Chapter 6

Impulse and Momentum

Continued

6.1 The Impulse-Momentum Theorem

Chapter 6 is about the COLLISION of TWO masses.
To understand the interaction, both masses must be considered.
Newton's 3rd Law plays a very important part.

Collisions involve two new concepts: Impulse and Momentum.
Impulse concept leads to the Momentum definition.

Also applied to two (or more) masses blown apart by an explosion.

DEFINITION OF IMPULSE

The impulse of a force is the product of the average force and the time interval during which the force acts:

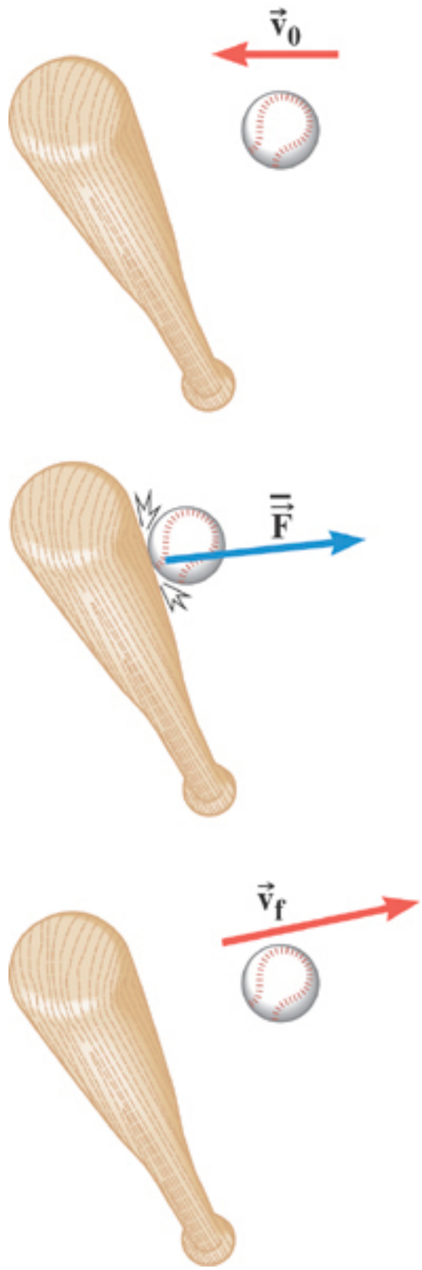
$$\vec{\mathbf{J}} = \overline{\vec{\mathbf{F}}}\Delta t$$

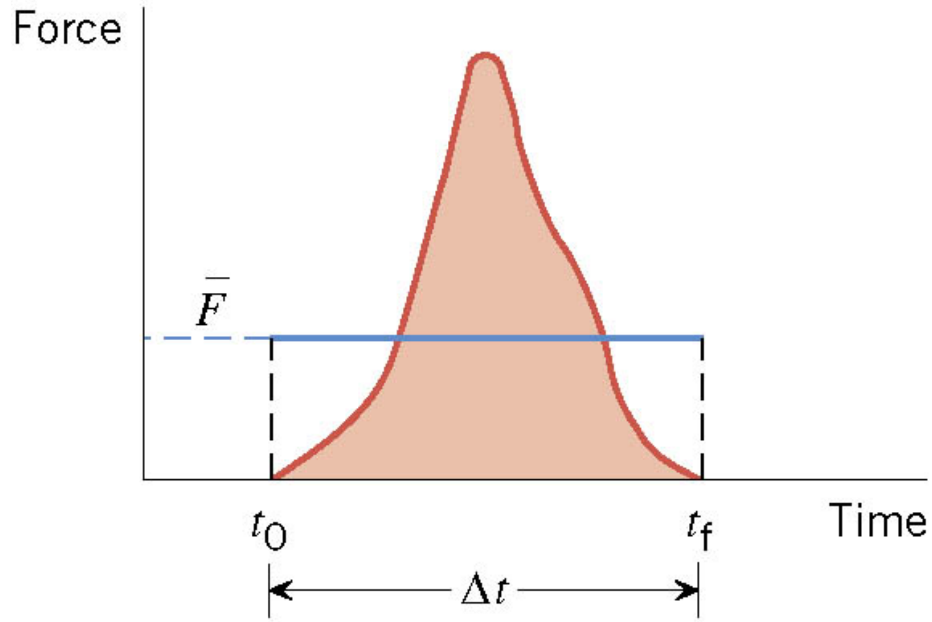
$\overline{\vec{\mathbf{F}}}$ = average force vector

Impulse is a vector quantity and has the same direction as the average force.

$$\text{newton} \cdot \text{seconds}\ (\text{N} \cdot \text{s})$$

6.1 The Impulse-Momentum Theorem





(b)

$$\vec{\mathbf{J}} = \overline{\vec{\mathbf{F}}}\Delta t$$

DEFINITION OF LINEAR MOMENTUM

The linear momentum of an object is the product of the object's mass times its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Linear momentum is a vector quantity and has the same direction as the velocity.

$$\text{kilogram} \cdot \text{meter/second} \ (\text{kg} \cdot \text{m/s})$$

IMPULSE-MOMENTUM THEOREM

When a net force acts on an object, the impulse of this force is equal to the change in the momentum of the object

impulse $\quad \vec{\mathbf{F}}_{\text{Net}} \Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{i}}$ $\quad$ Change in momentum

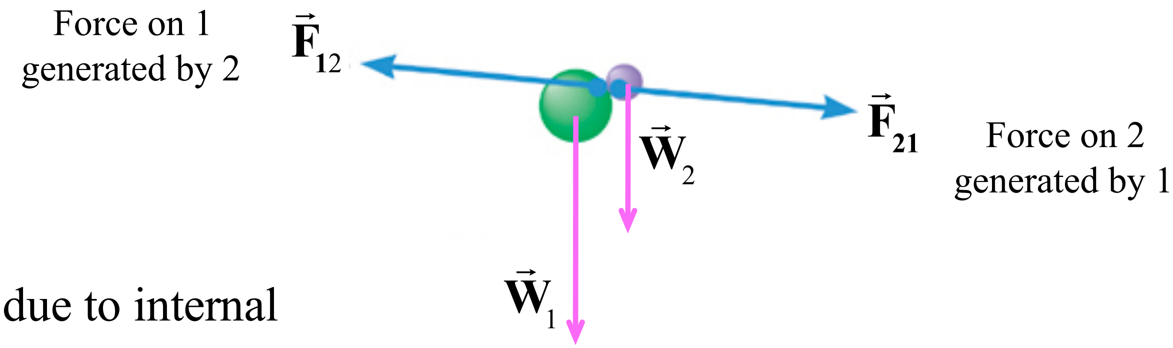
$$= \vec{\mathbf{p}}_{\mathbf{f}} - \vec{\mathbf{p}}_{\mathbf{i}}$$

$$= \Delta\vec{\mathbf{p}}$$

Newton's 2[nd] Law becomes: $\quad \vec{\mathbf{F}}_{\text{Net}} = \dfrac{\Delta\mathbf{p}}{\Delta t}$

6.2 The Principle of Conservation of Linear Momentum

During collision of two masses in free fall



Impulses due to internal forces cancel out in sum

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{f}} - (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{i}}$$

Impulses due only to external forces

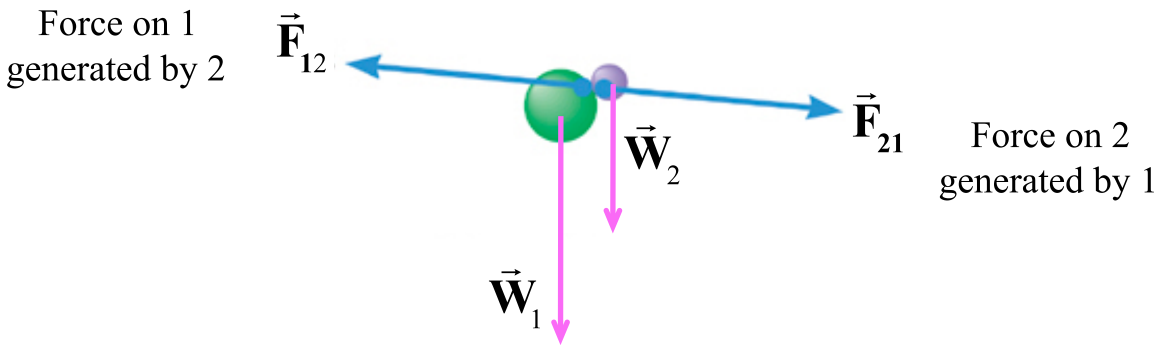
Total momentum in the final system

Total momentum in the initial system

Net EXTERNAL forces will change the Total Momentum of a system of masses

6.2 The Principle of Conservation of Linear Momentum

During collision of two masses in free fall



$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{f}} - (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{i}}$$

With only INTERNAL forces affecting motion (e.g., if external forces are balanced)

$$0 = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{f}} - (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{i}}$$

$$(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{f}} = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{i}}$$

Final value of total momentum

Initial value of total momentum

If only INTERNAL forces affect motion,
total momentum VECTOR of a system does not change

6.2 *The Principle of Conservation of Linear Momentum*

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

Most Important example

If the only external forces are gravitational forces that are balanced by normal forces, the total momentum VECTOR of a system is conserved in a collision.

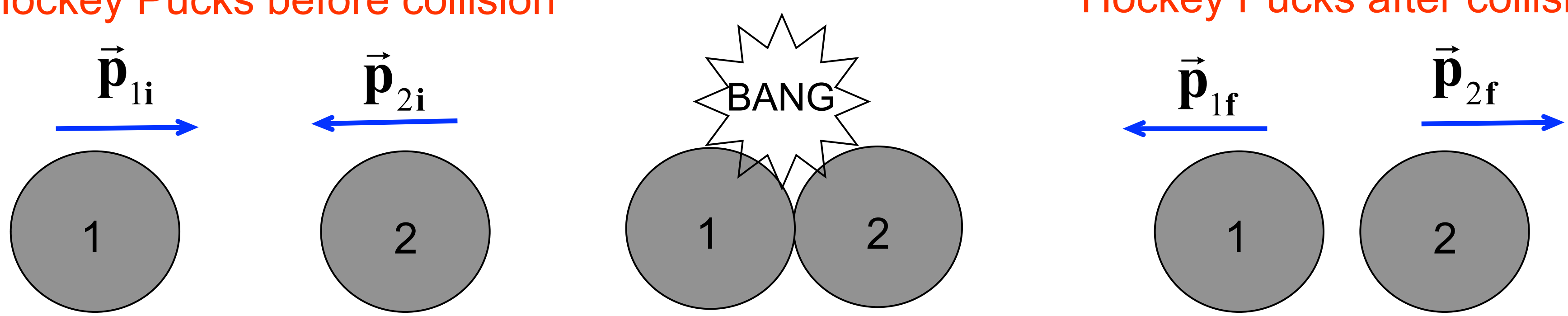
6.2 *The Principle of Conservation of Linear Momentum*

If the only external forces are gravitational forces that are balanced by normal forces, the total momentum VECTOR of a system is conserved in a collision

For Hockey Pucks on the ice, the gravitational force on each puck is balanced by the normal force of the ice.

Hockey Pucks before collision

Hockey Pucks after collision



$(\vec{p}_1 + \vec{p}_2)_i$

Only internal forces act

$(\vec{p}_1 + \vec{p}_2)_f$

Momentum Conservation: $(\vec{p}_1 + \vec{p}_2)_f = (\vec{p}_1 + \vec{p}_2)_i$

Clicker Question 6.2

Two hockey pucks bang into each other on frictionless ice. Each puck has a mass of 0.5 kg, and are moving directly toward each other each with a speed of 12 m/s. What is the total momentum of the system of two pucks?

a) $6.0 \text{ N} \cdot \text{s}$

b) $12 \text{ N} \cdot \text{s}$

c) $-6.0 \text{ N} \cdot \text{s}$

d) $-12 \text{ N} \cdot \text{s}$

e) $0.0 \text{ N} \cdot \text{s}$

Clicker Question 6.2

Two hockey pucks bang into each other on frictionless ice. Each puck has a mass of 0.5 kg, and are moving directly toward each other each with a speed of 12 m/s. What is the total momentum of the system of two pucks?

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d) $-12\ \text{N}\cdot\text{s}$

e) $0.0\ \text{N}\cdot\text{s}$

Clicker Question 6.3

After the pucks collide, what is the total momentum of the system?

a) $6.0\ \text{N}\cdot\text{s}$

b) $12\ \text{N}\cdot\text{s}$

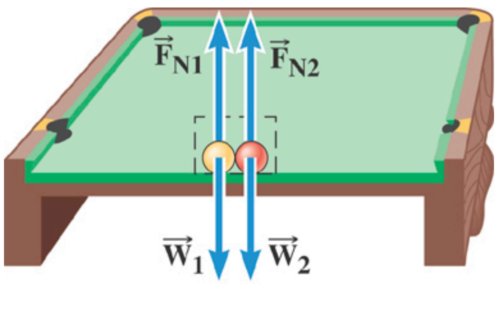
c) $-6.0\ \text{N}\cdot\text{s}$

d) $-12\ \text{N}\cdot\text{s}$

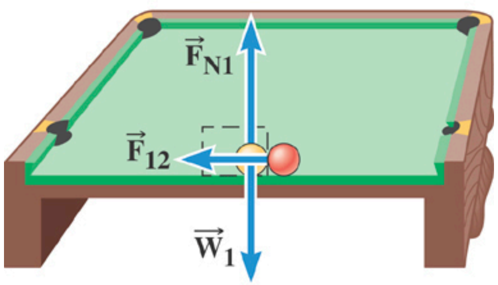
e) $0.0\ \text{N}\cdot\text{s}$

Conceptual Example Is the Total Momentum Conserved?

Imagine two balls colliding on a billiard table that is friction-free. Use the momentum conservation principle in answering the following questions. (a) Is the total momentum of the two-ball system the same before and after the collision? (b) Answer part (a) for a system that contains only one of the two colliding balls.



(*a*)



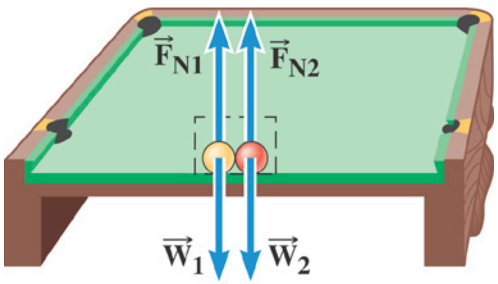
(*b*)

6.2 The Principle of Conservation of Linear Momentum

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

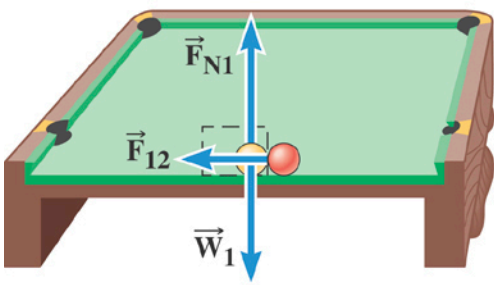
The total linear momentum of an isolated system is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

In the top picture the net external force on the system is zero.



(a)

In the bottom picture the net external force on the system is not zero.

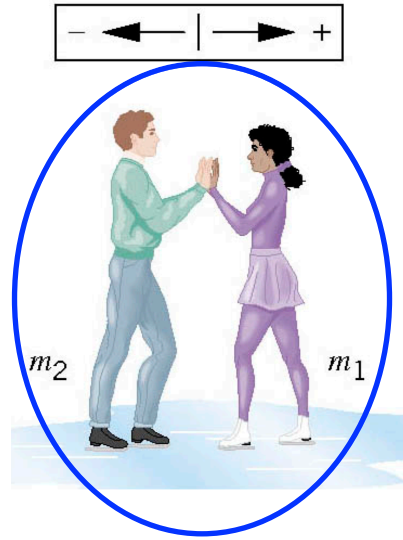


(b)

6.2 The Principle of Conservation of Linear Momentum

Skaters on the ice. Push off is an "explosion".

System of two masses



Before

Net External Force on system of two skaters is zero.

Total momentum is conserved

$$\vec{\mathbf{p}}_{Total,\mathbf{i}} = m_1\vec{\mathbf{v}}_{1\mathbf{i}} + m_2\vec{\mathbf{v}}_{2\mathbf{i}} + \ldots$$

initial momentum sum

$$= 0$$

Total momentum is conserved

$$\vec{\mathbf{p}}_{Total,\mathbf{f}} = \vec{\mathbf{p}}_{Total,\mathbf{i}}$$

$$\vec{\mathbf{p}}_{Total,\mathbf{f}} = 0$$

$$\vec{\mathbf{p}}_{Total,\mathbf{f}} = m_1\vec{\mathbf{v}}_{1\mathbf{f}} + m_2\vec{\mathbf{v}}_{2\mathbf{f}} = 0$$

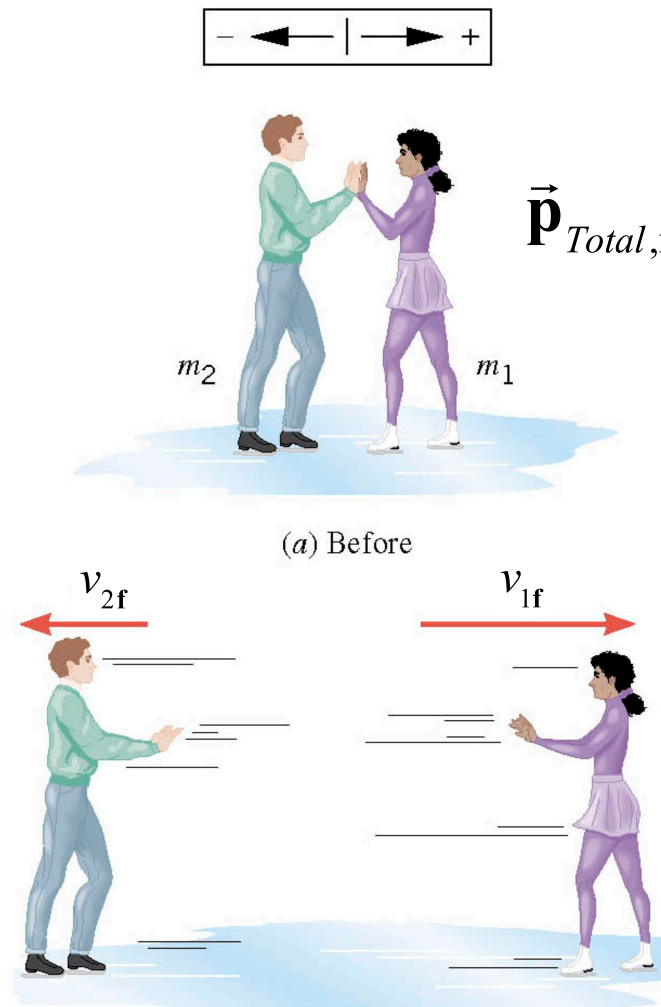
final momentum sum

Momentum vector of mass 2 $m_2\vec{\mathbf{v}}_{2\mathbf{f}} = -m_1\vec{\mathbf{v}}_{1\mathbf{f}}$ is opposite to Momentum vector of mass 1

Example: Ice Skaters

Starting from rest, two skaters push off against each other on ice where friction is negligible.

One is a 54-kg woman and one is a 88-kg man. The woman moves away with a speed of +2.5 m/s. Find the recoil velocity of the man.



(*a*) Before

(*b*) After

$$\vec{\mathbf{p}}_{Total,\mathbf{i}} = 0$$

Momentum Conservation: $\vec{\mathbf{p}}_{Total,\mathbf{f}} = \vec{\mathbf{p}}_{Total,\mathbf{i}}$

$$m_1 v_{1\mathbf{f}} + m_2 v_{2\mathbf{f}} = 0$$

$$v_{2\mathbf{f}} = -\frac{m_1 v_{1\mathbf{f}}}{m_2} = -\frac{(54\text{ kg})(+2.5\text{m/s})}{88\text{ kg}} = -1.5\text{m/s}$$

6.2 The Principle of Conservation of Linear Momentum

Applying the Principle of Conservation of Linear Momentum

1. Decide which objects are included in the system.

2. Relative to the system, identify the internal and external forces.

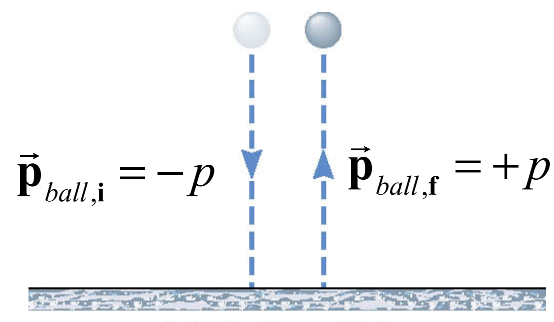
3. Verify that the system is isolated.

4. Set the final momentum of the system equal to its initial momentum.
Remember that momentum is a vector.

6.2 Collisions with the Earth

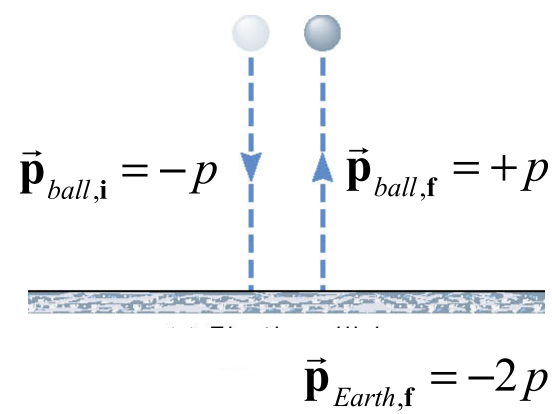
The total linear momentum is conserved when two objects collide, provided they constitute an isolated system.

Ball's momentum is NOT conserved



In this collision with the earth, the ball alone is not an isolated system. The ball's y-component of momentum changes in the collision from $-p$ to $+p$.

In the collision with the earth, the ball and the earth constitute an isolated system. After collision, what is the y-component of momentum for the earth?



$$\text{initial: } \vec{\mathbf{p}}_{Total,\mathbf{i}} = \vec{\mathbf{p}}_{ball,\mathbf{i}} + \vec{\mathbf{p}}_{earth,\mathbf{i}} = -p + 0$$

$$\text{final: } \vec{\mathbf{p}}_{Total,\mathbf{f}} = \vec{\mathbf{p}}_{ball,\mathbf{f}} + \vec{\mathbf{p}}_{earth,\mathbf{f}} = +p + \vec{\mathbf{p}}_{earth,\mathbf{f}}$$

<u>Momentum Conservation</u>

$$\vec{\mathbf{p}}_{Total,\mathbf{i}} = \vec{\mathbf{p}}_{Total,\mathbf{f}}$$

$$-p = +p + \vec{\mathbf{p}}_{earth,\mathbf{f}}$$

$$\vec{\mathbf{p}}_{earth,\mathbf{f}} = -2p$$

Due to the large mass of the earth, this momentum results in an imperceptible change in the earth's velocity.

6.2 Relationship between momentum and kinetic energy

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \quad \text{magnitude of momentum } p = mv$$

$$p^2 = m^2v^2; \qquad \frac{p^2}{2m} = \tfrac{1}{2}\frac{m^2v^2}{m} = \tfrac{1}{2}mv^2 \quad \text{kinetic energy} \quad \boxed{K = \frac{p^2}{2m}}$$

Compare the kinetic energies of a car, $m_C = 2.00\times10^3\,\text{kg},\ v_C = 30.0\,\text{m/s}$,
And the earth, $m_E = 6.00\times10^{24}\,\text{kg}$, with the same momentum as the car.

$$p_E = p_C = m_C v_C = (2.00\times10^3\,\text{kg})(30.0\,\text{m/s}) = 6.00\times10^4\,\text{kg}\cdot\text{m/s}$$

$$K_C = \frac{p_C^2}{2m_C} = \frac{(6.00\times10^4\,\text{kg}\cdot\text{m/s})^2}{4.00\times10^3\,\text{kg}} = 9.00\times10^5\,\text{J}$$

$$K_E = \frac{p_E^2}{2m_E} = \frac{(6.00\times10^4\,\text{kg}\cdot\text{m/s})^2}{6.00\times10^{24}\,\text{kg}} = 6.00\times10^{-16}\,\text{J} \quad \approx \text{zero}$$

The Earth can absorb a significant momentum, but it absorbs zero kinetic energy.

On ice, a puck hits a wall. The speed of puck hitting the wall and the speed coming off the wall are measured to be the same.

Clicker Question 6.4

Which statement below is true about this collision?

a) momentum of the puck is conserved

b) the system consists of the puck before and after the collision

c) kinetic energy is not conserved in the collision

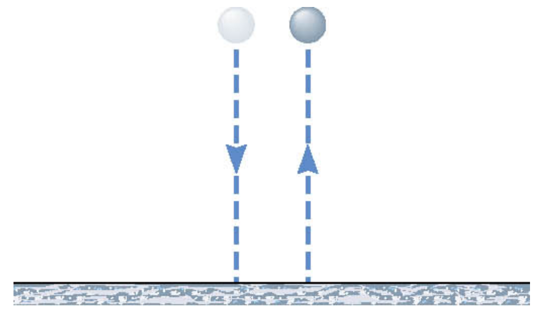
d) total energy is not conserved in the collision

e) momentum is conserved in a system containing the earth and puck.

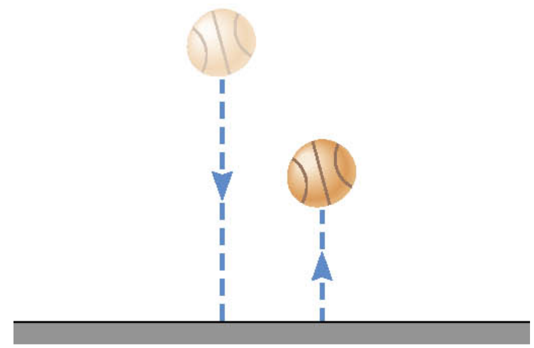
6.2 *Elastic and Inelastic collisions*

Elastic collision -- One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision.

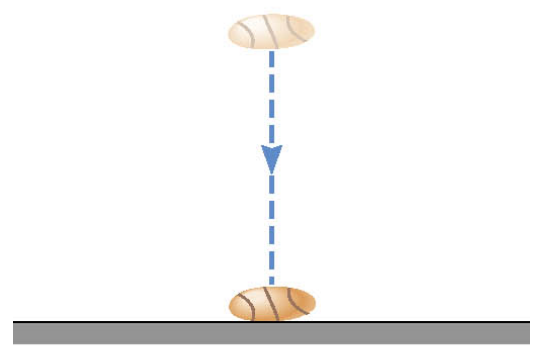
Inelastic collision -- One in which the total kinetic energy of the system after the collision is *not* equal to the total kinetic energy before the collision; if the objects stick together after colliding, the collision is said to be completely inelastic.



(a) Elastic collision



(b) Inelastic collision

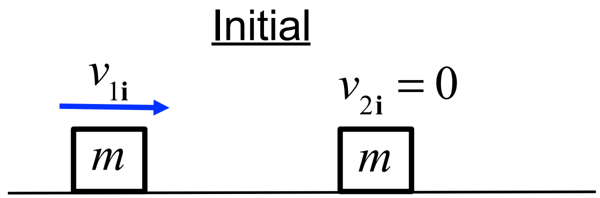


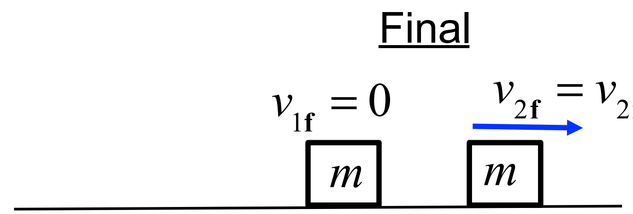
(c) Completely inelastic collision

6.3 Collisions in One Dimension

Elastic collisions $(x-\text{components of the velocities})$

Equal masses:





$(m_1 = m_2 = m)$ masses cancel in both equations

Momentum conservation: $v_{1\mathrm{i}} = v_{1\mathrm{f}} + v_{2\mathrm{f}}$

Energy conservation: $v_{1\mathrm{i}}^2 = v_{1\mathrm{f}}^2 + v_{2\mathrm{f}}^2$

$$v_{1\mathrm{f}}^2 + v_{2\mathrm{f}}^2 + 2v_{1\mathrm{f}}v_{2\mathrm{f}} = v_{1\mathrm{f}}^2 + v_{2\mathrm{f}}^2$$

$$2v_{1\mathrm{f}}v_{2\mathrm{f}} = 0$$

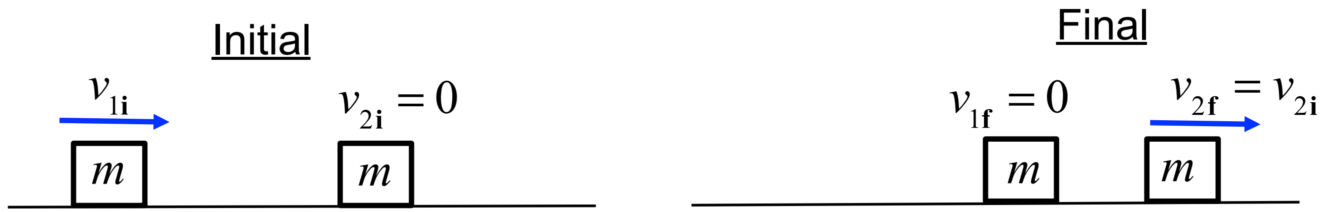
$$v_{1\mathrm{f}} = 0; \quad v_{2\mathrm{f}} = v_{1\mathrm{i}}$$

Incoming mass stops, target mass gets initial momentum

6.3 Collisions in One Dimension

Elastic collisions

Equal masses:



$(m_1 = m_2 = m)$ masses cancel in both equations

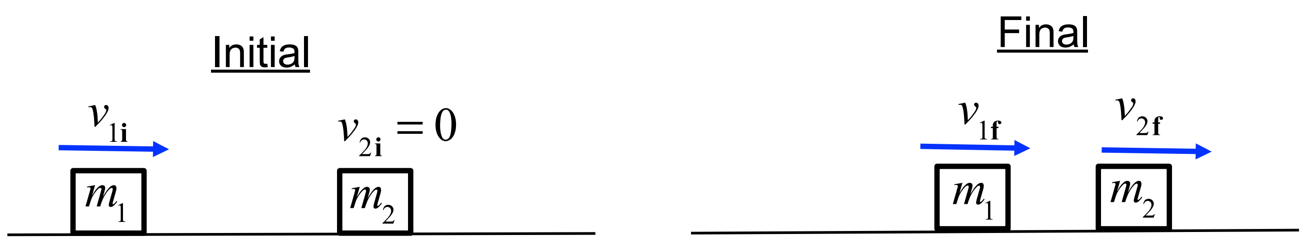
Momentum conservation: $v_{1i} = v_{1f} + v_{2f}$

Energy conservation: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

$$v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f} = v_{1f}^2 + v_{2f}^2$$
$$2v_{1f}v_{2f} = 0$$
$$v_{1f} = 0; \quad v_{2f} = v_{1i}$$

Incoming mass stops, target mass gets initial momentum

Unequal masses:



Momentum conservation: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \Rightarrow \quad v_{1i}^2 = v_{1f}^2 + 2\frac{m_2}{m_1} v_{1f} v_{2f} + \left(\frac{m_2}{m_1}\right)^2 v_{2f}^2$

Energy conservation: $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \Rightarrow \quad v_{1i}^2 = v_{1f}^2 + \frac{m_2}{m_1} v_{2f}^2$

6.3 Collisions in One Dimension

Elastic collisions with arbitrary masses (solve for final velocities)

Unequal masses:

Momentum conservation: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \Rightarrow \quad v_{1i}^2 = v_{1f}^2 + 2\frac{m_2}{m_1} v_{1f} v_{2f} + \left(\frac{m_2}{m_1}\right)^2 v_{2f}^2 \qquad (1)$

Energy conservation: $\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \quad \Rightarrow \quad v_{1i}^2 = v_{1f}^2 + \frac{m_2}{m_1} v_{2f}^2 \qquad (2)$

$$(1) = (2) \qquad \tfrac{m_2}{m_1} v_{2f}^2 = 2\tfrac{m_2}{m_1} v_{1f} v_{2f} + \left(\tfrac{m_2}{m_1}\right)^2 v_{2f}^2$$

$$v_{2f}^2 = 2 v_{1f} v_{2f} + \tfrac{m_2}{m_1} v_{2f}^2$$

$$v_{2f} = 2 v_{1f} + \tfrac{m_2}{m_1} v_{2f}$$

$$v_{1f} = \tfrac{1}{2}\left[1 - \tfrac{m_2}{m_1}\right] v_{2f} = \left[\tfrac{m_1 - m_2}{2 m_1}\right] v_{2f} \quad \Rightarrow \quad v_{2f} = \left[\frac{2 m_1}{m_1 + m_2}\right] v_{1i} \qquad (3)$$

Momentum conservation with (3) $\quad v_{1i} = v_{1f} + \tfrac{m_2}{m_1} v_{2f} \Rightarrow v_{1f} + \tfrac{m_2}{m_1}\left[\tfrac{2 m_1}{m_1 + m_2}\right] v_{1i}$

$$v_{1f} = \left(1 - \tfrac{2 m_2}{m_1 + m_2}\right) v_{1i} \quad \Rightarrow \quad v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} \qquad (4)$$

6.3 Collisions in One Dimension

Elastic collisions with arbitrary masses

$$v_{1\mathbf{f}} = \frac{m_1 - m_2}{m_1 + m_2} v_{1\mathbf{i}} \quad \text{and} \quad v_{2\mathbf{f}} = \frac{2m_1}{m_1 + m_2} v_{1\mathbf{i}}$$

Equal mass solution is here too

6.3 Collisions in One Dimension

Elastic collisions with arbitrary masses $(x-\text{components of the velocities})$

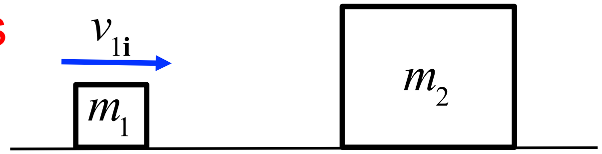
$$v_{1\mathrm{f}} = \frac{m_1 - m_2}{m_1 + m_2} v_{1\mathrm{i}} \quad \text{and} \quad v_{2\mathrm{f}} = \frac{2m_1}{m_1 + m_2} v_{1\mathrm{i}}$$

Little mass hits big mass

$m_2 >> m_1$

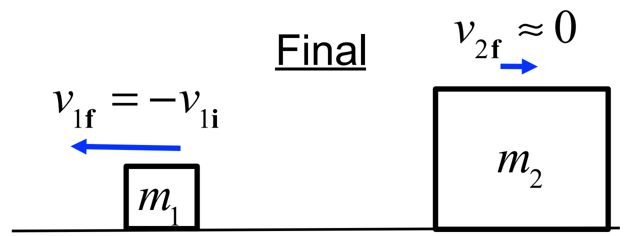
Let $m_1 \to 0$

Initial



Bounces back

Final



$$v_{1\mathrm{f}} = -v_{1\mathrm{i}} \quad \text{and} \quad v_{2\mathrm{f}} \approx 0$$

6.3 Collisions in One Dimension

Elastic collisions with arbitrary masses

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \quad \text{and} \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

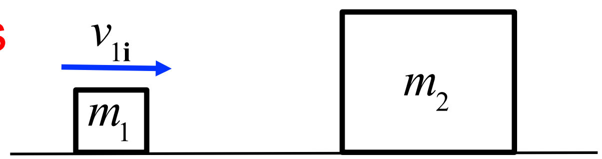
Equal mass solution is here too

Little mass hits big mass

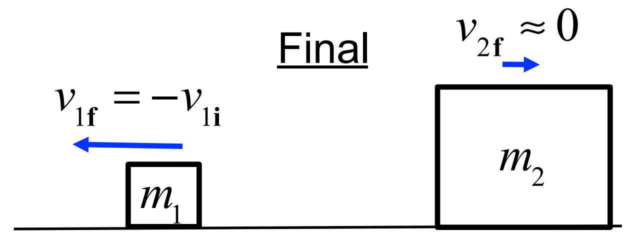
$m_2 >> m_1$

Let $m_1 \to 0$

Initial



Final



Bounces back

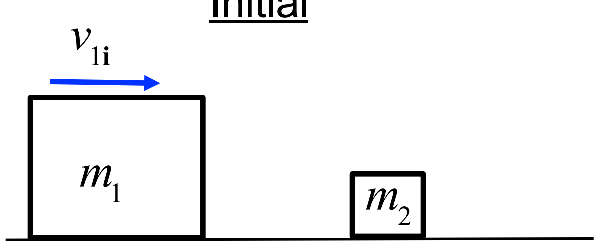
$$v_{1f} = -v_{1i} \quad \text{and} \quad v_{2f} \approx 0$$

Big mass hits little mass

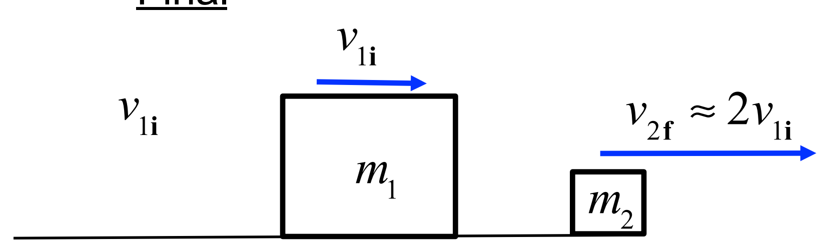
$m_1 >> m_2$

Let $m_1 \to \infty$

Initial



Final



Small mass ejected at twice the speed

$$v_{1f} \approx v_{1i} \quad \text{and} \quad v_{2f} \approx 2v_{1i}$$