Chapter 5

Work and Energy

conclusion

Chaper 5 Review: Work and Energy – Forces and Displacements

Effect of forces acting over a displacement

Work

$$W = (F\cos\theta)\Delta x$$

Kinetic Energy

$$K = \tfrac{1}{2}mv^2$$

Work changes the Kinetic Energy of a mass

Work - Energy Theorem (true always)

$$W = K - K_0$$

Conservative Force	Potential Energy
Gravity	$U_G = mgy$
Ideal Spring	$U_S = \tfrac{1}{2}kx^2$

Non-Conservative Forces doing work

W_{NC} Humans, Friction, Explosions

Work - Energy Theorem (w/potential energy U)

$$W_{NC} = \left(K - K_0\right) + \left(U - U_0\right)$$

All of these quantities are scalars.

(magnitude of a vector is a scalar)

Clicker Question 5.10

A ball is thrown upward with an initial speed v from the roof of a building. An identical ball is thrown downward with the same initial speed v. When the balls reach the ground, how do the kinetic energies of the two balls compare? Ignore air resistance effects.

a) The kinetic energies of the two balls are the same.

b) The first ball has twice the kinetic energy as the second ball.

c) The first ball has one half the kinetic energy as the second ball.

d) The first ball has four times the kinetic energy as the second ball.

e) The first ball has three times the kinetic energy as the second ball.

5.6 Power

DEFINITION OF AVERAGE POWER

Average power is the rate at which work is done, and it is obtained by dividing the work by the time required to perform the work.

$$\bar{P} = \frac{\text{Work}}{\text{Time}} = \frac{W_{\text{NC}}}{t}$$

$$\text{Power units: joule/s} = \text{watt (W)}$$

$$\text{Note: 1 horsepower} = 745.7 \text{ watts}$$

$$\bar{P} = \frac{\Delta E}{t} \quad \Rightarrow \quad \Delta E = \bar{P}\Delta t$$

$$\text{Work - Energy Theorem: } W_{\text{NC}} = \Delta E$$

5.6 Power

Example: A 1.0-hp motor runs for 1 minute.

How much energy has it delivered?

$$P = 1.0 \text{ horsepower} = 745.7 \text{ watts} = 745.7 \text{ J/s}$$

$$\Delta E = P\Delta t = (745.7 \text{ J/s})(60 \text{ s}) = 45 \text{ kJ}$$

Also, relating power to force and motion:

$$\bar{P} = \frac{W}{t} = \frac{F_x \Delta x}{t} = F_x\left(\frac{\Delta x}{t}\right)$$

$$= F_x \bar{v}_x$$

Power = (force) (average velocity)

Clicker Question 5.11

If the amount of energy needed to operate a 100 W light bulb for one minute were used to launch a 2-kg projectile what maximum height could the projectile reach? Ignore air friction. (1 W = 1 J/s)

a) 20 m

b) 50 m

c) 100 m

d) 200 m

e) 300 m

Table of **Human Metabolic Rates**[a]

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

[a]For a young 70-kg male.

6.8 Other Forms of Energy and the Conservation of Energy

THE PRINCIPLE OF CONSERVATION OF ENERGY

Energy can neither be created not destroyed, but can only be converted from one form to another.

Heat energy is the kinetic or vibrational energy of molecules. The result of a non-conservative force is often to remove mechanical energy and transform it into heat.

Examples of heat generation: sliding friction, muscle forces.

Chapter 6

Impulse and Momentum

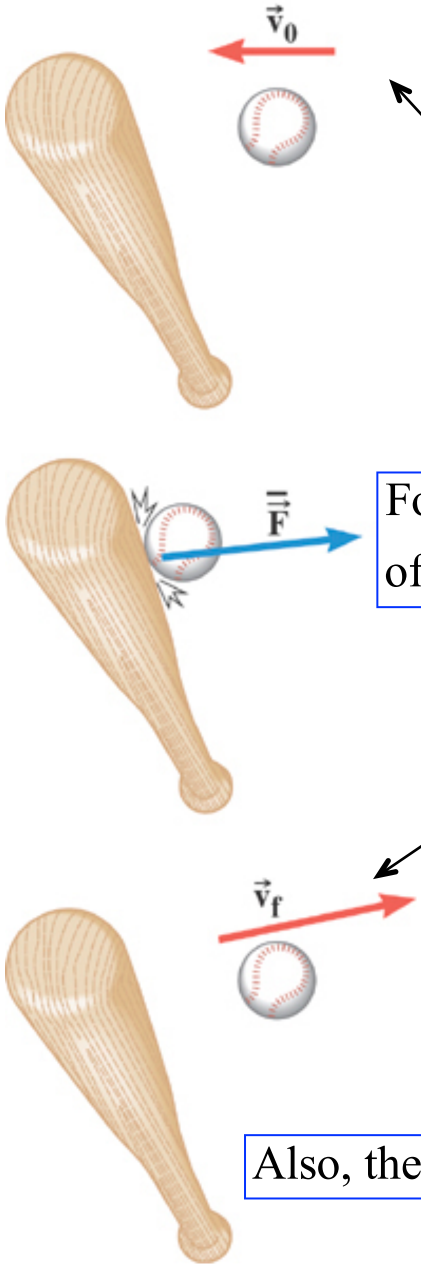
6.1 The Impulse-Momentum Theorem

Chapter 6 is about the COLLISION of TWO masses.
To understand the interaction, both masses must be considered.
Newton's 3rd Law plays a very important part.

Collisions involve two new concepts: Impulse and Momentum.
Impulse concept leads to the Momentum definition.

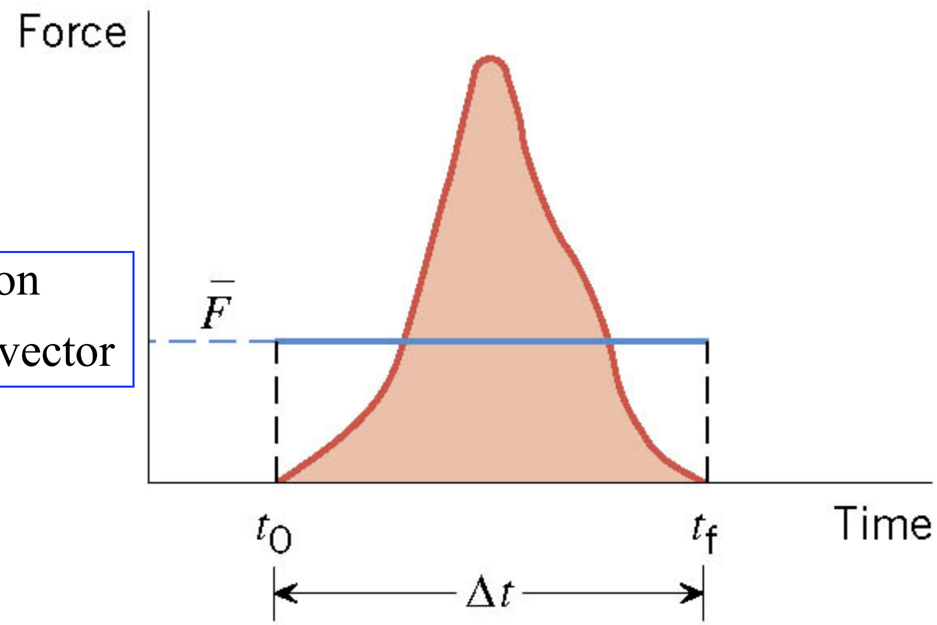
Also applied to two (or more) masses blown apart by an explosion.

6.1 The Impulse-Momentum Theorem



What is the effect of force acting over a short time?

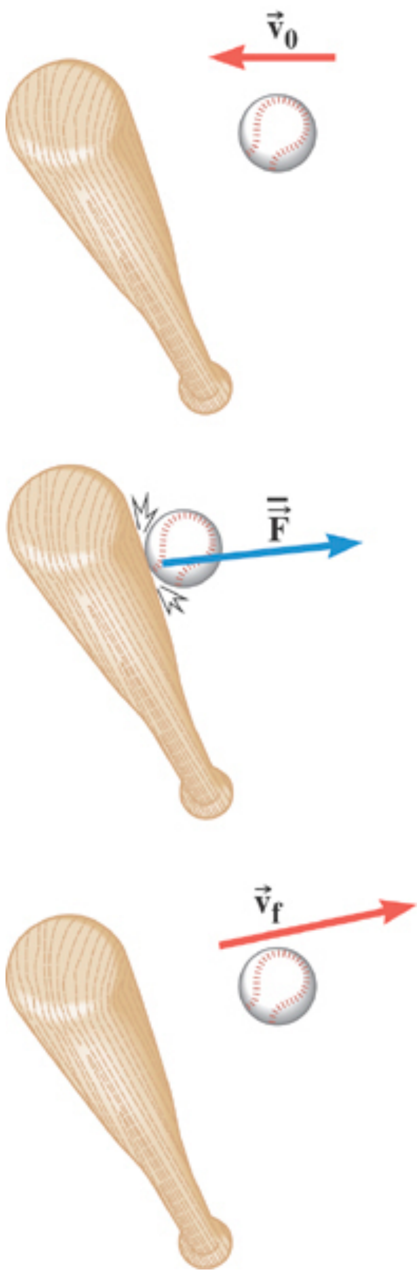
Force changes the direction of the baseball's velocity vector



The bat/ball force is not constant and the mass makes a very short displacement while it acts.

Also, the reaction force changes the magnitude of the bat's velocity vector

6.1 The Impulse-Momentum Theorem



$\vec{F}_{Net}$ acts on the Baseball

$m, \vec{v}$, and $\vec{a}$ are of the Baseball

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_o}{\Delta t}$$

$$\vec{F}_{Net} = m\vec{a}$$

$$\vec{F}_{Net} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

Momentum

$$\vec{p} = m\vec{v}$$

on the BALL — of the BALL

$$\vec{F}_{Net}\,\Delta t = \mathbf{p}_f - \mathbf{p}_i$$

Impulse $\Rightarrow$ changes BALL's momentum

Impulse

$$\vec{F}_{Net}\,\Delta t$$

reaction — Newton's 3rd Law — action

$$(\vec{F}_{Net})_{\text{on the BAT}} = -(\vec{F}_{Net})_{\text{on the BALL}}$$

DEFINITION OF IMPULSE

The impulse of a force is the product of the average force and the time interval during which the force acts:

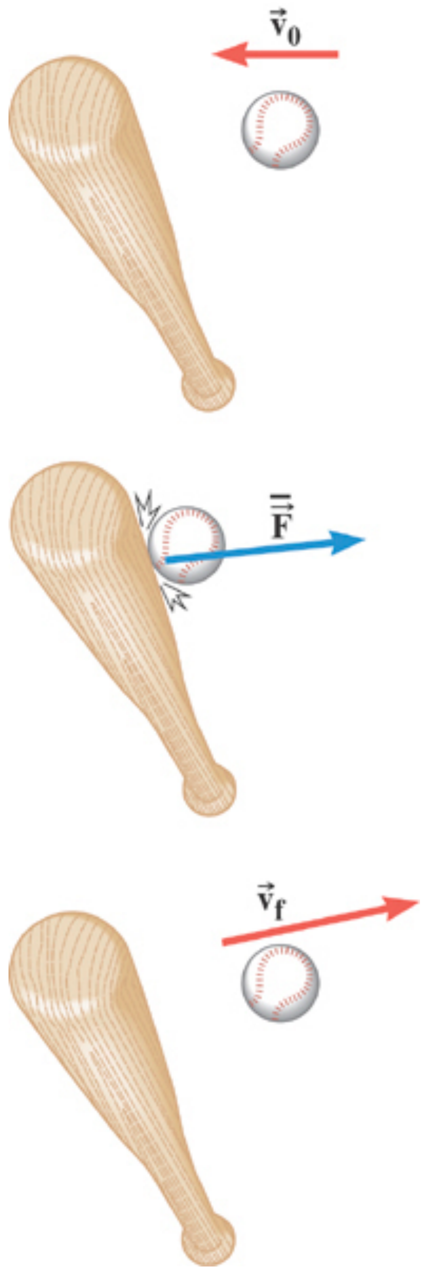
$$\vec{\mathbf{J}} = \overline{\vec{\mathbf{F}}}_{\text{Net}}\,\Delta t$$

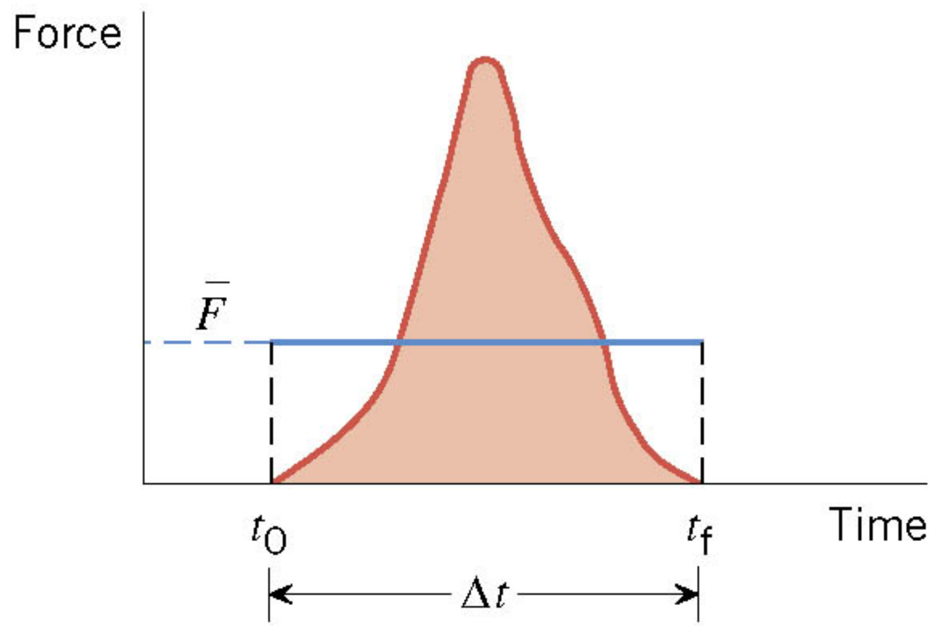
$$\overline{\vec{\mathbf{F}}}_{\text{Net}} = \text{average net force vector}$$

Impulse is a vector quantity and has the same direction as the average force.

$$\text{newton} \cdot \text{seconds}\ (\text{N} \cdot \text{s})$$

6.1 The Impulse-Momentum Theorem





(b)

$$\vec{\mathbf{J}} = \overline{\vec{\mathbf{F}}}\Delta t$$

Impulse

DEFINITION OF LINEAR MOMENTUM

The linear momentum of an object is the product of the object's mass times its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Linear momentum is a vector quantity and has the same direction as the velocity.

$$\text{kilogram} \cdot \text{meter/second (kg} \cdot \text{m/s)}$$

IMPULSE-MOMENTUM THEOREM

When a net force acts on an object, the impulse of this force is equal to the change in the momentum of the object

impulse — final momentum — minus — initial momentum

$$\bar{\vec{\mathbf{F}}}_{\mathrm{Net}}\,\Delta t = m\vec{\mathbf{v}}_{\mathrm{f}} - m\vec{\mathbf{v}}_{\mathrm{i}}$$

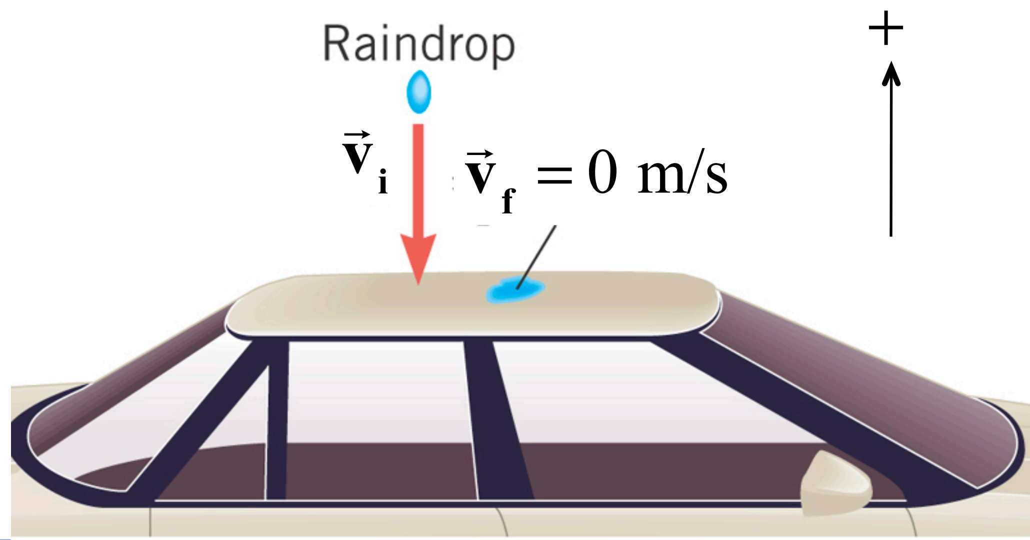
Time averaged force acting on a mass.

Changes the momentum of the mass.

Example 2 A Rain Storm

Rain comes down with a velocity of -15 m/s and hits the roof of a car. The mass of rain per second that strikes the roof of the car is 0.060 kg/s. Assuming that rain comes to rest upon striking the car, find the average force exerted by the rain on the roof.

$$\bar{\vec{\mathbf{F}}}_{\text{Net}}\,\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{i}}$$

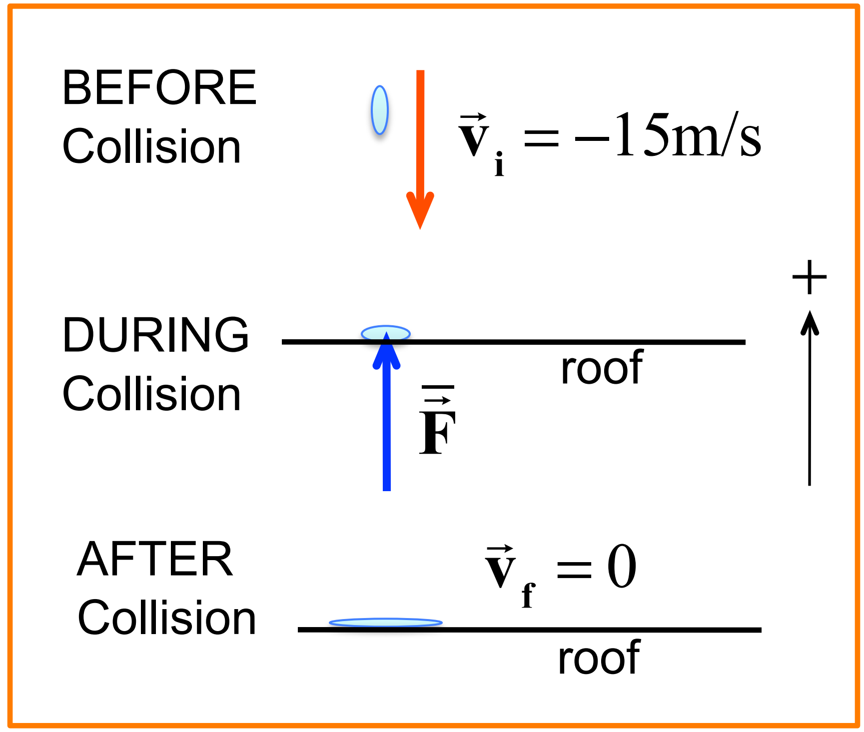


Using this, you will determine the average force on the raindrops.

But, using Newton's 3rd law you can get the average force on the roof .

6.1 The Impulse-Momentum Theorem

Neglecting the raindrop's weight, the average net force on the raindrops caused by the collisions with the roof is obtained.



Impulse of roof on raindrops

Changes momentum of the raindrops

$$\vec{\mathbf{F}}\Delta t = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$$

$\vec{\mathbf{v}}_f = 0$

$$\vec{\mathbf{F}} = -\left(\frac{m}{\Delta t}\right)\vec{\mathbf{v}}_o$$

mass of rain per second $\left(\frac{m}{\Delta t}\right) = 0.060\text{ kg/s}$

$$\vec{\mathbf{F}} = -(0.060\,\text{kg/s})(-15\,\text{m/s})$$
$$= +0.90\text{ N}$$

By Newton's 3rd Law average force of raindrops on the roof is

$$\vec{\mathbf{F}} = -0.90\,\text{N}$$

Clicker Question 6.1 Hailstones versus raindrops

Instead of rain, suppose hail has velocity of –15 m/s and one hailstone with a mass 0.060 kg of hits the roof and bounces off with a velocity of +10 m/s. In the collision, what is the change of the momentum vector of the hailstone?

a) $+0.3\ \mathrm{N \cdot s}$

b) $-0.3\ \mathrm{N \cdot s}$

c) $0.0\ \mathrm{N \cdot s}$

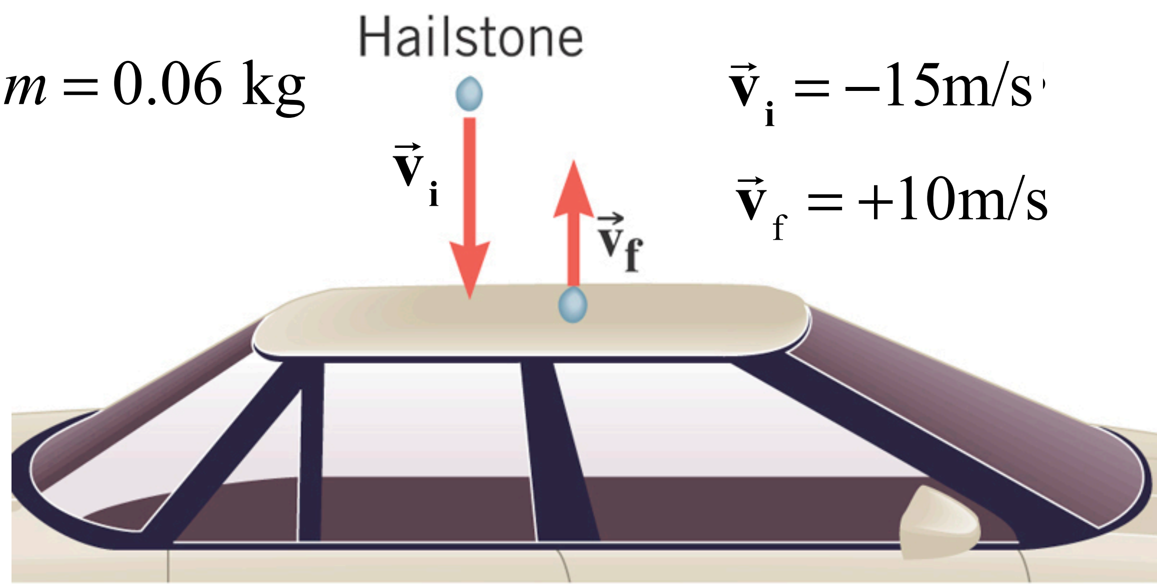
d) $+1.5\ \mathrm{N \cdot s}$

e) $-1.5\ \mathrm{N \cdot s}$

$m = 0.06\ \mathrm{kg}$

$\vec{\mathbf{v}}_i = -15\mathrm{m/s}$

$\vec{\mathbf{v}}_f = +10\mathrm{m/s}$



6.2 The Principle of Conservation of Linear Momentum

WORK-ENERGY THEOREM ⇔ CONSERVATION OF ENERGY

IMPULSE-MOMENTUM THEOREM ⇔ ???

Apply the impulse-momentum theorem to the midair collision between two objects while falling due to gravity.

Distinguish the EXTERNAL forces and INTERNAL forces

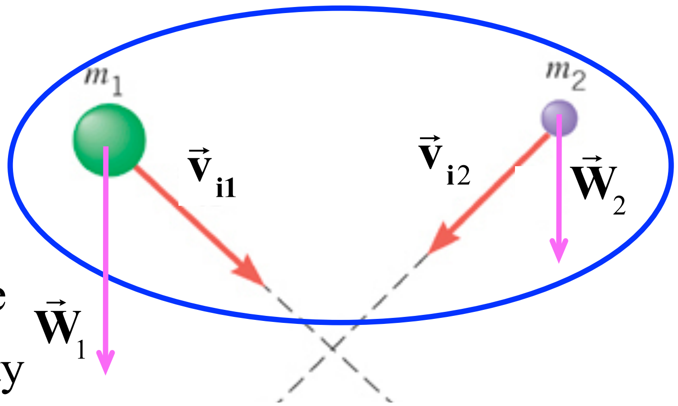
6.2 The Principle of Conservation of Linear Momentum

External forces – Forces exerted on the objects by agents external to the system. Net force changes the velocity (& momentum) of the masses.

Newton's 2nd Law

$\vec{W}$(weight vectors), the external force of gravity

System of two masses



Before the collision

6.2 The Principle of Conservation of Linear Momentum

External forces – Forces exerted on the objects by agents external to the system. Net force changes the velocity (& momentum) of the masses.

Newton's 2[nd] Law

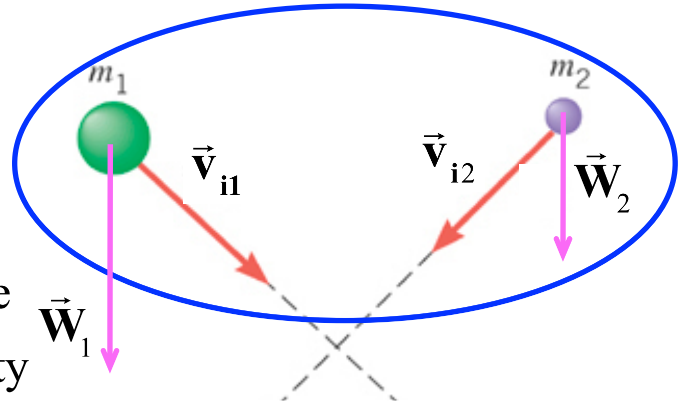
$\vec{\mathbf{W}}$(weight vectors), the external force of gravity

Internal forces – Forces **within the system** that objects exert on each other. These forces have equal magnitudes and opposite directions.

Newton's 3[rd] Law

forces at contact point

System of two masses



Before the collision

also external $\vec{\mathbf{W}}_1$ and $\vec{\mathbf{W}}_2$

During the collision

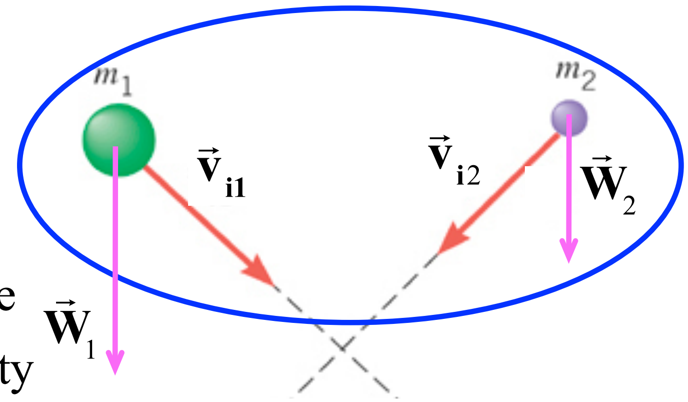
6.2 *The Principle of Conservation of Linear Momentum*

System of two masses

External forces – Forces exerted on the objects by agents external to the system. Net force changes the velocity (& momentum) of the masses.

Newton's 2nd Law

$\vec{\mathbf{W}}$(weight vectors), the external force of gravity



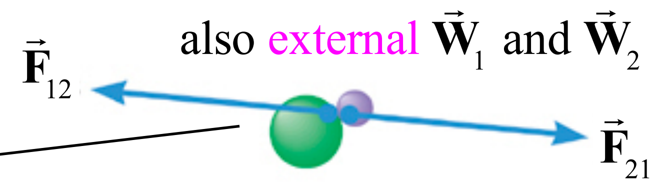
Before the collision

Internal forces – Forces **within the system** that objects exert on each other. These forces have equal magnitudes and opposite directions.

Newton's 3rd Law

forces at contact point

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

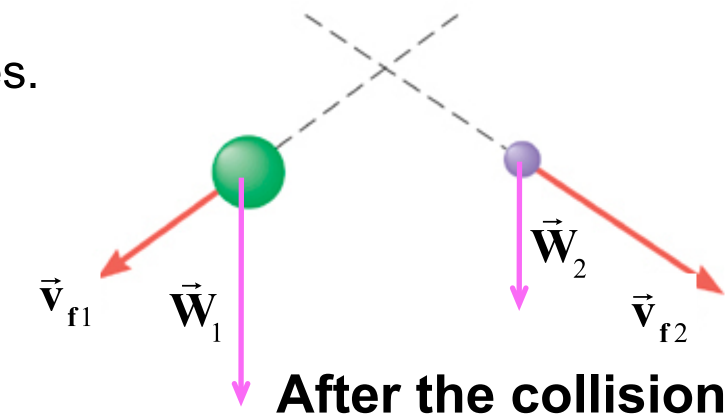


During the collision

External forces – Forces exerted on the objects by agents external to the system. Net force changes the velocity (and momentum) of the masses.

Newton's 2nd Law

$\vec{\mathbf{W}}$(weight vectors), the external force of gravity



After the collision

6.2 The Principle of Conservation of Linear Momentum

During the collision (Δt)

Impulse-Momentum Theorem

Mass 1

Weight of mass 1.

$$\left(\vec{\mathbf{W}}_1 + \vec{\bar{\mathbf{F}}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{i}1}$$

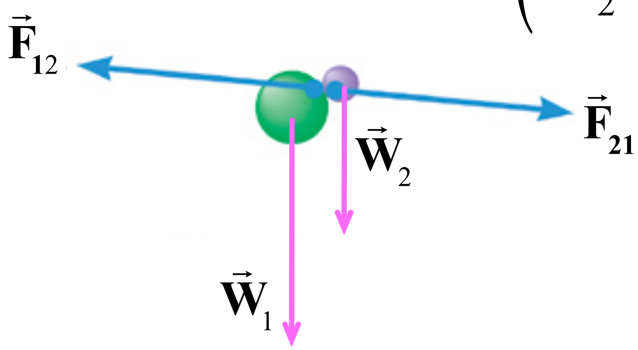
Force on 1 generated by 2

Impulse-Momentum Theorem

Mass 2

Weight of mass 2.

$$\left(\vec{\mathbf{W}}_2 + \vec{\bar{\mathbf{F}}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{i}2}$$



Force on 2 generated by 1

Net effect on the system of two masses $\Rightarrow$ add the equations together

6.2 The Principle of Conservation of Linear Momentum

During the collision (Δt)

Impulse-Momentum Theorem

Weight of mass 1.

Mass 1

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{F}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{f1} - m_1\vec{\mathbf{v}}_{i1}$$

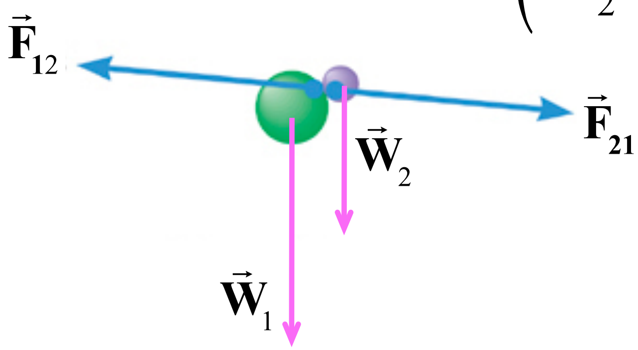
Impulse-Momentum Theorem

Weight of mass 2.

Mass 2

$$\left(\vec{\mathbf{W}}_2 + \vec{\mathbf{F}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{f2} - m_2\vec{\mathbf{v}}_{i2}$$

Force on 1 generated by 2



Force on 2 generated by 1

Net effect on the system of two masses $\Rightarrow$ add the equations together

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{F}}_{12} + \vec{\mathbf{W}}_2 + \vec{\mathbf{F}}_{21}\right)\Delta t = \left(m_1\vec{\mathbf{v}}_{f1} - m_1\vec{\mathbf{v}}_{i1}\right) + \left(m_2\vec{\mathbf{v}}_{f2} - m_2\vec{\mathbf{v}}_{i2}\right)$$

At contact point: $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$

put final values together & initial values together

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = \left(m_1\vec{\mathbf{v}}_{f1} + m_2\vec{\mathbf{v}}_{f2}\right) - \left(m_1\vec{\mathbf{v}}_{i1} + m_2\vec{\mathbf{v}}_{i2}\right)$$

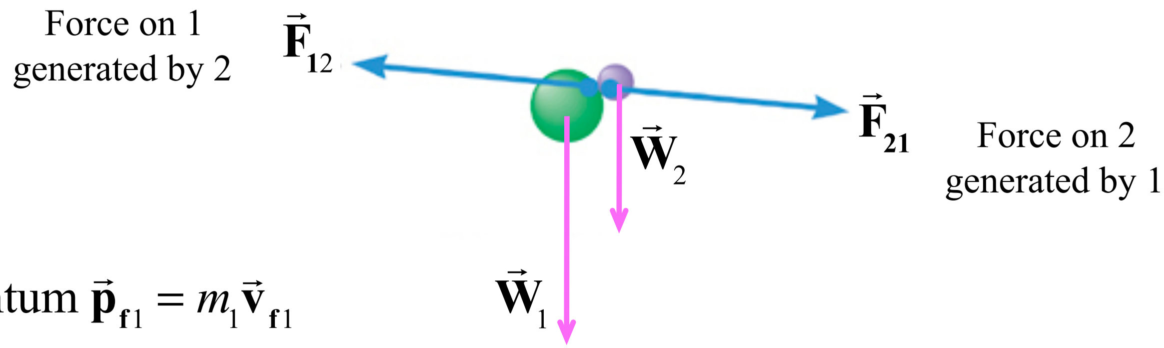
Impulses due only to external forces

Total momentum in the final system

Total momentum in the initial system

6.2 The Principle of Conservation of Linear Momentum

During the collision (Δt)



Using momentum $\vec{\mathbf{p}}_{f1} = m_1\vec{\mathbf{v}}_{f1}$

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = (\vec{\mathbf{p}}_{f1} + \vec{\mathbf{p}}_{f2}) - (\vec{\mathbf{p}}_{i1} + \vec{\mathbf{p}}_{i2})$$

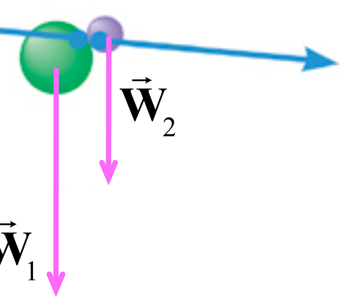
Impulses due only to external forces	Total momentum in the final system	Total momentum in the initial system

Only EXTERNAL forces can change the momentum of a system of masses

If only INTERNAL forces act (as they do in a collision without gravity)

6.2 The Principle of Conservation of Linear Momentum

During the collision (Δt)



Using momentum $\vec{\mathbf{p}}_{f1} = m_1\vec{\mathbf{v}}_{f1}$

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = (\vec{\mathbf{p}}_{f1} + \vec{\mathbf{p}}_{f2}) - (\vec{\mathbf{p}}_{i1} + \vec{\mathbf{p}}_{i2})$$

Impulses due only to external forces	Total momentum in the final system	Total momentum in the initial system

Only EXTERNAL forces can change the Total Momentum of a system of masses

With only INTERNAL forces affecting motion (e.g., if external forces are balanced)

$$0 = (\vec{\mathbf{p}}_{f1} + \vec{\mathbf{p}}_{f2}) - (\vec{\mathbf{p}}_{i1} + \vec{\mathbf{p}}_{i2})$$

$$(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{f}} = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{i}}$$

Final value of total momentum	Initial value of total momentum

If only INTERNAL forces affect motion,
total momentum VECTOR of a system does not change

6.2 The Principle of Conservation of Linear Momentum

If only INTERNAL forces affect the motion,
total momentum VECTOR of a system does not change

$$(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \ldots)_{\mathbf{f}} = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \ldots)_{\mathbf{i}}$$

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system of masses is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

Most Important example

If there are NO external forces affecting the motion,
e.g., gravitational forces are balanced by normal forces,
the total momentum VECTOR of the system is conserved.

Clicker Question 6.2

Two hockey pucks bang into each other on frictionless ice. Each puck has a mass of 0.5 kg, and are moving directly toward each other each with a speed of 12 m/s. What is the total momentum vector of the system of two pucks?

a) $6.0\ \text{N}\cdot\text{s}$

b) $12\ \text{N}\cdot\text{s}$

c) $-6.0\ \text{N}\cdot\text{s}$

d) $-12\ \text{N}\cdot\text{s}$

e) $0.0\ \text{N}\cdot\text{s}$

Clicker Question 6.3

After the pucks collide, what is the total momentum of the system?

a) $6.0\ \text{N}\cdot\text{s}$

b) $12\ \text{N}\cdot\text{s}$

c) $-6.0\ \text{N}\cdot\text{s}$

d) $-12\ \text{N}\cdot\text{s}$

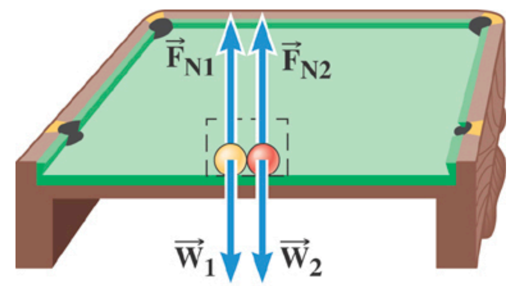
e) $0.0\ \text{N}\cdot\text{s}$

Conceptual Example: Is the Total Momentum Conserved?

Imagine two balls colliding on a billiard table that is friction-free. Use the momentum conservation principle in answering the following questions.

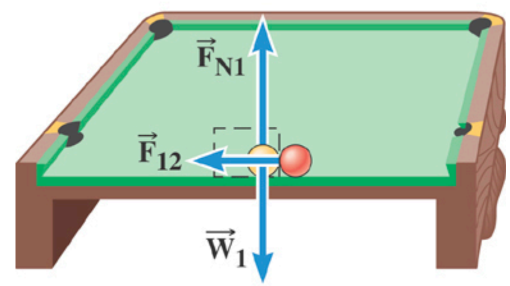
Showing only force vectors, velocity or momentum vectors are not shown.

(a) Is the total momentum of the two-ball system the same before and after the collision?



(*a*)

(b) Answer part (a) for a system that contains only the ball on the left of the two colliding balls.



(*b*)

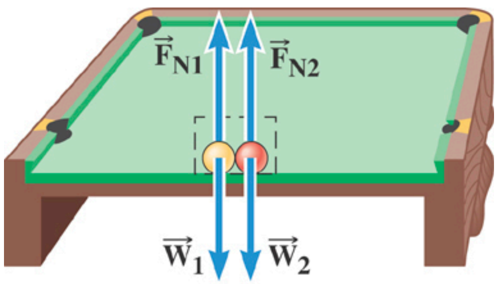
6.2 *The Principle of Conservation of Linear Momentum*

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

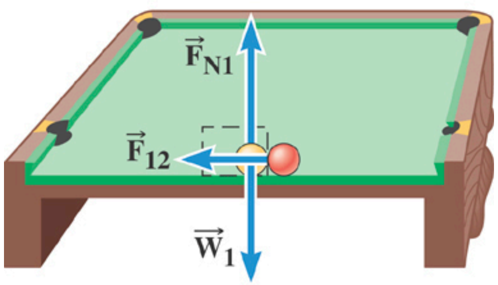
Showing only force vectors, velocity or momentum vectors are not shown.

In the top picture the net external force on the system is zero.



(a)

In the bottom picture the net external force on the system (of only the left billiard ball) is not zero.

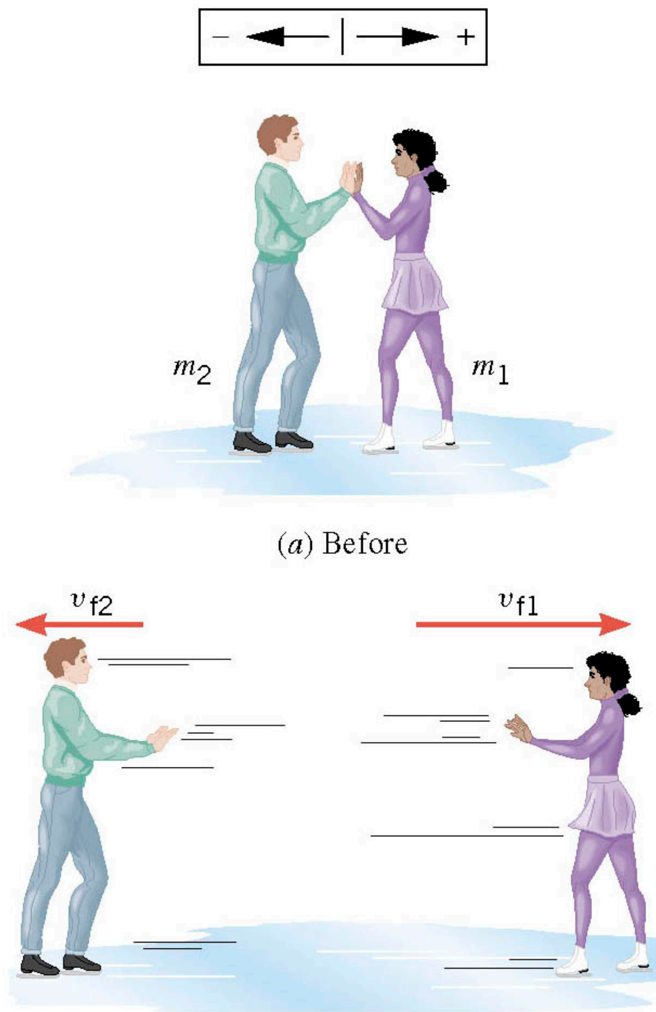


(b)

Example: Ice Skaters

Starting from rest, two skaters push off against each other on ice where friction is negligible.

One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.



(*a*) Before

(*b*) After

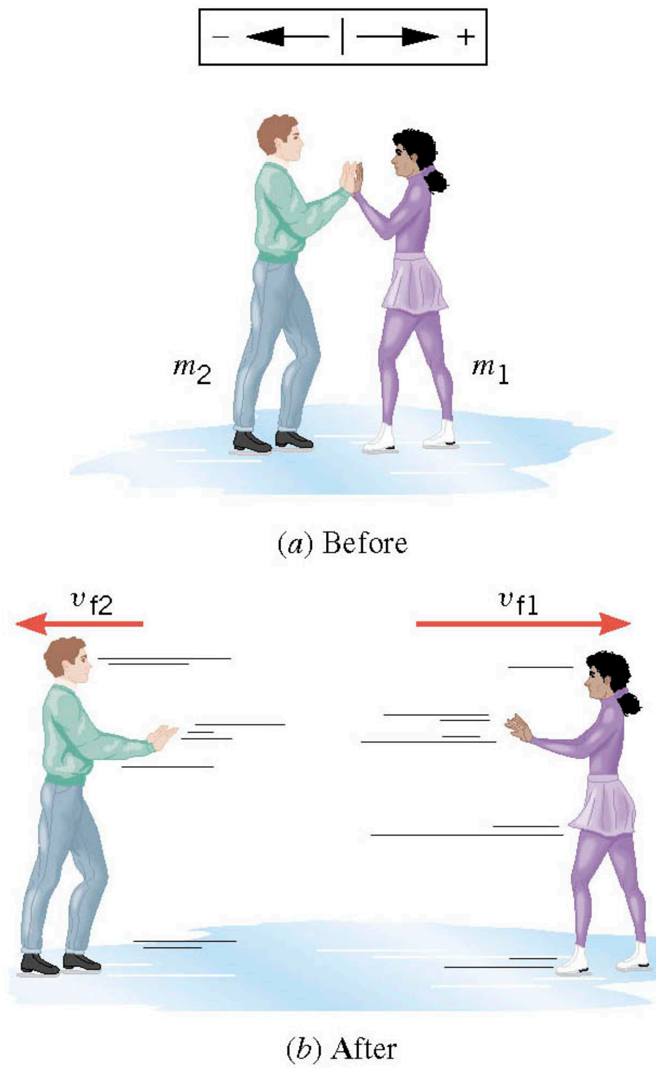
6.2 The Principle of Conservation of Linear Momentum

$$\vec{\mathbf{P}}_{\mathbf{f}} = \vec{\mathbf{P}}_{\mathbf{o}}$$

$$m_1 v_{f1} + m_2 v_{f2} = 0$$

$$v_{f2} = -\frac{m_1 v_{f1}}{m_2}$$

$$v_{f2} = -\frac{(54\text{ kg})(+2.5\text{m/s})}{88\text{ kg}} = -1.5\text{m/s}$$



Applying the Principle of Conservation of Linear Momentum

1. Decide which objects are included in the system.

2. Relative to the system, identify the internal and external forces.

3. Verify that the system is isolated.

4. Set the final momentum of the system equal to its initial momentum. Remember that momentum is a vector.