Exam 3
Solutions

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Centripetal acceleration: $a_c = \dfrac{v^2}{r} \Rightarrow v = \sqrt{a_c r}$

Period of motion: $T = \dfrac{2\pi r}{v}$

$$T = \frac{2\pi r}{\sqrt{a_c r}} = \frac{2\pi\sqrt{r}}{\sqrt{a_c}} = \frac{2\pi\sqrt{50\,\mathrm{m}}}{\sqrt{0.548\,\mathrm{m/s^2}}} = 60.0\,\mathrm{s}$$

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2. The mass and radius of the moon are 7.40×10^{22} kg and 1.70×10^{6} m, respectively. What is the weight of a 1.0-kg object on the surface of the moon?

$$g_{\text{moon}} = \frac{GM_{\text{moon}}}{R_{\text{moon}}^2} = \frac{(6.67\times10^{-11}\,\text{Nm}^2/\text{kg}^2)(7.40\times10^{22}\,\text{kg})}{(1.70\times10^{6}\,\text{m})^2} = 1.71\,\text{N/kg}$$

$$W = mg_{\text{moon}} = 1\,\text{kg}(1.71\,\text{N/kg}) = 1.71\,\text{N}$$

Clicker Question 13.3

3. A spaceship is in orbit around the earth at an altitude of 19,290 km. Which one of the following statements best explains why an astronaut experiences "weightlessness"?

A) The centripetal force of the earth on the astronaut in orbit is zero newtons.
B) The pull of the earth on the spaceship is canceled by the pull of the other planets.
C) The spaceship is in free fall so its floor cannot press upward on the astronaut.
D) The force decreases as the inverse square of the distance from the earth's center.
E) The force of the earth on the spaceship and the force of the spaceship on the earth cancel because they are equal in magnitude but opposite in direction.

4. A fan rotating with an initial angular velocity of 1000 rev/min is switched off. In 2 seconds, the angular velocity decreases to 200 rev/min. Assuming the angular acceleration is constant, how many revolutions does the blade undergo during this time?

$2\,\mathrm{s} = \frac{1}{30}\,\mathrm{min}$ (OR convert all angular velocities to rev/s or rad/s)

$$\boxed{\begin{aligned} \Delta\theta &= \tfrac{1}{2}(\omega + \omega_0)t \\ &= (600\,\mathrm{rev/min})(\tfrac{1}{30}\,\mathrm{min}) \\ &= 20.0\ \mathrm{revolutions} \end{aligned}}$$

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$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{-800\,\mathrm{rev/min}}{(\frac{1}{30}\,\mathrm{min})} = -24{,}000\ \mathrm{rev/min^2}$$

Get the same answer !

5. A hollow sphere of radius 0.25 m is rotating about an axis that passes through its center. The mass of the sphere is 3.8 kg. A constant net torque is applied to the sphere and 13.4 J of work is required to bring the sphere to a stop. What was the initial angular speed? Note: moment of inertia of a hollow sphere, $I_S = \frac{2}{3} MR^2$.

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$$\boxed{\begin{aligned} W &= \Delta K \qquad K_0 = \tfrac{1}{2} I\omega_0^2, \quad K = 0 \text{ (final kinetic energy)} \\ &= \tfrac{1}{2} I\omega_0^2 \qquad (W \ \underline{\text{and}}\ \Delta K \text{ are negative}) \\ &= \tfrac{1}{3} MR^2\omega_0^2 \\ \omega_0 &= \sqrt{3W / MR^2} = \sqrt{3(13.4\,\mathrm{J})/(3.8\,\mathrm{kg})(0.25\,\mathrm{m})^2} = 13.0\,\mathrm{rad/s} \end{aligned}}$$

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$$\omega_0 = \sqrt{3W / MR^2} = \sqrt{3(13.4\,\text{J})/(3.8\,\text{kg})(0.25\,\text{m})^2} = 13.0\,\text{rad/s}$$

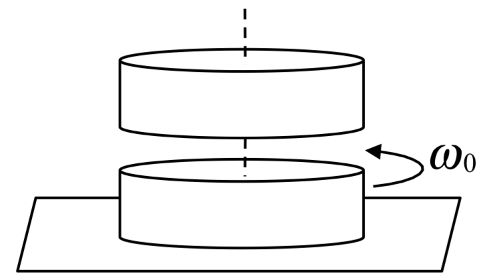
6. A string is wrapped around a pulley of radius 0.20 m and moment of inertia 0.40 kg · m^2. The string is pulled with a force of 28.0 N. What is the magnitude of the resulting angular acceleration of the pulley?

$$\alpha = \frac{\tau}{I} \quad \text{from: } \tau = I\alpha$$

$$= \frac{Fr}{I} = \frac{(28\,\text{N})(0.20\,\text{m})}{0.40\,\text{kg}\cdot\text{m}^2} = 14 \text{ rad/s}^2$$

7. A solid disk with a mass of 0.50 kg is rotating on a frictionless surface with an angular speed of 15.0 rad/s. Another disk just above the first with the same radius and a mass of 1.00 kg is dropped onto the lower disk. Kinetic friction between the disks brings both disks to what common angular speed?

No external torques – angular momentum conserved



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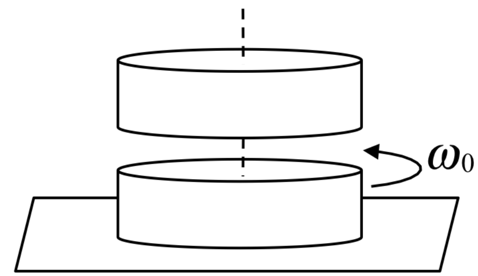

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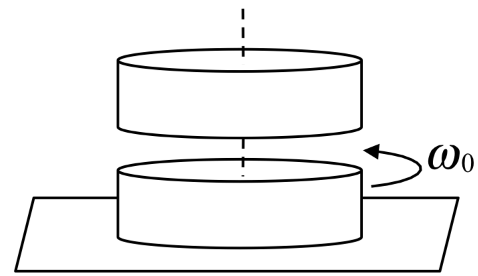

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initial: $L_0 = I_0\omega_0$ final: $L = (I_0 + 2I_0)\omega$

angular momentum conservation: $L_0 = L$

$$I_0\omega_0 = (I_0 + 2I_0)\omega$$

$$\omega = I_0\omega_0 / (I_0 + 2I_0)$$

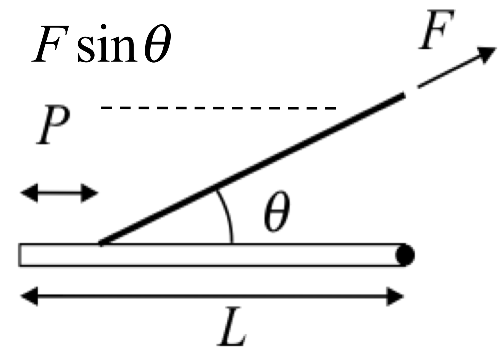
$$= \omega_0 / 3$$

$$= 5\,\mathrm{rad/s}$$

8. A board in equilibrium has length of 25 m is pivoted about one end. A rope tied a distance of 5 m from the other end makes an angle of 15° with the board and is pulled with a force of 540 N. What is the mass of the board?

Equilibrium – torques must sum to zero

$\boxed{\tau = rF\sin\theta \qquad \text{(+ cntr-clockwise)(– clockwise)}}$



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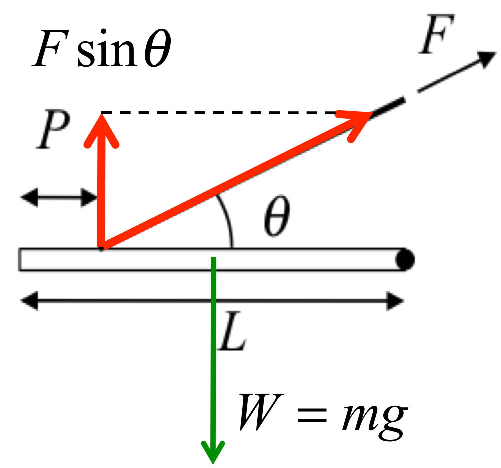

$$\vec{\tau}_{\text{Net}} = \vec{\tau}_1 + \vec{\tau}_2 = 0 \quad \Rightarrow \quad \vec{\tau}_1 = -\vec{\tau}_2$$

$$\vec{\tau}_1 = (L/2)mg \qquad (+\text{ cntr-clockwise})$$

$$\vec{\tau}_2 = -(L-P)F\sin\theta \quad (-\text{ clockwise})$$

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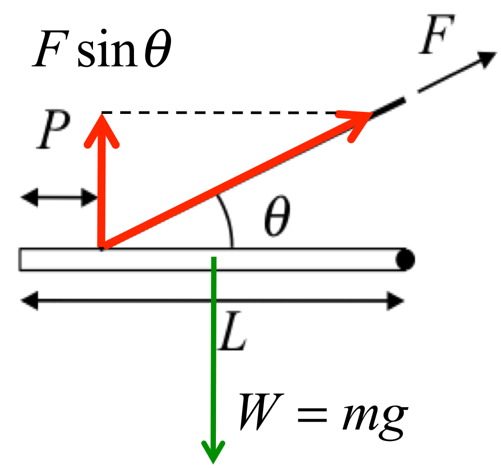

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$$(L/2)mg = (L-P)F\sin\theta$$

$$m = \frac{(L-P)F\sin\theta}{gL/2} = \frac{2(20\,\text{m})(540\,\text{N})\sin(15°)}{(9.81\,\text{N/kg})(25\,\text{m})} = 22.8\text{ kg}$$

9. Young's modulus of nylon is 3.70×10^9 N/m^2. A force of 6.00×10^5 N is applied to a 1.50-m length of nylon of cross sectional area 0.250 m^2. By what amount does the nylon stretch?

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$\Delta L = \frac{FL}{YA} = \frac{(6.00\times 10^5\,\text{N})(1.5\,\text{m})}{(3.70\times 10^9\,\text{Pa})(0.25\,\text{m}^2)} = 0.000973\,\text{m}$$

$$= 0.973\,\text{mm}$$

10. A force of 250 N is applied to a hydraulic jack piston that is 0.02 m in diameter. A mass of 1400 kg can be lifted by the jack. Ignoring any difference in height between the pistons, the piston that supports the load has what diameter?

Hydraulic pressure on pistons the same

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow A_2 = \left(\frac{F_2}{F_1}\right)A_1$$

Area in terms of diameter

$$A_1 = \tfrac{\pi}{4} d_1^2$$

$$A_2 = \tfrac{\pi}{4} d_2^2$$

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Area in terms of diameter

$$A_1 = \tfrac{\pi}{4}d_1^2$$

$$A_2 = \tfrac{\pi}{4}d_2^2$$

$$d_2^2 = \left(\frac{F_2}{F_1}\right)d_1^2$$

$$d_2 = \sqrt{\frac{mg}{F_1}}d_1 = \sqrt{\frac{(1400\,\mathrm{kg})(9.81\,\mathrm{N/kg})}{250\,\mathrm{N}}}(0.02\,\mathrm{m}) = 0.15\,\mathrm{m}$$

11. A balloon inflated with a gas (density = 0.5 kg/m^3) has a volume of 6.00×10^{-3} m^3. If the density of air is 1.30 kg/m^3, what is the buoyant force (F_B) exerted on the balloon?

$$\boxed{\begin{aligned} F_B &= \rho_f V g = (1.30\,\mathrm{kg/m^3})(6.00\times10^{-3}\,\mathrm{m^3})(9.81\,\mathrm{N/kg}) \\ &= 7.65\times10^{-2}\,\mathrm{N} \end{aligned}}$$

12. Water flows through a pipe of diameter 8.0 cm with a speed of 10.0 m/s. It then enters a smaller pipe of diameter 3.0 cm. What is the speed of the water as it flows through the smaller pipe?

$$\boxed{\begin{aligned} &v_1 A_1 = v_2 A_2 \quad v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{d_1^2}{d_2^2} \\ &v_2 = (10.0\,\mathrm{m/s})\frac{(8.0)^2}{(3.0)^2} = 71.1\,\mathrm{m/s} \end{aligned}}$$

13. The surface area of *each* wing of an airplane is 16.0 m^2. In level flight the air speed over the top of each wing is 62.0 m/s and the air speed beneath each wing is 54.0 m/s. If the density of the air at this altitude is 1.29 kg/m^3, what is the weight of the airplane?

Bernoulli's Equation: $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$$\frac{F_{\text{Net}}}{A_{2\,\text{wings}}} = P_2 - P_1 = \tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

$$F_{\text{Net}} = A_{2\,\text{wings}}\,\tfrac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

$$= 16\,\text{m}^2(1.29\,\text{kg/m}^3)(62^2 - 54^2)(\text{m}^2/\text{s}^2) = 1.92\times10^4\ \text{N}$$

14. Steel has a Young's modulus 2.00×10^{11} N/m^2 and coefficient of thermal expansion 12.0×10^{-6} (°C)$^{-1}$. A steel beam at 10 °C is constrained to a length of 2.50 m. If the temperature of the beam is increased from 10 °C to 40.0 °C, what pressure is generated at each end of the beam.

Stress-Strain: $F = YA\dfrac{\Delta L}{L}$ Thermal Expansion: $\Delta L = \alpha L \Delta T$

$$P = \frac{F}{A} = Y\alpha\Delta T = (2.00\times10^{11}\ \text{N/m}^2)(12.0\times10^{-6}\ {}^\circ\text{C}^{-1})(30^\circ\text{C})$$

$$= 7.20\times10^{7}\ \text{N/m}^2$$

15. If there are 1.20×10^{24} molecules in 0.088 kg of a substance, what is its atomic mass?

$$m \text{ (in u)} = \frac{m}{n} \ (m \text{ in g}) \quad \text{u} = \text{atomic mass unit}$$

$$= \frac{m}{(N / N_A)} = (88) / [(1.2 \times 10^{24}) / (6.02 \times 10^{23})] = 44.1\text{u}$$

16. Helium atoms at 450 K have an RMS speed of 1675 m/s. At what temperature does the speed increase to 2372 m/s?

$$\text{Average Kinetic Energy: } \bar{K} = \tfrac{1}{2} m \overline{v^2} = \tfrac{3}{2} k_B T$$

$$v_{RMS} = \sqrt{\overline{v^2}} \Rightarrow v_{RMS}^2 = \overline{v^2}$$

$$\frac{\frac{1}{2} m \overline{v_1^2}}{\frac{1}{2} m \overline{v_2^2}} = \frac{\frac{3}{2} k_B T_1}{\frac{3}{2} k_B T_2} \quad \Rightarrow$$

$$T_2 = T_1 \frac{\overline{v_2^2}}{\overline{v_1^2}} = (450\,\text{K}) \frac{(2372)^2}{(1675)^2} = 902\,\text{K}$$

Chapter 13

The Transfer of Heat

continued

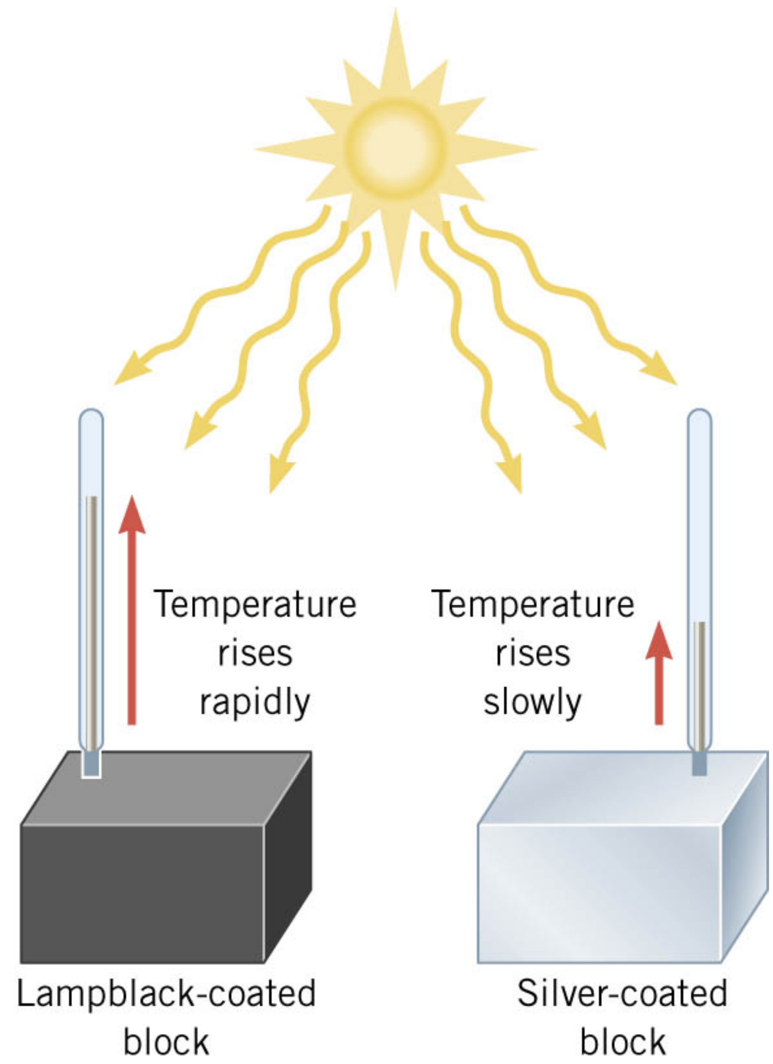
RADIATION

RADIATION

Radiation is the process in which energy is transferred by means of electromagnetic waves.

A material that is a good absorber is also a good emitter.

A material that absorbs completely is called a ***perfect blackbody***.



THE STEFAN-BOLTZMANN LAW OF RADIATION

The radiant energy *Q*, emitted in a time *t* by an object that has a Kelvin temperature *T*, a surface area *A*, and an emissivity *e*, is given by

$$\boxed{Q = e\sigma T^4 At}$$

emissivity $e =$ constant between 0 to 1

$e = 1$ (perfect black body emitter)

Stefan-Boltzmann constant

$\sigma = 5.67\times10^{-8}\ \mathrm{J}/\left(\mathrm{s}\cdot\mathrm{m}^2\cdot\mathrm{K}^4\right)$

Example A Supergiant Star

The supergiant star Betelgeuse has a surface temperature of about 2900 K and emits a power of approximately 4x10^{30} W. Assuming Betelgeuse is a perfect emitter and spherical, find its radius.

power, $P = \dfrac{Q}{t}$

with $A = 4\pi r^2$ (surface area of sphere with radius r)

$$r = \sqrt{\frac{Q/t}{4\pi e\sigma T^4}} = \sqrt{\frac{4\times10^{30}\,\mathrm{W}}{4\pi(1)\left[5.67\times10^{-8}\ \mathrm{J}/\left(\mathrm{s}\cdot\mathrm{m}^2\cdot\mathrm{K}^4\right)\right](2900\ \mathrm{K})^4}}$$

$$= 3\times10^{11}\,\mathrm{m}$$

Chapter 14

Thermodynamics

THE FIRST LAW OF THERMODYNAMICS

The internal energy of a system changes due to heat and work:

$$\boxed{\Delta U = U_f - U_i = Q + W}$$

$Q > 0$ system gains heat

$W > 0$ if work done on the system

The internal energy (U) of an Ideal Gas depends only on the temperature:

$$\boxed{\text{Ideal Gas (only):} \quad U = \tfrac{3}{2} nRT \ \text{ or } U = \tfrac{3}{2} Nk_B T}$$

$$\boxed{\begin{aligned} \Delta U &= U_f - U_i \\ &= \tfrac{3}{2} nR(T_f - T_i) \end{aligned}}$$

Otherwise, values for both Q and W are needed to determine ΔU

Clicker Question 14.1

An insulated container is filled with a mixture of water and ice at zero °C. An electric heating element inside the container is used to add 1680 J of heat to the system while a paddle does 450 J of work by stirring. What is the increase in the internal energy of the ice-water system?

$$\boxed{\Delta U = Q + W}$$

a) 450 J

b) 1230 J

c) 1680 J

d) 2130 J

e) zero J

14.1 Thermal Processes

Work done on a gas

$(\Delta P = 0)$ ***isobaric:*** constant pressure: $W = -P\Delta V$

$(\Delta V = 0)$ ***isochoric:*** constant volume: $W = -P\Delta V = 0$

For an Ideal Gas only

$(\Delta T = 0)$ ***isothermal:*** constant temperature: $W = nRT\ln\left(V_i/V_f\right)$

$(Q = 0)$ ***adiabatic:*** no transfer of heat: $W = \frac{3}{2}nR\left(T_f - T_i\right)$

An ***isobaric*** process is one that occurs slowly at constant pressure.

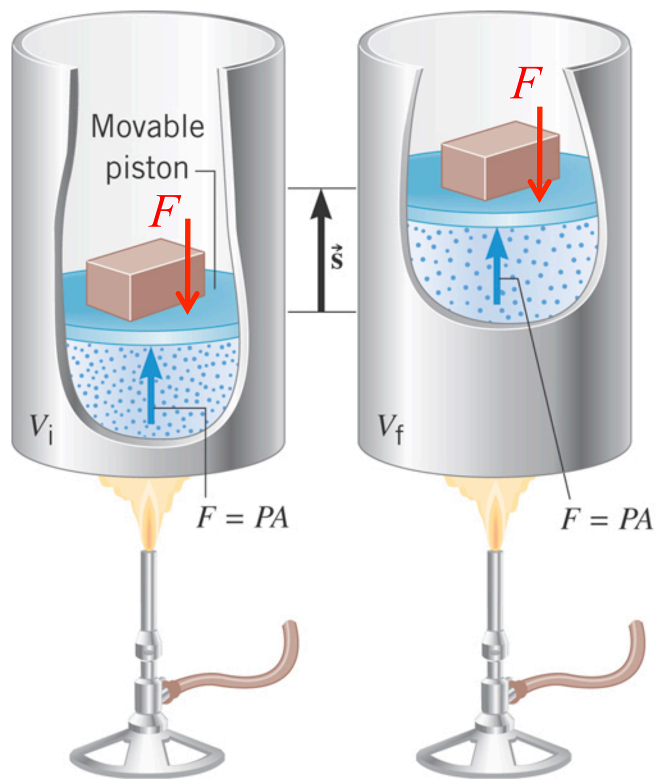
$$\boxed{\begin{array}{l}\cos\theta = +1 \\ \cos\theta = -1\end{array}}$$

$$\boxed{\begin{array}{l}\text{If piston is pushed down by mass, } W_{\text{on gas}} > 0. \\ \text{If piston is pushed upward by pressure, } W_{\text{on gas}} < 0\end{array}}$$

$$W = Fs\cos\theta = -P\left(As\right)$$

$$= -P\Delta V$$

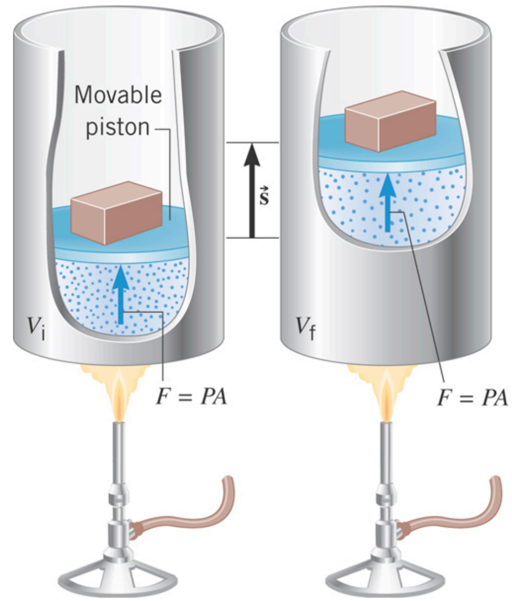
$$= -P\left(V_f - V_i\right)$$



Example Isobaric Expansion of Water (Liquid)

One gram of water is placed in the cylinder and the pressure is maintained at 2.0×10^{5} Pa. The temperature of the water is raised by 31°C. The water is in the liquid phase and expands by a very small amount, 1.0×10^{-8} m^3.

Find the work done and the change in internal energy.



$$\boxed{\begin{aligned} W &= -P\Delta V \\ &= -\left(2.0\times10^{5}\,\text{Pa}\right)\left(1.0\times10^{-8}\,\text{m}^3\right) = -0.0020\text{J}\end{aligned}}$$

$$\boxed{\text{Liquid water } \Delta V \sim 0}$$

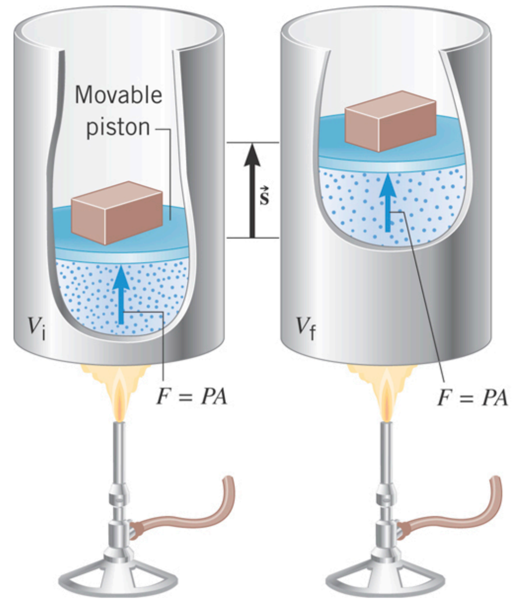
$$Q = mc\Delta T$$

$$c_{\text{water}} = 4186\,\text{J}/\left(\text{kg}\cdot\text{C}^\circ\right)$$

$$= \left(0.0010\text{ kg}\right)\left[4186\,\text{J}/\left(\text{kg}\cdot\text{C}^\circ\right)\right]\left(31\text{ C}^\circ\right) = 130\text{ J}$$

$$\boxed{\Delta U = Q - W = 130\text{ J} - 0.0020\text{ J} = 130\text{ J}}$$

Example Isobaric Expansion of Water (Vapor)



One gram of water vapor is placed in the cylinder and the pressure is maintained at 2.0×10^5 Pa. The temperature of the vapor is raised by 31°C, and the gas expands by 7.1×10^{-5} m^3. Heat capacity of the gas is 2020 J/(kg-C°).

Find the work done and the change in internal energy.

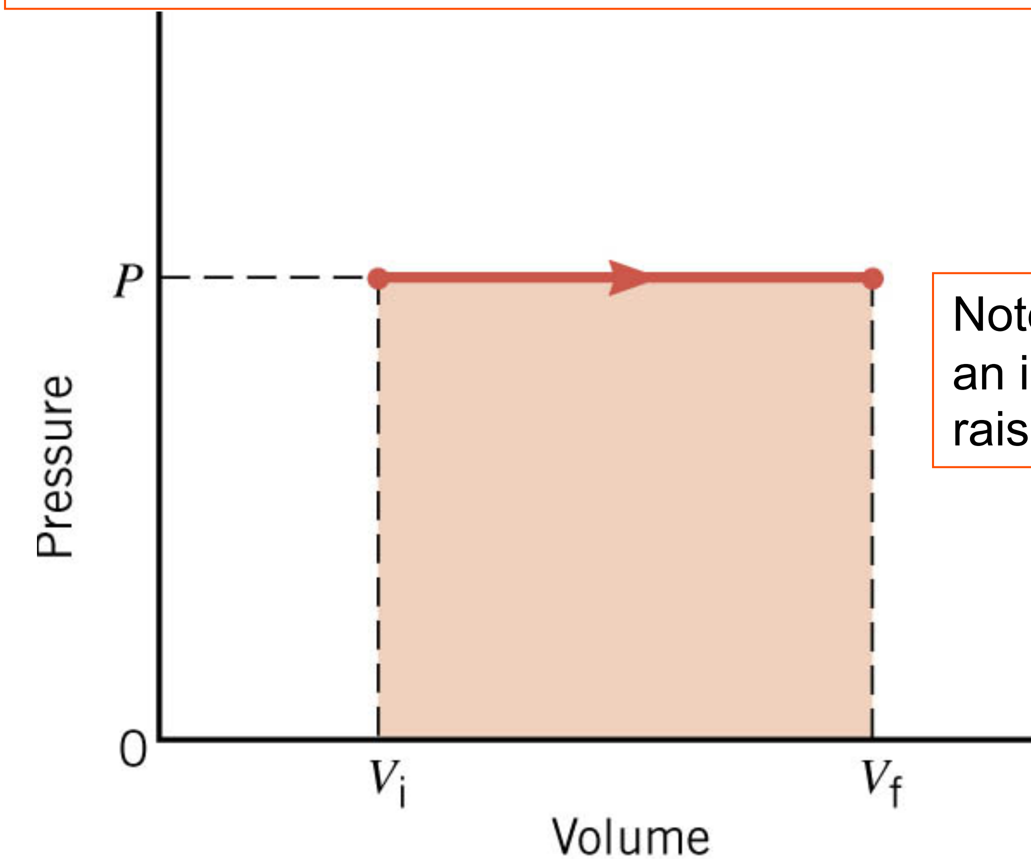
$$W = -P\Delta V = -(2.0\times10^{5}\ \text{Pa})(7.1\times10^{-5}\text{m}^{3})$$
$$= -14.2\ \text{J}$$

$$Q = mc\Delta T$$
$$= (0.0010\ \text{kg})\left[2020\,\text{J}/(\text{kg}\cdot\text{C}^{\circ})\right](31\ \text{C}^{\circ}) = 63\ \text{J}$$

$$\Delta U = Q + W = 63\text{J} + (-14\ \text{J}) = 49\ \text{J}$$

$$W = -P\Delta V = -P\left(V_f - V_i\right)$$

The work done on a gas at constant pressure - the work done is (minus) the area under a P-V diagram.



Note: the temperature of an ideal gas must be raised to do this.

Clicker Question 14.2

An ideal gas at a constant pressure of 1×10^5 Pa is reduced in volume from 1.00 m^3 to 0.25 m^3. What work was done on the gas?

$$\boxed{W = -P\Delta V}$$

a) zero J

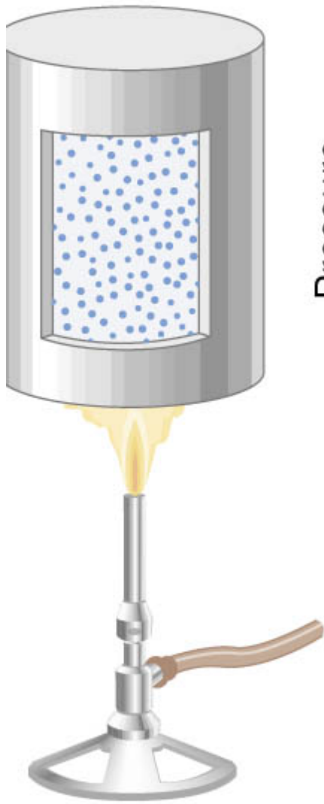
b) 0.25×10^5 J

c) 0.50×10^5 J

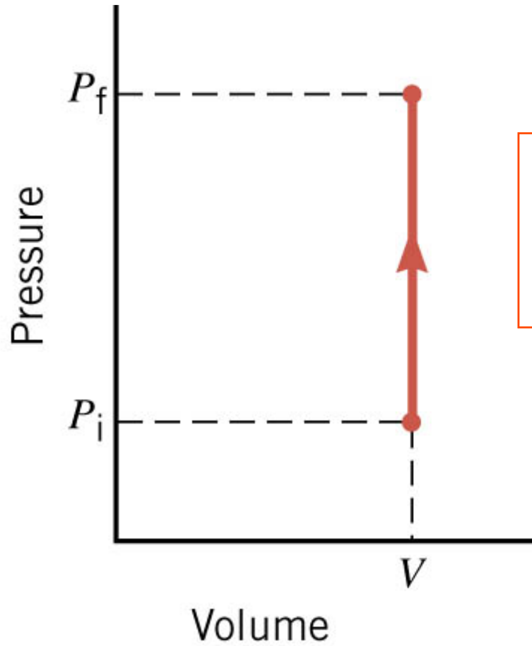
d) 0.75×10^5 J

e) 4.00×10^5 J

isochoric: constant volume



(a)



(b)

The work done at constant volume is the area under a P-V diagram. The area is zero!

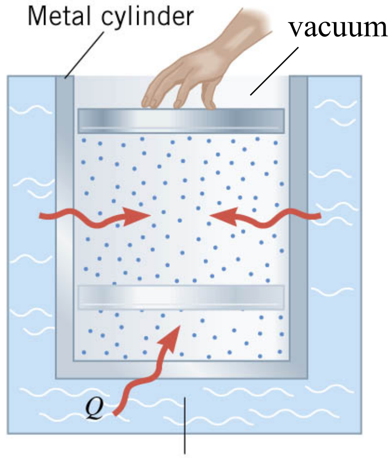
$$W = 0$$

$$\Delta U = Q + W = Q$$

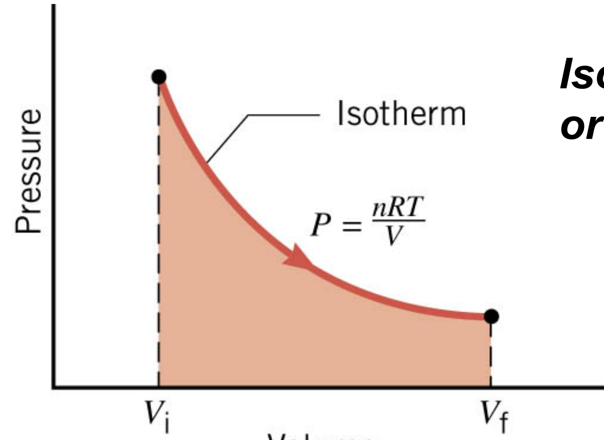
Change in internal energy is equal to the heat added.

14.2 Thermal Processes Using and Ideal Gas

ISOTHERMAL EXPANSION OR COMPRESSION



(a)



(b)

Isothermal expansion or compression of an ideal gas

$$W_{\text{on gas}} = nRT\ln\left(\frac{V_i}{V_f}\right)$$

Example 5 Isothermal Expansion of an Ideal Gas

Two moles of argon (ideal gas) expand isothermally at 298K, from initial volume of 0.025m³ to a final volume of 0.050m³. Find (a) the work done by the gas, (b) change in gas internal energy, and (c) the heat supplied.

$$\text{a) } W_{\text{on gas}} = nRT\ln\left(V_i/V_f\right)$$

$$= (2.0\text{ mol})\left(8.31\text{J}/(\text{mol}\cdot\text{K})\right)(298\text{ K})\ln\left(\frac{0.025}{0.050}\right)$$

$$= -3400\text{ J}$$

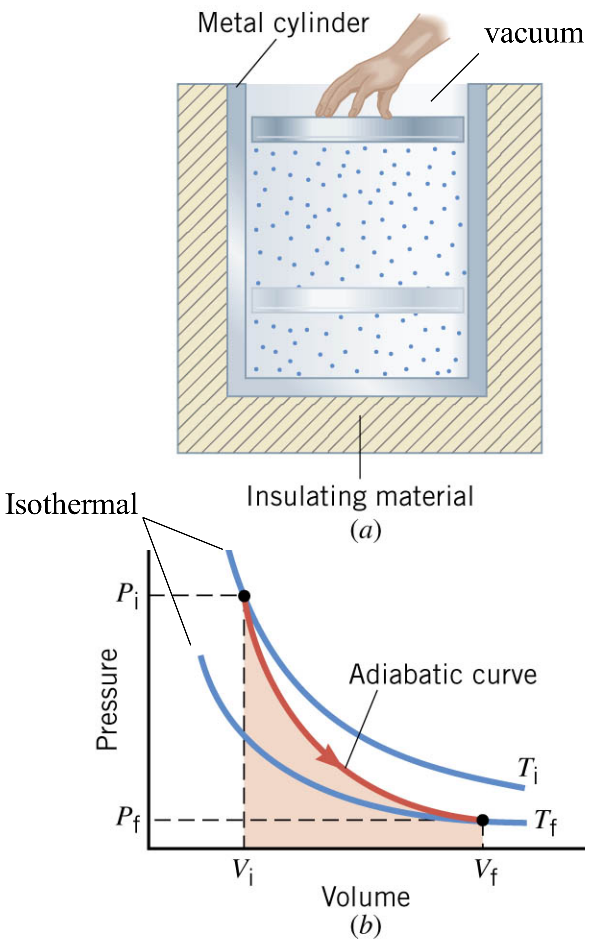
$$\text{b) } \Delta U = U_f - U_i = \tfrac{3}{2}nR\Delta T$$

$$\Delta T = 0 \text{ therefore } \Delta U = 0$$

$$\text{c) } \Delta U = Q + W = 0$$

$$Q = -W = 3400\text{J}$$

14.2 Thermal Processes Using and Ideal Gas



ADIABATIC EXPANSION OR COMPRESSION

Adiabatic expansion or compression of a monatomic ideal gas

$$W_{\text{on gas}} = \tfrac{3}{2} nR\left(T_i - T_f\right)$$

Adiabatic expansion or compression of a monatomic ideal gas

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$\gamma = c_P / c_V$$

Ratio of heat capacity at constant P over heat capacity at constant V.

These are needed to understand basic operation of refrigerators and engines

ADIABATIC EXPANSION OR COMPRESSION

ISOTHERMAL EXPANSION OR COMPRESSION

14.2 Specific Heat Capacities

To relate heat and temperature change in solids and liquids (mass in kg), use:

$$\boxed{Q = mc\Delta T} \quad \text{specific heat capacity, } c \ \left[\mathrm{J}/\left(\mathrm{kg}\cdot{}^\circ\mathrm{C}\right)\right]$$

For gases, the amount of gas is given in moles, use molar heat capacities:

$$\boxed{Q = nC\Delta T} \quad \text{molar heat capacity, } C \ \left[\mathrm{J}/\left(\mathrm{mole}\cdot{}^\circ\mathrm{C}\right)\right]$$

$$\boxed{C = \left(m/n\right)c = m_u c; \quad m_u = \text{mass/mole (kg)}}$$

ALSO, for gases it is necessary to distinguish between the molar specific heat capacities at constant pressure and at constant volume:

$$C_P, C_V$$

14.2 Specific Heat Capacities

$$\text{Ideal Gas: } PV = nRT; \quad \Delta U = \tfrac{3}{2} nR\Delta T$$

$$\text{1st Law of Thermodynamics: } \Delta U = Q + W_{\text{on gas}}$$

$$\underline{\text{Constant Pressure } (\Delta P = 0)}$$

$$W_P = -P\Delta V = -nR\Delta T$$

$$Q_P = \Delta U - W = \tfrac{3}{2} nR\Delta T + nR\Delta T = \tfrac{5}{2} nR\Delta T$$

Constant pressure for a monatomic ideal gas

$$Q_P = nC_P\Delta T$$

$$C_P = \tfrac{5}{2} R$$

$$\underline{\text{Constant Volume } (\Delta V = 0)}$$

$$W_V = -P\Delta V = 0$$

$$Q_V = \Delta U - W = \tfrac{3}{2} nR\Delta T = \tfrac{3}{2} nR\Delta T$$

Constant volume for a monatomic ideal gas

$$Q_V = nC_V\Delta T$$

$$C_V = \tfrac{3}{2} R$$

monatomic ideal gas

$$\gamma = C_P / C_V = \tfrac{5}{2} R \big/ \tfrac{3}{2} R = 5/3$$

any ideal gas

$$C_P - C_V = R$$

14.3 The Second Law of Thermodynamics

The second law is a statement about the natural tendency of heat to flow from hot to cold, whereas the first law deals with energy conservation and focuses on both heat and work.

THE SECOND LAW OF THERMODYNAMICS: THE HEAT FLOW STATEMENT

Heat flows spontaneously from a substance at a higher temperature to a substance at a lower temperature and does not flow spontaneously in the reverse direction.

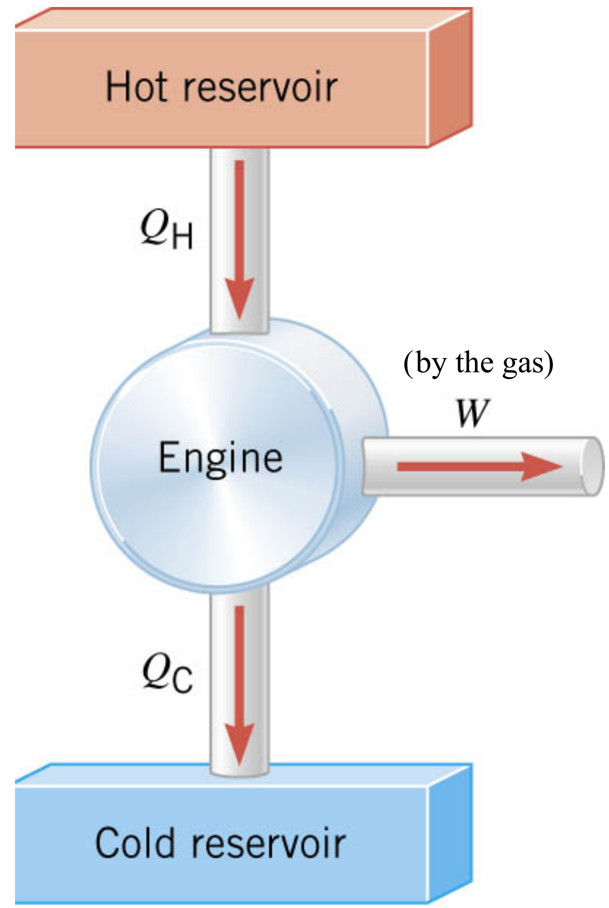
14.3 ***Heat Engines***

A ***heat engine*** is any device that uses heat to perform work. It has three essential features.

1. Heat is supplied to the engine at a relatively high temperature from a place called the *hot reservoir.*

2. Part of the input heat is used to perform work by the *working substance* of the engine.

3. The remainder of the input heat is rejected to a place called the *cold reservoir.*



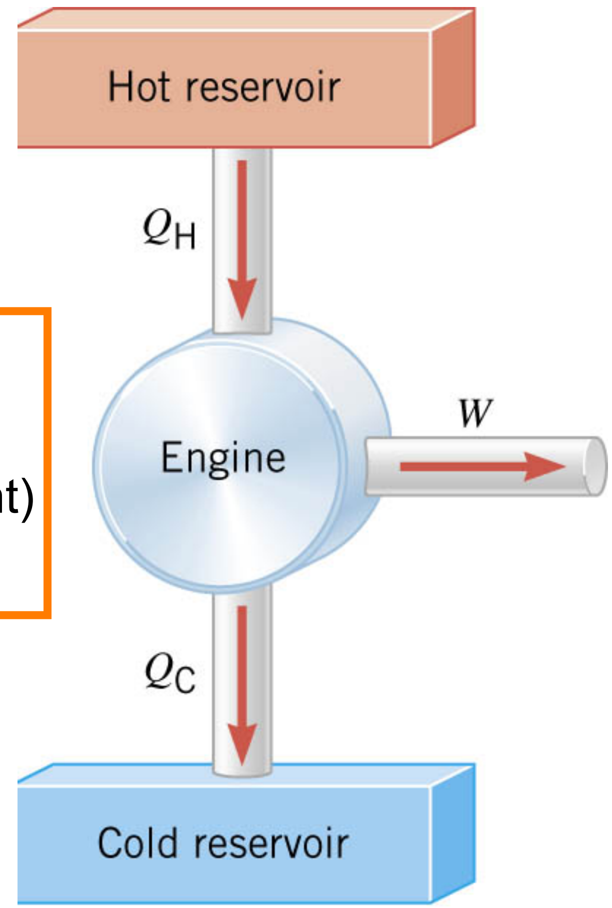
$$|Q_H| = \text{magnitude of input heat}$$

$$|Q_C| = \text{magnitude of rejected heat}$$

$$|W| = \text{magnitude of the work done}$$

Carnot Engine Working with an Ideal Gas

1. ISOTHERMAL EXPANSION ($Q_{in}=Q_H$, T_{Hot} constant)
2. ADIABATIC EXPANSION ($Q=0$, T drops to T_{Cold})
3. ISOTHERMAL COMPRESSION ($Q_{out}=Q_C$, T_{Cold} constant)
4. ADIABATIC COMPRESSION ($Q=0$, T rises to T_{Hot})



$$|Q_H| = \text{magnitude of input heat}$$

$$|Q_C| = \text{magnitude of rejected heat}$$

$$|W| = \text{magnitude of the work done}$$

14.3 Heat Engines

The ***efficiency*** of a heat engine is defined as the ratio of the work done to the input heat:

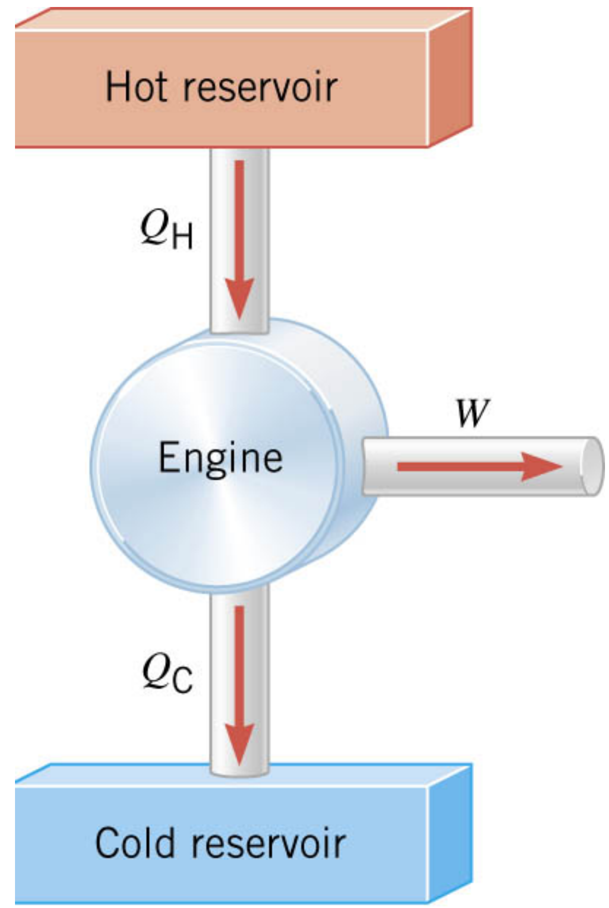
$$e = \frac{|W|}{|Q_H|}$$

If there are no other losses, then

$$|Q_H| = |W| + |Q_C|$$

$$\Downarrow$$

$$e = 1 - \frac{|Q_C|}{|Q_H|}$$



Example An Automobile Engine

An automobile engine has an efficiency of 22.0% and produces 2510 J of work. How much heat is rejected by the engine?

$$e = \frac{|W|}{|Q_H|}$$

$$= \frac{|W|}{|Q_C| + |W|} \quad \Rightarrow \quad e\left(|Q_C| + |W|\right) = |W|$$

$$|Q_C| = \frac{|W| - e|W|}{e} = |W|\left(\frac{1}{e} - 1\right) = 2510\,\mathrm{J}\left(\frac{1}{0.22} - 1\right)$$

$$= 8900\ \mathrm{J}$$

14.3 Carnot's Principle and the Carnot Engine

A reversible process is one in which both the system and the environment can be returned to exactly the states they were in before the process occurred.

CARNOT'S PRINCIPLE: AN ALTERNATIVE STATEMENT OF THE SECOND LAW OF THERMODYNAMICS

No irreversible engine operating between two reservoirs at constant temperatures can have a greater efficiency than a reversible engine operating between the same temperatures. Furthermore, all reversible engines operating between the same temperatures have the same efficiency.

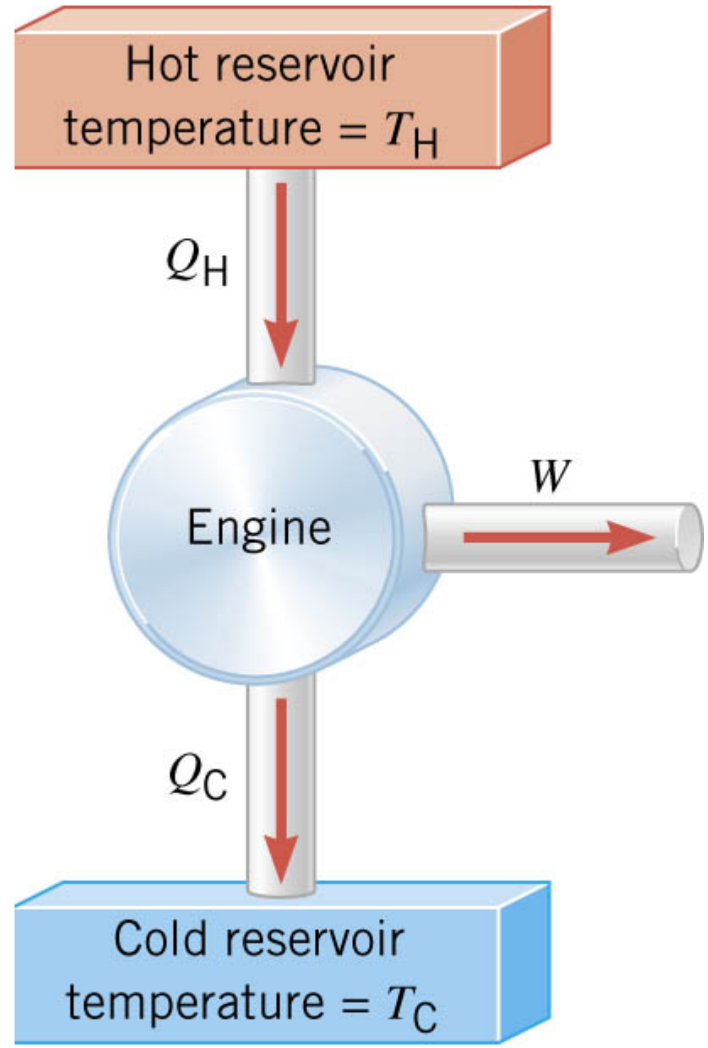
14.3 Carnot's Principle and the Carnot Engine

The ***Carnot engine*** is useful as an idealized model.

All of the heat input originates from a single temperature, and all the rejected heat goes into a cold reservoir at a single temperature.

Since the efficiency can only depend on the reservoir temperatures, the ratio of heats can only depend on those temperatures.

$$e = 1 - \frac{|Q_C|}{|Q_H|} = 1 - \frac{T_C}{T_H}$$



Example A Tropical Ocean as a Heat Engine

Surface temperature is 298.2 K, whereas 700 meters deep, the temperature is 280.2 K. Find the maximum efficiency for an engine operating between these two temperatures.

$$e_{\text{carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{280.2\ \text{K}}{298.2\ \text{K}} = 0.060$$

Maximum of only 6% efficiency.
Real life will be worse.

Conceptual Example 8 Natural Limits on the Efficiency of a Heat Engine

Consider a hypothetical engine that receives 1000 J of heat as input from a hot reservoir and delivers 1000J of work, rejecting no heat to a cold reservoir whose temperature is above 0 K. Decide whether this engine violates the first or second law of thermodynamics.

If $T_H > T_C > 0$

$$e_{\text{carnot}} = 1 - \frac{T_C}{T_H} \text{ must be less than 1}$$

$$e_{hypothetical} = \frac{|W|}{|Q_H|} = \frac{1000\,\text{J}}{1000\,\text{J}} = 1$$

Violates 2nd law of thermodynamics

14.3 Entropy

In general, irreversible processes cause us to lose some, but not necessarily all, of the ability to do work. This partial loss can be expressed in terms of a concept called ***entropy***.

Carnot engine

$$\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H} \quad \Longrightarrow \quad \frac{|Q_C|}{T_C} = \frac{|Q_H|}{T_H}$$

entropy change

$$\Delta S = \left(\frac{Q}{T}\right)_{\mathrm{R}}$$

reversible

14.3 Entropy

Entropy, like internal energy, is a function of the state of the system.

$$\Delta S = \left(\frac{Q}{T}\right)_R$$

Consider the entropy change of a Carnot engine. The entropy of the hot reservoir decreases and the entropy of the cold reservoir increases.

$$\Delta S = +\frac{|Q_C|}{T_C} - \frac{|Q_H|}{T_H} = 0$$

Reversible processes do not alter the entropy of the universe.