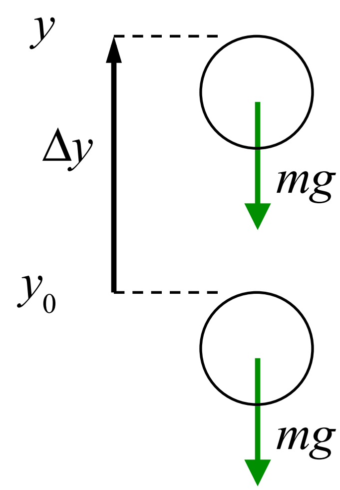
e) – 980 J

$$U = mgy = (10\text{kg})(9.8\ \text{m/s}^2)(-10\text{m})$$
$$= -980\ \text{J}$$

5.4 Gravitational Potential Energy

Thrown upward

Gravitational work is negative.



$$W_G = \left(F\cos 180^\circ\right)\Delta y$$
$$= -mg(y - y_0)$$

Gravitational Potential Energy increases.

$$U - U_0 = mg\left(y - y_0\right)$$
$$= -W_G$$

Work-Energy Theorem becomes:

$$K - K_0 = W_G$$
$$= -(U - U_0)$$

$$\boxed{K + U = K_0 + U_0 \quad \text{Conservation of Energy}}$$

5.4 Gravitational Potential Energy

GRAVITATIONAL POTENTIAL ENERGY

Energy of mass m due to its position relative to the surface of the earth.
Position measured by the height y of mass relative to an arbitrary zero level:

$$U = mgy$$

U replaces Work by gravity in the Work-Energy Theorem

Work-Energy Theorem becomes Mechanical Energy Conservation:

$$K + U = K_0 + U_0$$
$$E = E_0$$

Initial total energy, $E_0 = K_0 + U_0$ doesn't change.
It is the same as final total energy, $E = K + U$.

Another way (equivalent) to look at Mechanical Energy Conservation:

$$(K - K_0) + (U - U_0) = 0$$
$$\Delta K + \Delta U = 0$$

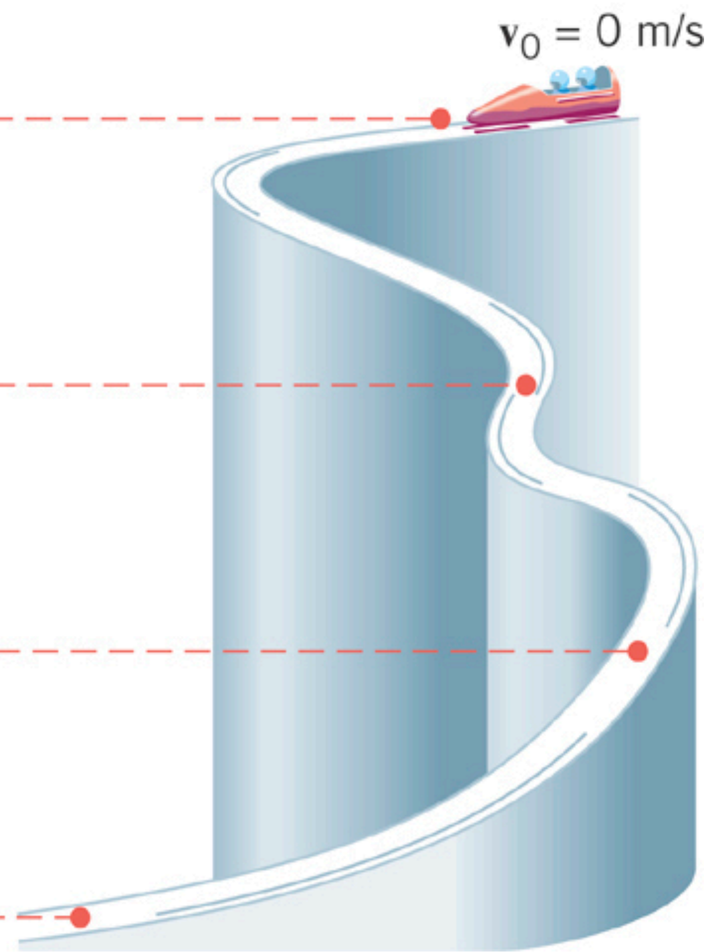
Any increase (decrease) in K is balanced by a decrease (increase) in U.

These are used if only Conservative Forces act on the mass.
(Gravity, Ideal Springs, Electric forces)

Sliding without friction: only gravity does work.

Normal force of ice is always perpendicular to displacements.

K	U	$E = K + U$
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J



Clicker Question 5.9

You are investigating the safety of a playground slide. You are interested in finding out what the maximum speed will be of children sliding on it when the conditions make it very slippery (assume frictionless). The height of the slide is 2.5 m. What is that maximum speed of a child if she starts from rest at the top?

a) 1.9 m/s

b) 2.5 m/s

c) 4.9 m/s

d) 7.0 m/s

e) 9.8 m/s

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You are investigating the safety of a playground slide. You are interested in finding out what the maximum speed will be of children sliding on it when the conditions make it very slippery (assume frictionless). The height of the slide is 2.5 m. What is that maximum speed of a child if she starts from rest at the top?

a) 1.9 m/s

b) 2.5 m/s

c) 4.9 m/s

d) 7.0 m/s

e) 9.8 m/s

$$E_0 = U_0 = mgy; \quad E = K + U = \tfrac{1}{2}mv^2 + mg(0)$$

$$E_0 = E \Rightarrow mgy = \tfrac{1}{2}mv^2$$

$$v = \sqrt{2gy} = \sqrt{2(9.8)(2.5)}\ \text{m/s}$$

$$= 7.0\ \text{m/s}$$

5.5 Conservative Versus Nonconservative Forces

In many situations both conservative and non-conservative forces act simultaneously on an object, so the work done by the net external force can be written as

$$W_{\text{Net}} = W_{\text{C}} + W_{\text{NC}}$$

W_{C} = work by conservative force such as work by gravity W_{G}

But replacing W_{C} with $-(U - U_0)$

Work-Energy Theorem becomes:

$$K + U = K_0 + U_0 + W_{\text{NC}}$$

$$\text{E}_{\text{f}} = \text{E}_0 + W_{\text{NC}}$$

work by non-conservative forces will add or remove energy from the mass

$$E = K + U \neq E_0 = K_0 + U_0$$

Another (equivalent) way to think about it:

$$\left(K - K_0\right) + \left(U - U_0\right) = W_{\text{NC}}$$

$$\Delta K + \Delta U = W_{\text{NC}}$$

if non-conservative forces do work on the mass, energy changes will not sum to zero

But all you need is this:

$$K + U = K_0 + U_0 + W_{NC}$$

non-conservative forces add or remove energy

$$\text{If } W_{NC} \neq 0, \text{ then } E \neq E_0$$

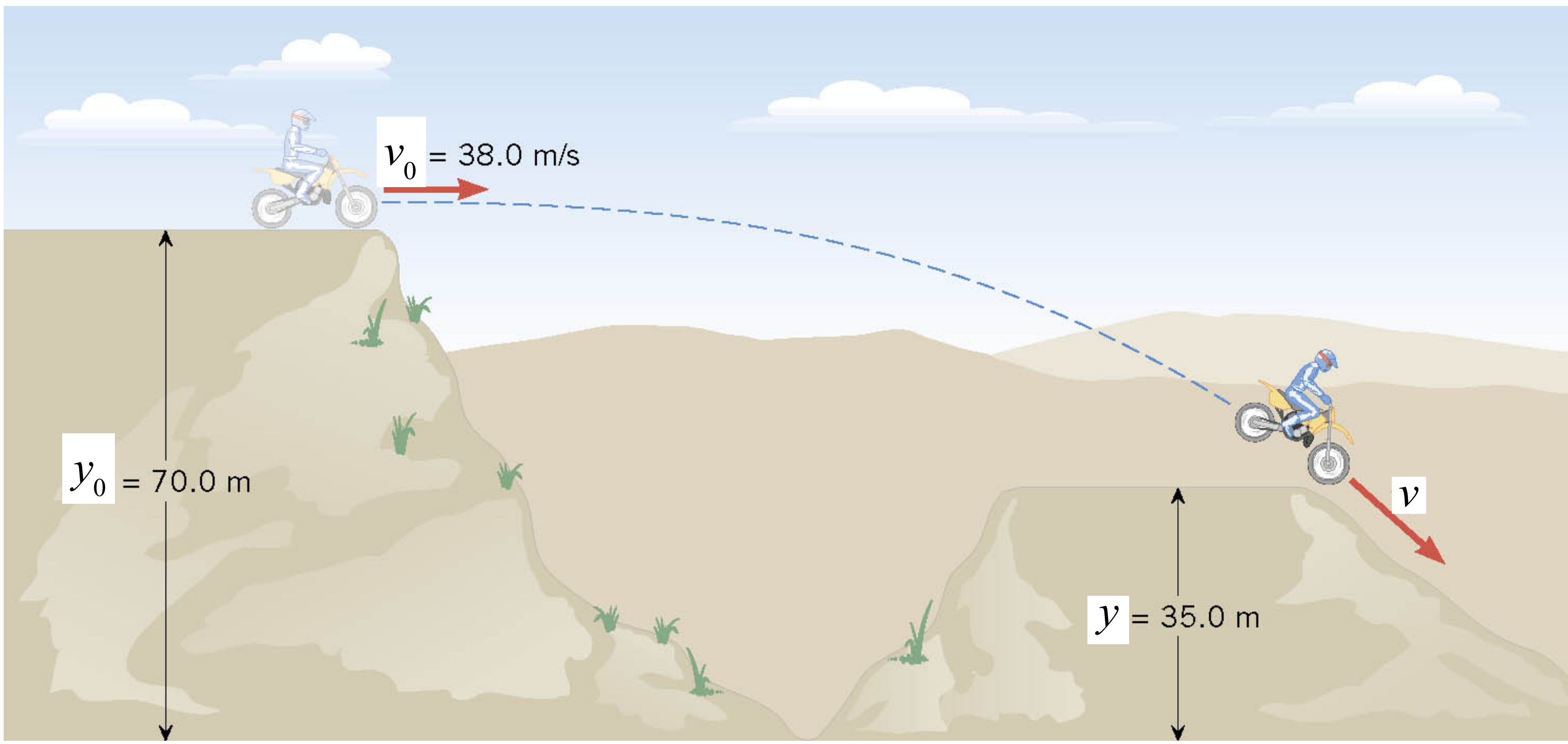
If the net work on a mass by non-conservative forces is zero, then its total energy does not change:

$$\text{If } W_{NC} = 0, \text{ then } E = E_0$$

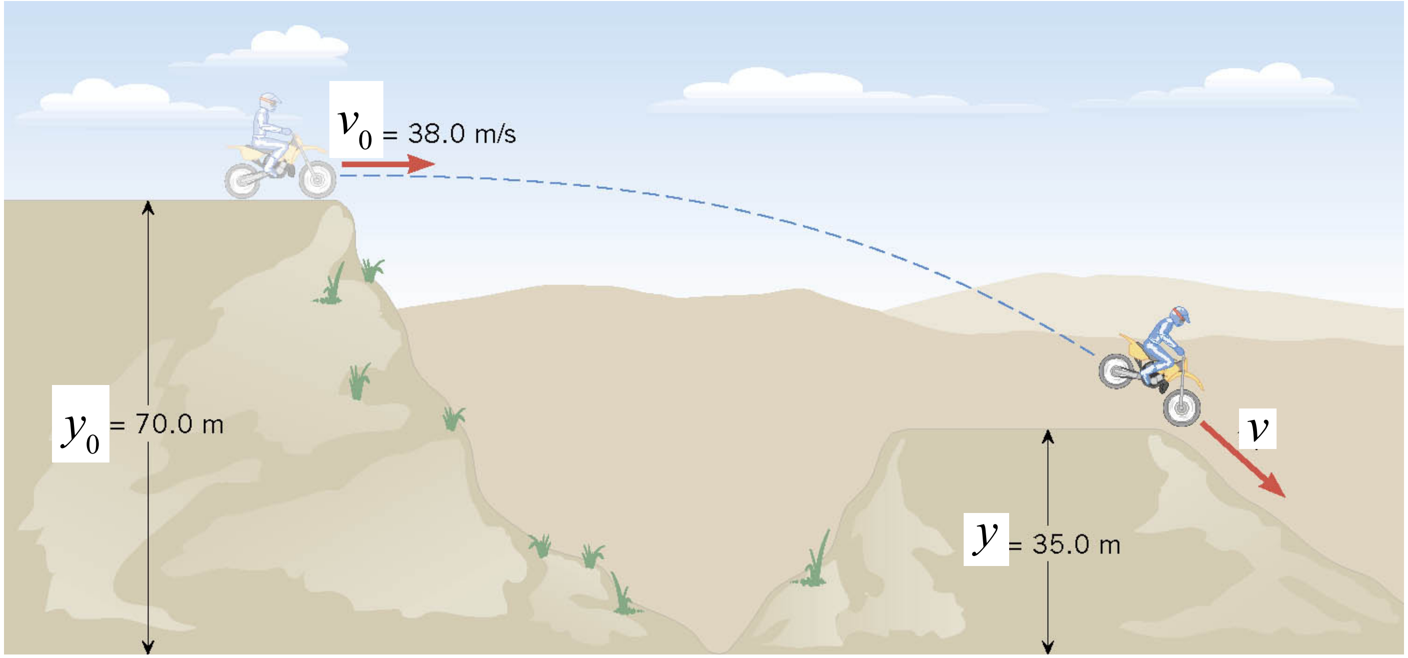
$$K + U = K_0 + U_0$$

Example: A Daredevil Motorcyclist

A motorcyclist is trying to leap across the canyon by driving horizontally off a cliff 38.0 m/s. Ignoring air resistance, find the speed with which the cycle strikes the ground on the other side.



5.5 The Conservation of Mechanical Energy

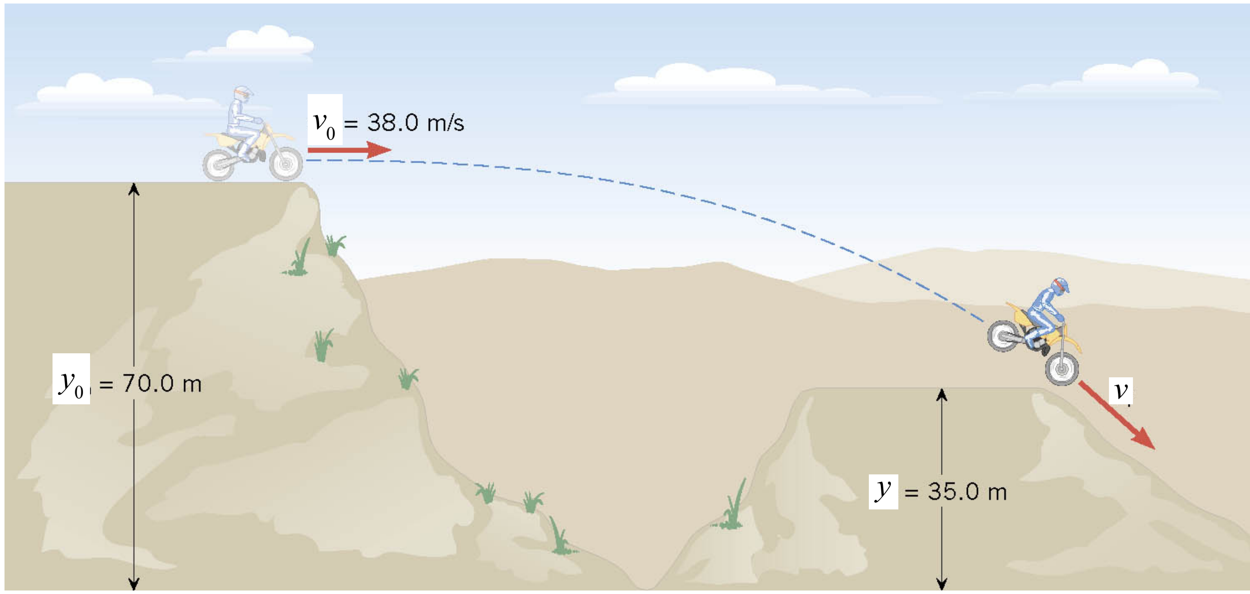


$$E = E_o$$

$$mgy + \tfrac{1}{2}mv^2 = mgy_0 + \tfrac{1}{2}mv_0^2$$

$$gy + \tfrac{1}{2}v^2 = gy_0 + \tfrac{1}{2}v_0^2$$

5.5 The Conservation of Mechanical Energy



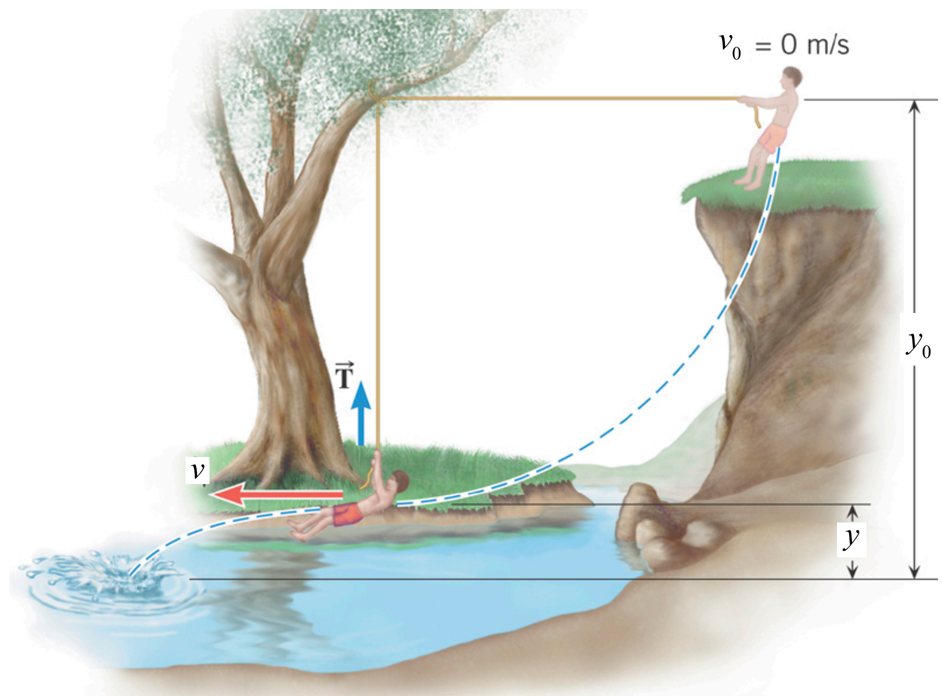
$$gy + \tfrac{1}{2}v^2 = gy_0 + \tfrac{1}{2}v_0^2$$

$$v = \sqrt{2g\left(y_0 - y\right) + v_0^2}$$

$$v = \sqrt{2\left(9.8\,\mathrm{m/s^2}\right)\left(35.0\,\mathrm{m}\right) + \left(38.0\,\mathrm{m/s}\right)^2} = 46.2\,\mathrm{m/s}$$

Conceptual Example: The Favorite Swimming Hole

The person starts from rest, with the rope held in the horizontal position, swings downward, and then lets go of the rope, with no air resistance. Two forces act on him: gravity and the tension in the rope.

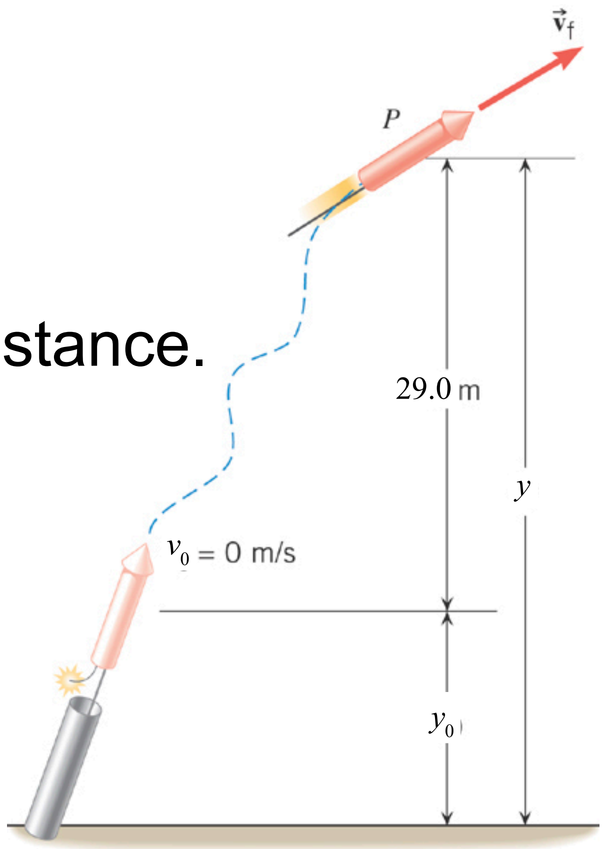


Note: tension in rope is always perpendicular to displacement, and so, does no work on the mass.

The principle of conservation of energy can be used to calculate his final speed.

Example 11 Fireworks

Assuming that the nonconservative force generated by the burning propellant does 425 J of work, what is the final speed of the rocket (m = 0.20kg). Ignore air resistance.



$$E = E_0 + W_{NC}$$

$$W_{\text{NC}} = \left(mgy + \tfrac{1}{2}mv_f^2\right) - \left(mgy_0 + \tfrac{1}{2}mv_0^2\right)$$

$$= mg\left(y - y_0\right) + \tfrac{1}{2}mv^2 + 0$$

$$v^2 = 2W_{\text{NC}}/m - 2g\left(y - y_0\right)$$

$$= 2\left(425\ \text{J}\right)/\left(0.20\ \text{kg}\right) - 2\left(9.81\text{m/s}^2\right)\left(29.0\ \text{m}\right)$$

$$v = 60.7\ \text{m/s}$$

5.6 Power

DEFINITION OF AVERAGE POWER

Average power is the rate at which work is done, and it is obtained by dividing the work by the time required to perform the work.

$$\overline{P} = \frac{\text{Work}}{\text{Time}} = \frac{W}{t}$$

$$\text{joule/s} = \text{watt (W)}$$

Note: 1 horsepower = 745.7 watts

$$\overline{P} = \frac{\text{Change in energy}}{\text{Time}}$$

$$\overline{P} = \frac{W}{t} = \frac{F_x \Delta x}{t} = F_x\left(\frac{\Delta x}{t}\right) = F_x \overline{v}_x$$

5.6 Power

Table of **Human Metabolic Rates**[a]

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

[a]For a young 70-kg male.

THE PRINCIPLE OF CONSERVATION OF ENERGY

Energy can neither be created not destroyed, but can only be converted from one form to another.

Heat energy is the kinetic or vibrational energy of molecules. The result of a non-conservative force is often to remove mechanical energy and transform it into heat.

Examples of heat generation: sliding friction, muscle forces.