Chapter 7

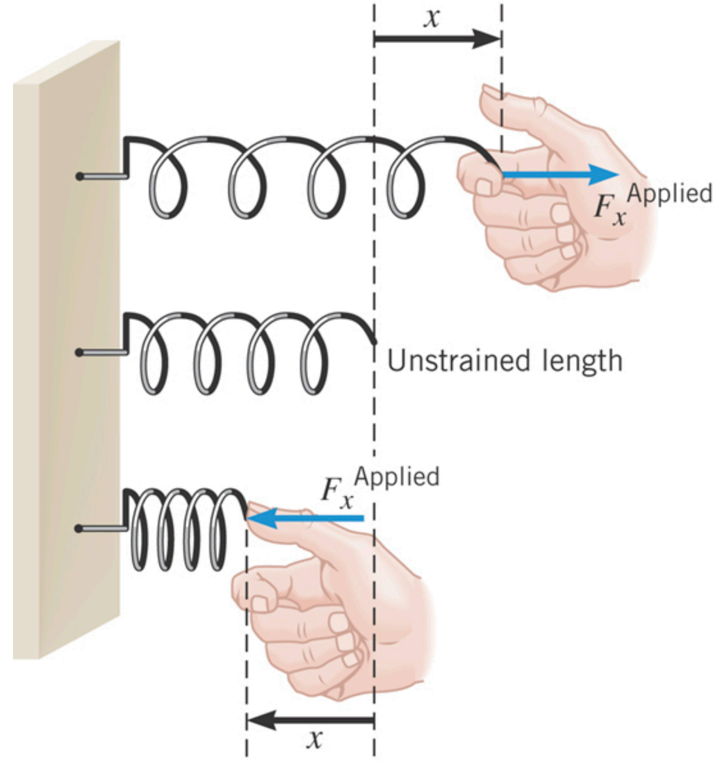
Simple Harmonic Motion

5.2 The Ideal Spring and Simple Harmonic Motion

$$F_x^{Applied} = kx$$

spring constant

Units: N/m



$F_x^{Applied}$ is the applied force in the x direction

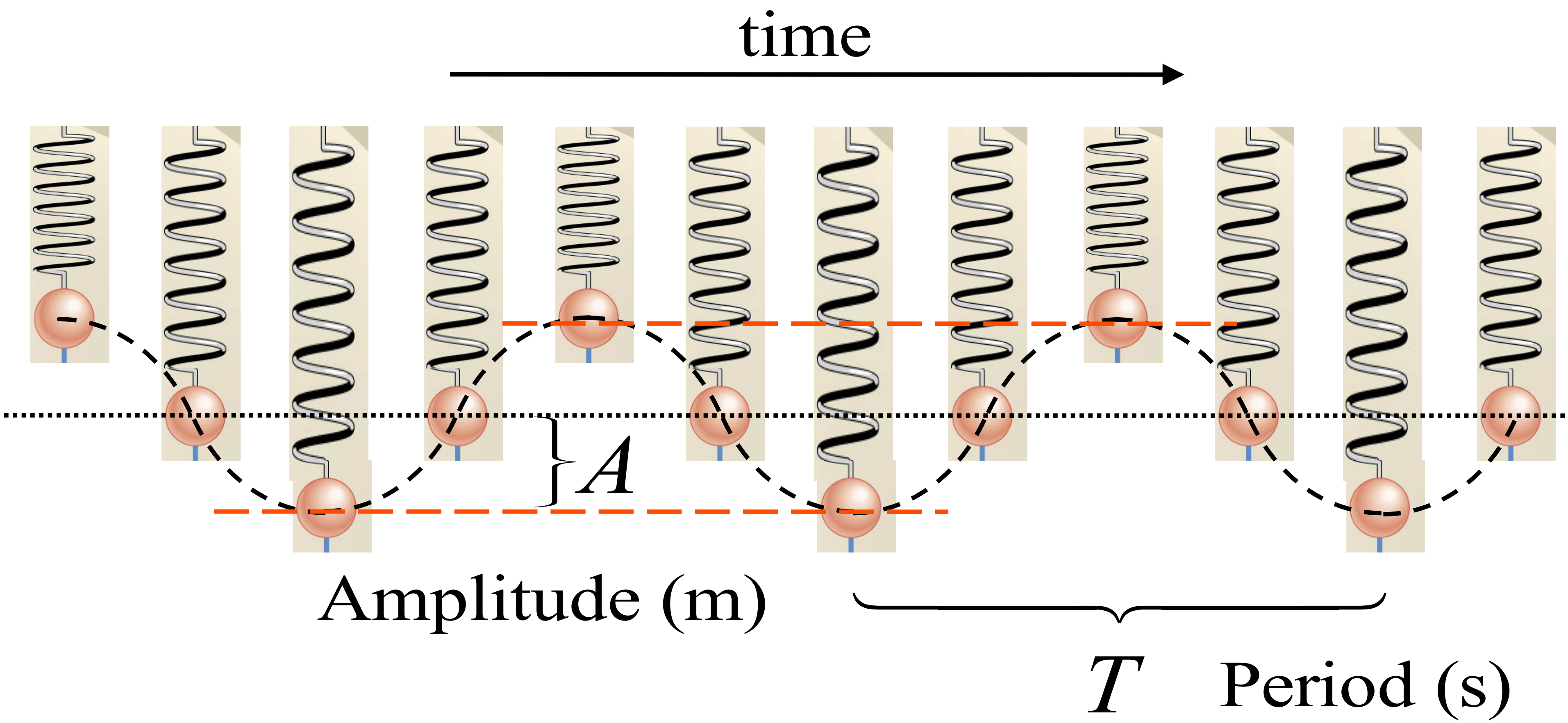
x is the spring displacement

k is the spring constant (strength of the spring)

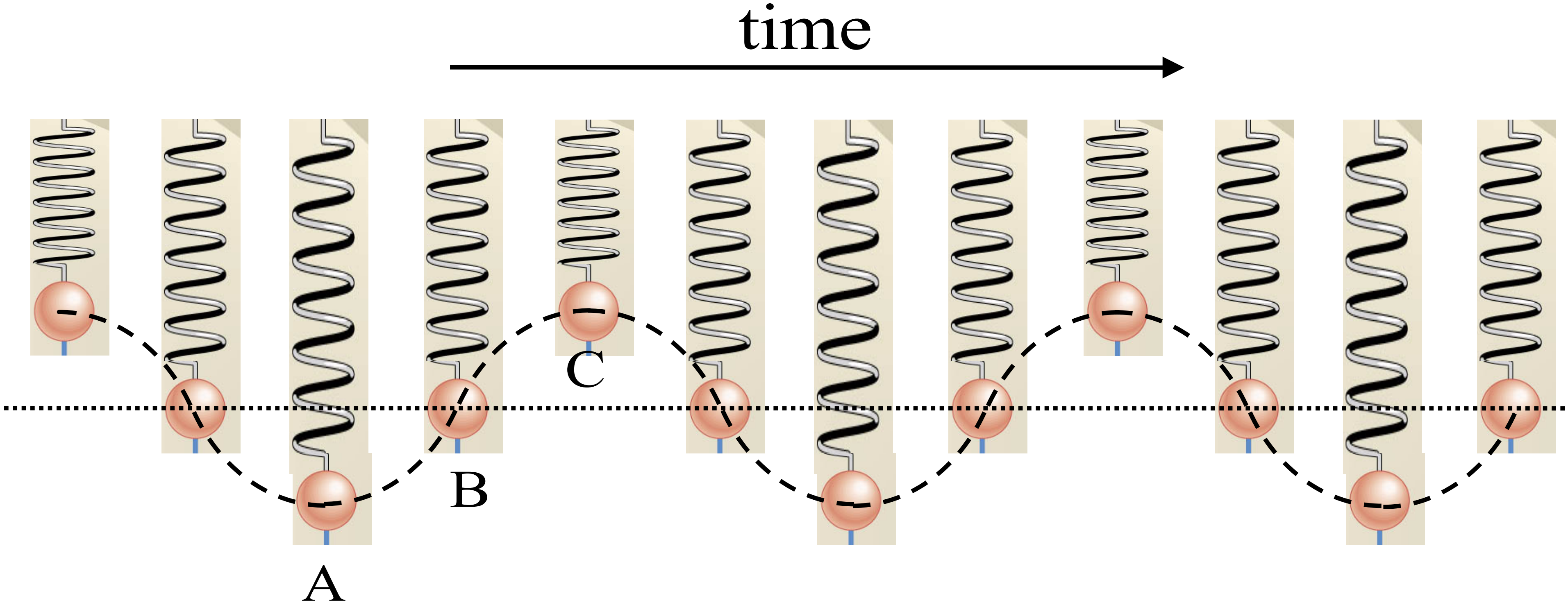
$$F_x^{Spring} = -kx$$

F_x^{Spring} is the spring's force in the x direction

Simple Harmonic Motion



Clicker Question 7.1



Where is the mass moving vertically at the highest speed?

A) only at A

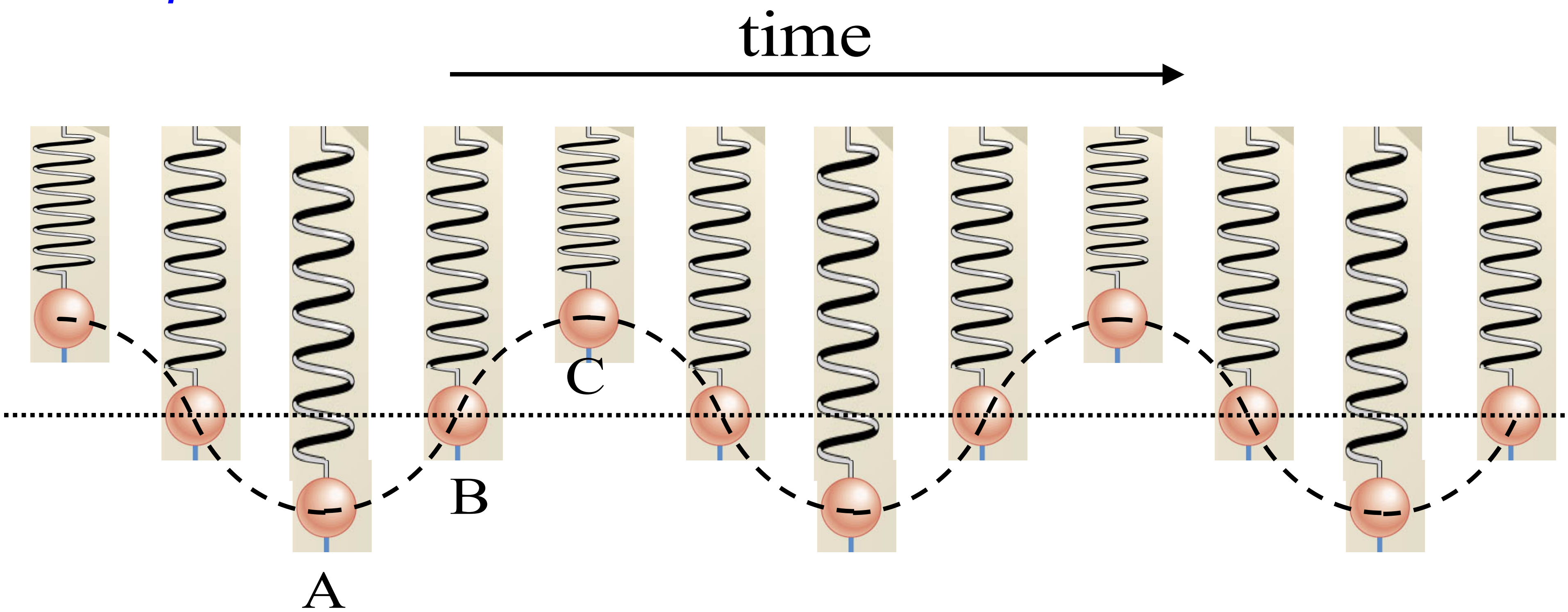
B) only at B

C) only at C

D) at both A and C

E) none of the above

7.1 Simple Harmonic Motion and the Reference Circle

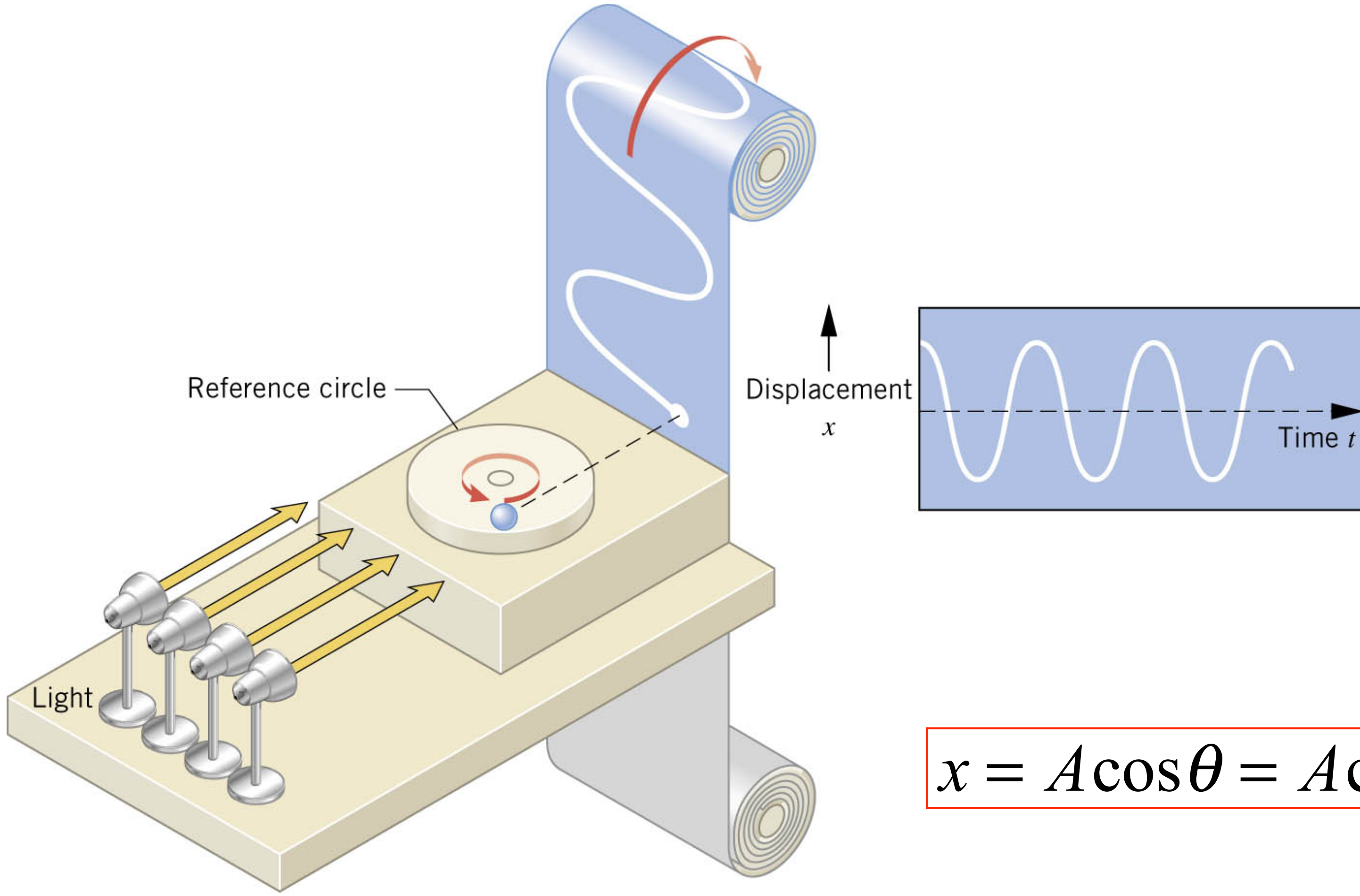


mass has the greatest magnitude of acceleration when
at points A and C - when it is turning around (speed is slowest)

acceleration vector is positive at point A and negative at C

mass has zero acceleration at point B - when speed is the greatest.

DISPLACEMENT

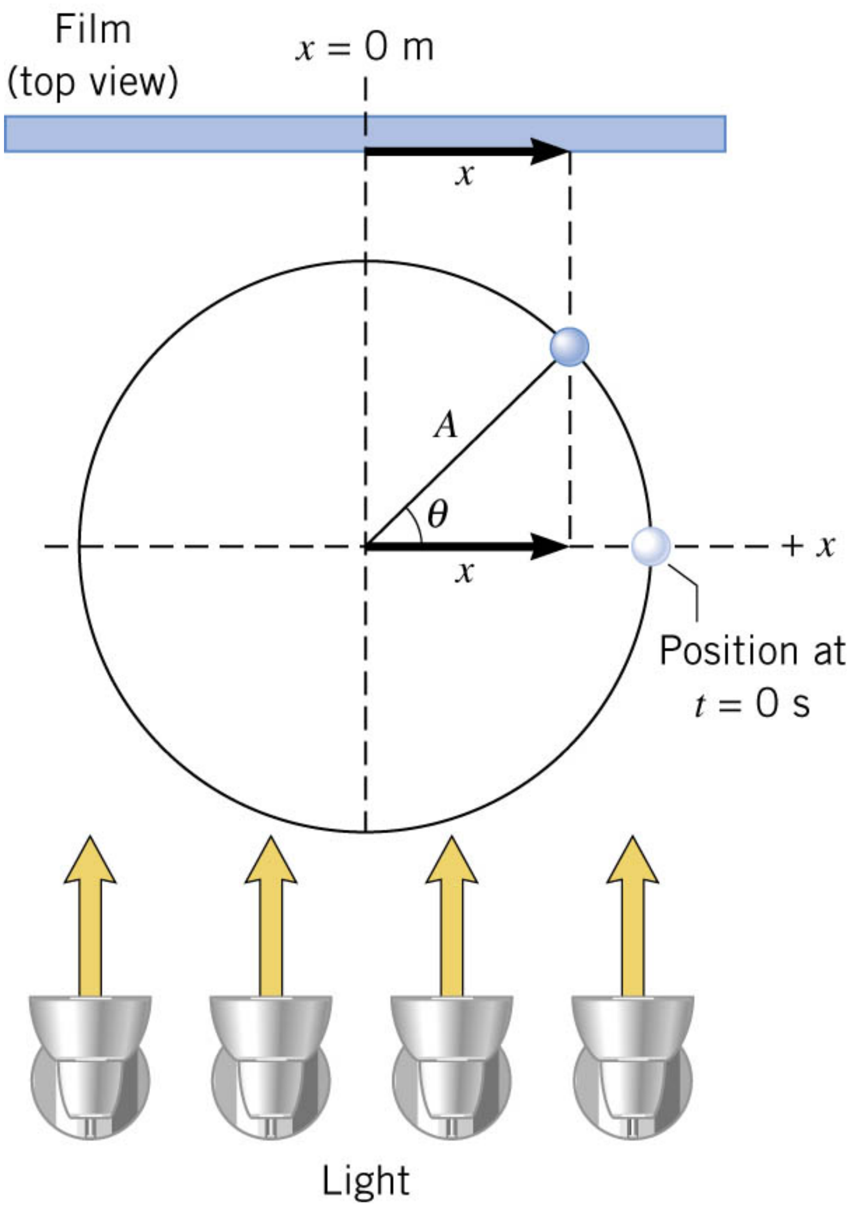


$$\boxed{x = A\cos\theta = A\cos\omega t}$$

Angular velocity, ω (unit: rad/s)

Angular displacement, $\theta = \omega t$ (unit: radians)

7.2 Simple Harmonic Motion and the Reference Circle



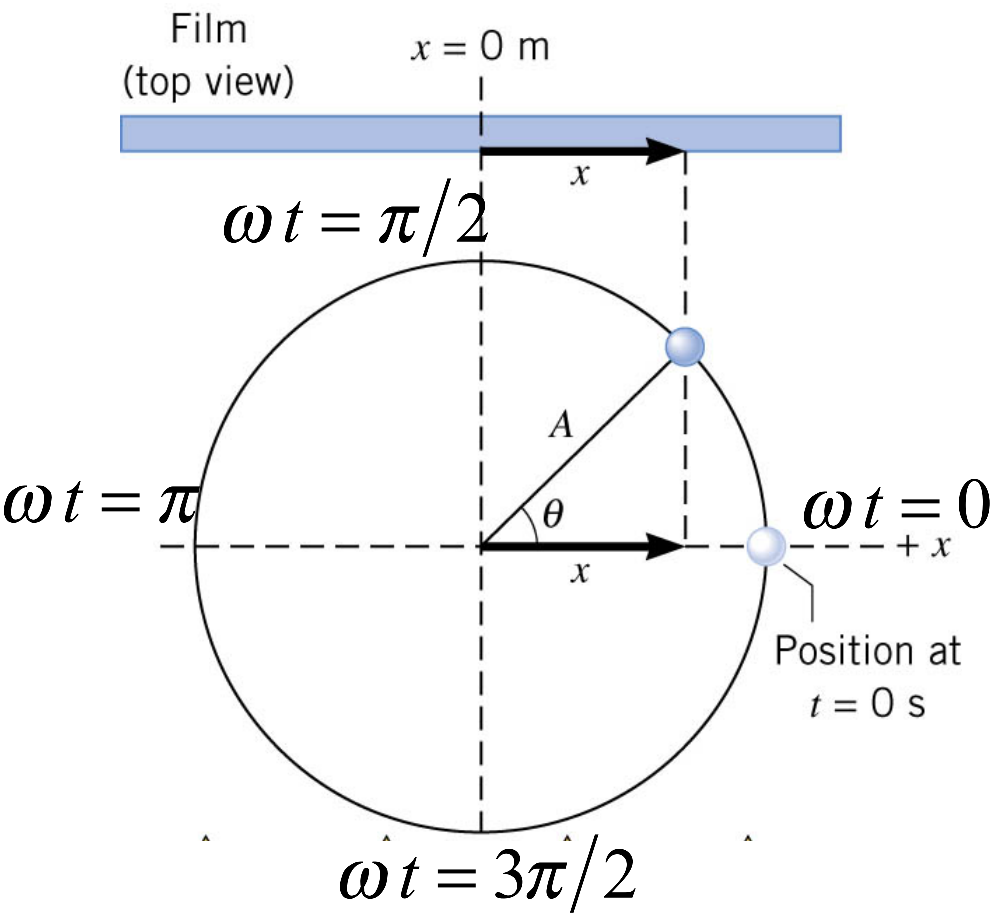
<u>uniform circular motion</u>

$$\theta = \omega t + \tfrac{1}{2}\alpha t^2 \text{ with } \alpha = 0$$

$$\theta = \omega t$$

$$\boxed{\begin{aligned} x &= A\cos\theta \\ &= A\cos(\omega t) \end{aligned}}$$

7.2 Simple Harmonic Motion and the Reference Circle



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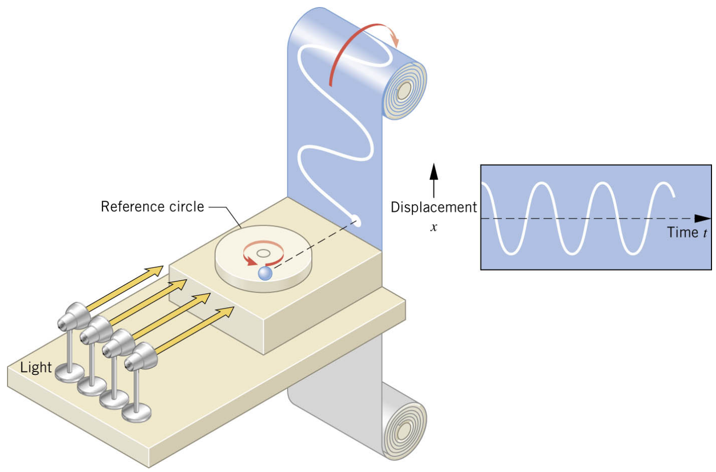
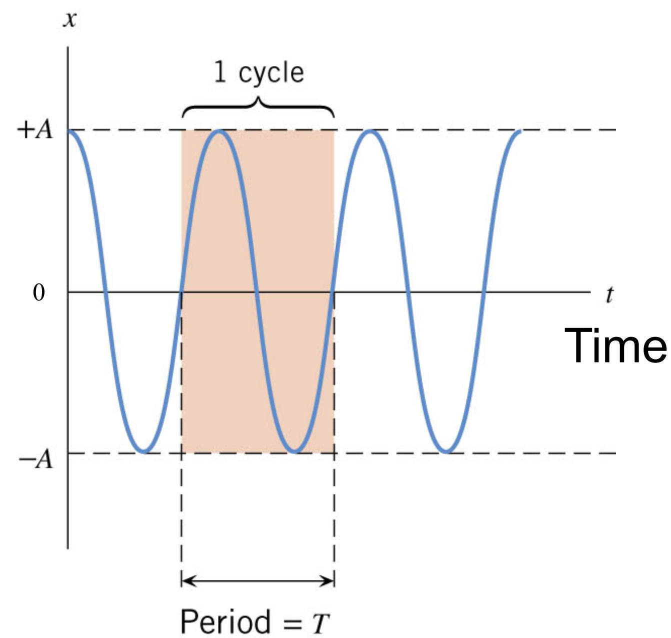
7.2 Simple Harmonic Motion and the Reference Circle



amplitude A:



amplitude A: the maximum displacement

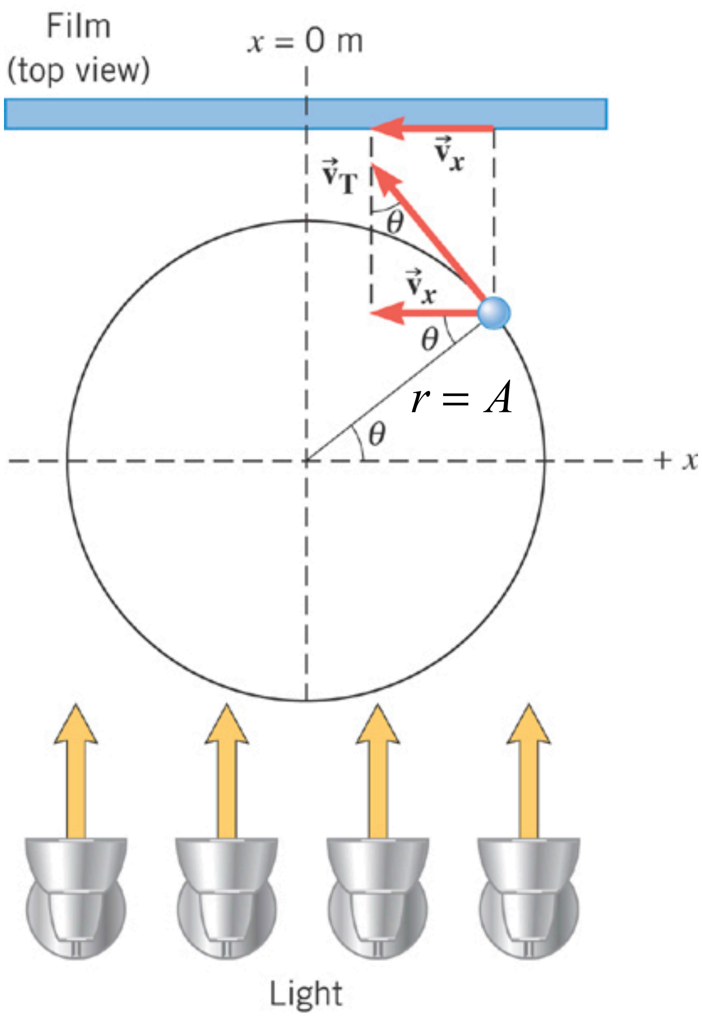
period T: the time required to complete one "cycle"

frequency f: the number of "cycles" per second (measured in Hz = 1/s)

frequency f: $$\boxed{f = \frac{1}{T}}$$

angular frequency: $$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$ (Radians per second)

VELOCITY



Note: $\sin(\omega t)$

$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{max}} \sin(\omega t)$$

Maximum velocity: $\mp A\omega$ (units, m/s)

$$\boxed{\begin{array}{l} v_x = -A\omega\sin(\omega t) = \mp A\omega \\ \text{when } \omega t = \pi/2,\ 3\pi/2 \text{ radians} \end{array}}$$

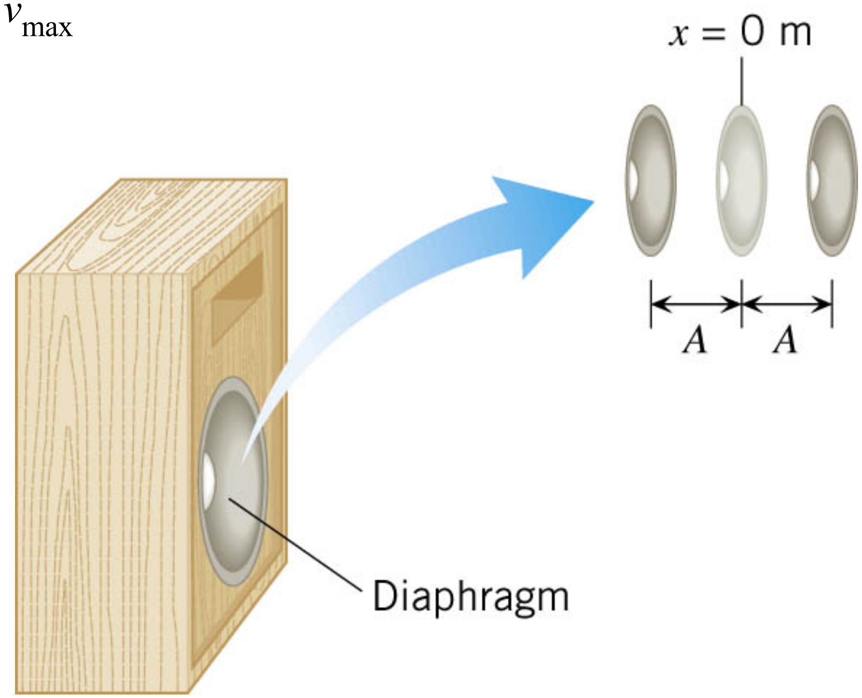
Maximum velocity occurs at

$$\boxed{\begin{aligned} x &= A\cos(\omega t) \\ &= A\cos(\pi/2) = 0 \end{aligned}}$$

Example The Maximum Speed of a Loudspeaker Diaphragm

The frequency of motion is 1.0 KHz and the amplitude is 0.20 mm.
(a) What is the maximum speed of the diaphragm?
(b) Where in the motion does this maximum speed occur?

$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{\max}} \sin\omega t$$



a) $v_{\max} = A\omega = A(2\pi f)$

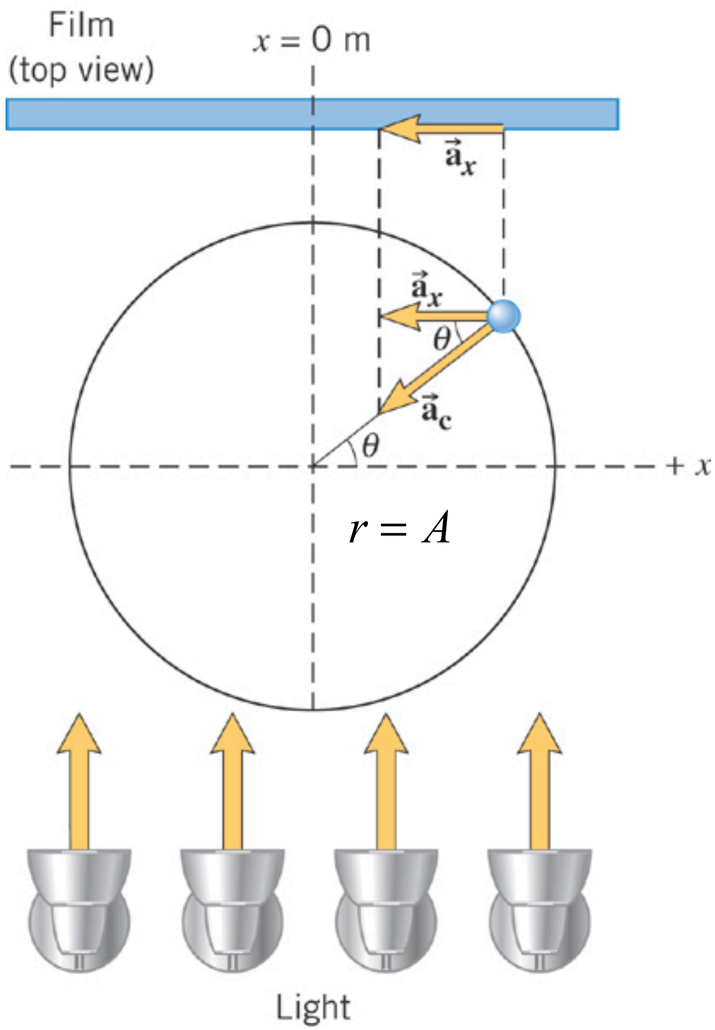
$= (0.20\times10^{-3}\text{m})(2\pi)(1.0\times10^{3}\text{Hz})$

$= 1.3\text{m/s}$

b) The maximum speed occurs midway between the ends of its motion.

ACCELERATION

$$\boxed{a_c = \frac{v^2}{r} = r\omega^2}$$



$$a_x = -a_c\cos\theta = -\underbrace{A\omega^2}_{a_{max}}\cos\omega t$$

Maximum a_x: $\mp A\omega^2$ (units, m/s^2)

$$\boxed{\begin{array}{l} a_x = -A\omega^2\cos(\omega t) = \mp A\omega^2 \\ \text{when } \omega t = 0, \pi \text{ radians} \end{array}}$$

Maximum a_x occurs at

$$\boxed{\begin{aligned} x &= A\cos(\omega t) \\ &= A\cos(0) = A \\ &= A\cos(\pi) = -A \end{aligned}}$$

FREQUENCY OF VIBRATION

$$\sum F_x = ma_x$$

$$-kx = ma_x$$

$$x = A\cos\omega t$$

$$a_x = -A\omega^2 \cos\omega t$$

$$-Ak\cos\omega t = -Am\omega^2 \cos\omega t$$

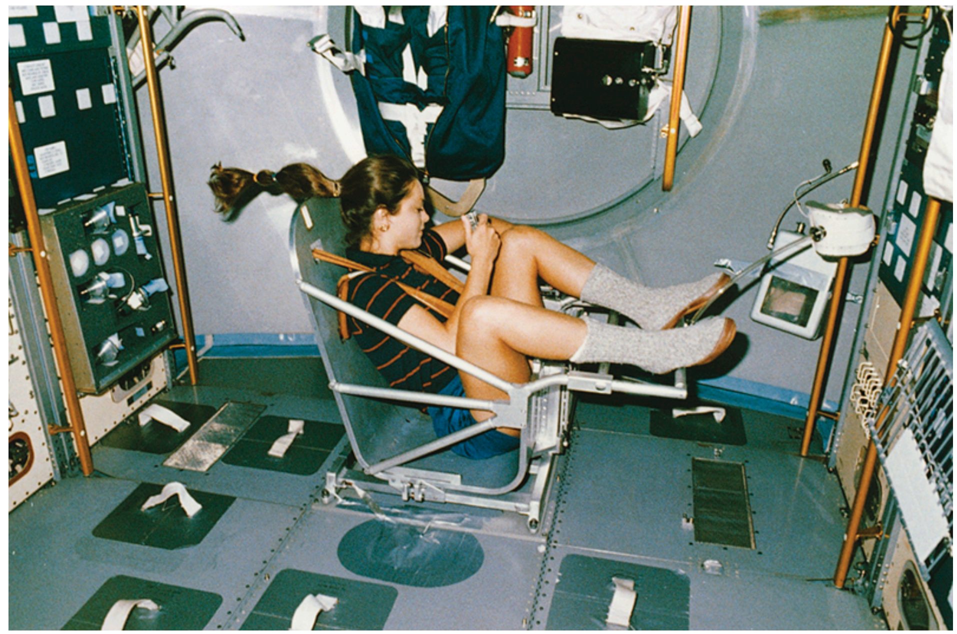
$$k = m\omega^2$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

angular frequency for oscillations of a mass (m) on a spring (k)

Example A Body Mass Measurement Device

The device consists of a spring-mounted chair in which the astronaut sits. The spring has a spring constant of 606 N/m, and the mass of the chair is 12.0 kg. The measured period is 2.41 s. Find the mass of the astronaut.



spring constant: $k = 606\,\text{N/m}$

chair mass: $m_{\text{chair}} = 12.0\,\text{kg}$

oscillation period: $T = 2.41\,\text{s}$

$$\omega = \sqrt{\frac{k}{m_{\text{total}}}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$m_{\text{total}} = \frac{k}{\omega^2} = \frac{kT^2}{4\pi^2} = 89.2\text{ kg}$$

$$m_{\text{astro}} = m_{\text{total}} - m_{\text{chair}} = 77.2\,\text{kg}$$

Summary: spring constants & oscillations

Hooke's Law $\boxed{F_A = kx}$ Displacement proportional to applied force

Oscillations $\boxed{\omega = \sqrt{\frac{k}{m}}}$ Angular frequency $(\omega = 2\pi f = 2\pi / T)$

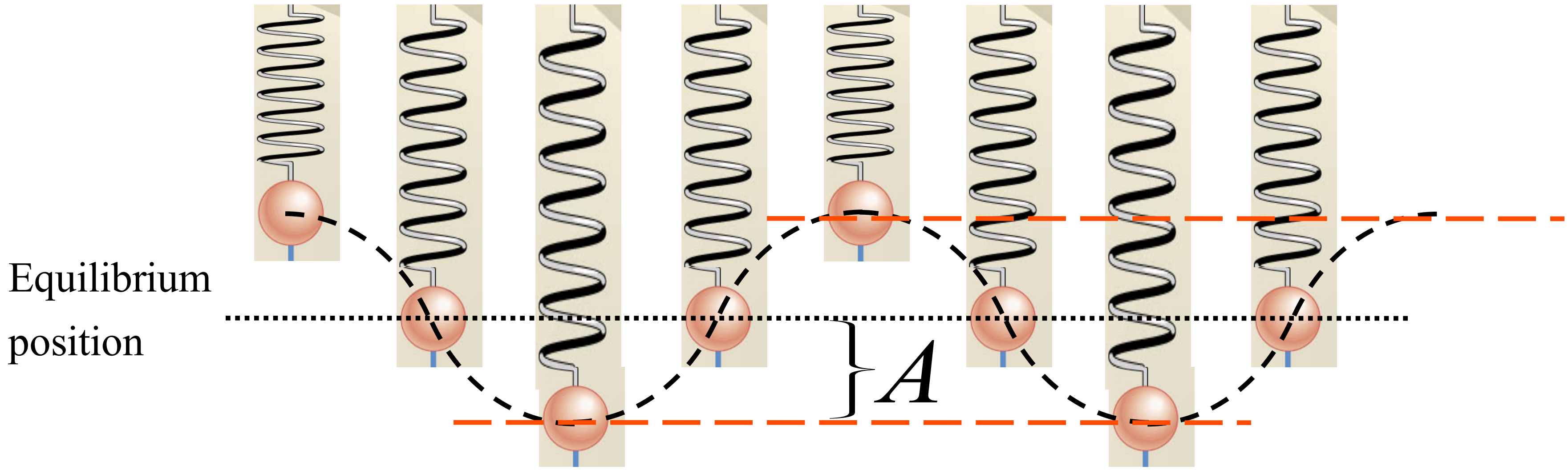
position: $\boxed{x = A\cos(\omega t)}$

velocity: $\boxed{v_x = -\underbrace{A\omega}_{v_{\max}}\sin(\omega t)}$

acceleration: $\boxed{a_x = -\underbrace{A\omega^2}_{a_{\max}}\cos\omega t}$

7.3 Energy in Simple Harmonic Motion

Consider this motion taking place far from the Earth



Speed maximum at equilibrium position

Energy all in kinetic energy: $E = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}mA^2\omega^2$

At highest and lowest point energy is

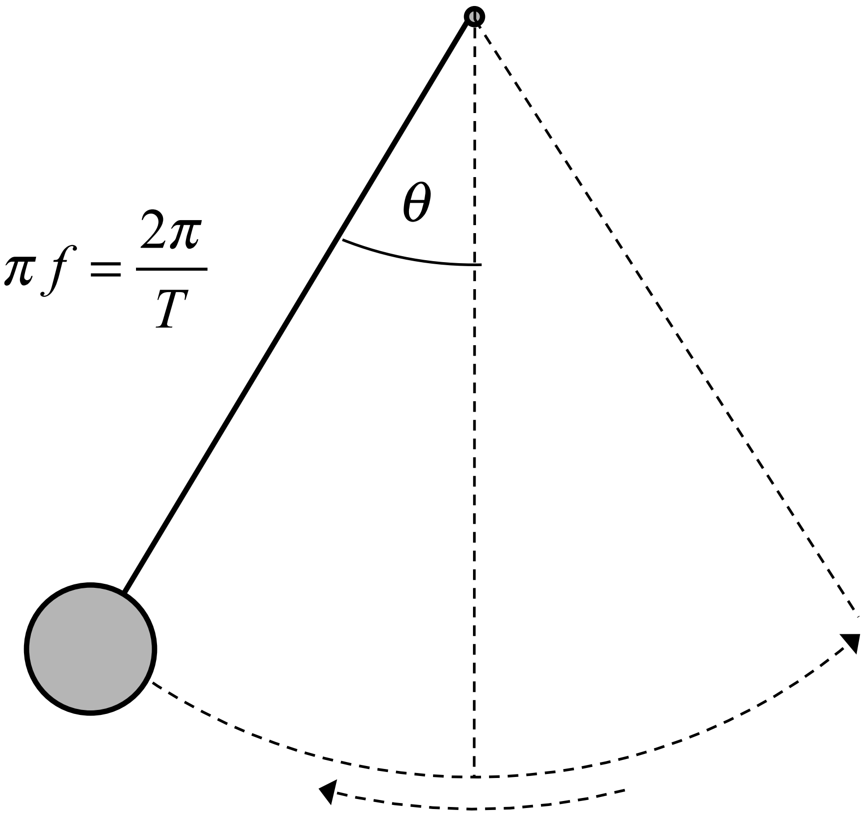
all in spring potential energy: $U_S = \frac{1}{2}kA^2 = E_{Total}$

At intermediate points total energy

$$E_{Total} = \frac{1}{2}kA^2 = K + U_S = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

7.5 The Pendulum

A ***simple pendulum*** consists of a particle attached to a frictionless pivot by a cable of negligible mass.



Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{g}{L}} \quad \text{(small angles only)}$$

$$I = mL^2$$

$$\frac{1}{L} = \frac{mL}{I}$$

$$\omega = \sqrt{\frac{mgL}{I}} \quad \text{(small angles only)}$$

Works for objects with moment of inertia, I and distance to center of mass, L_{CM}

Clicker Question 7.2

At the surface of Mars, the acceleration due to gravity is 3.71 m/s^2 . What is the length of a pendulum on Mars that oscillates with a period of one second?

a) 0.0940 m

b) 0.143 m

c) 0.248 m

d) 0.296 m

e) 0.655 m

$$\omega_{\text{pendulum}} = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

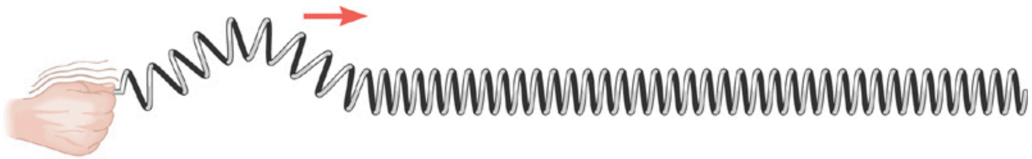
Chapter 11

Waves & Sound

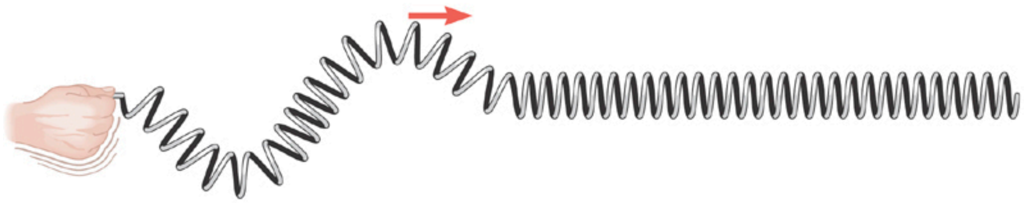
11.1 The Nature of Waves

1. A wave is a traveling disturbance.

2. A wave carries energy from place to place.



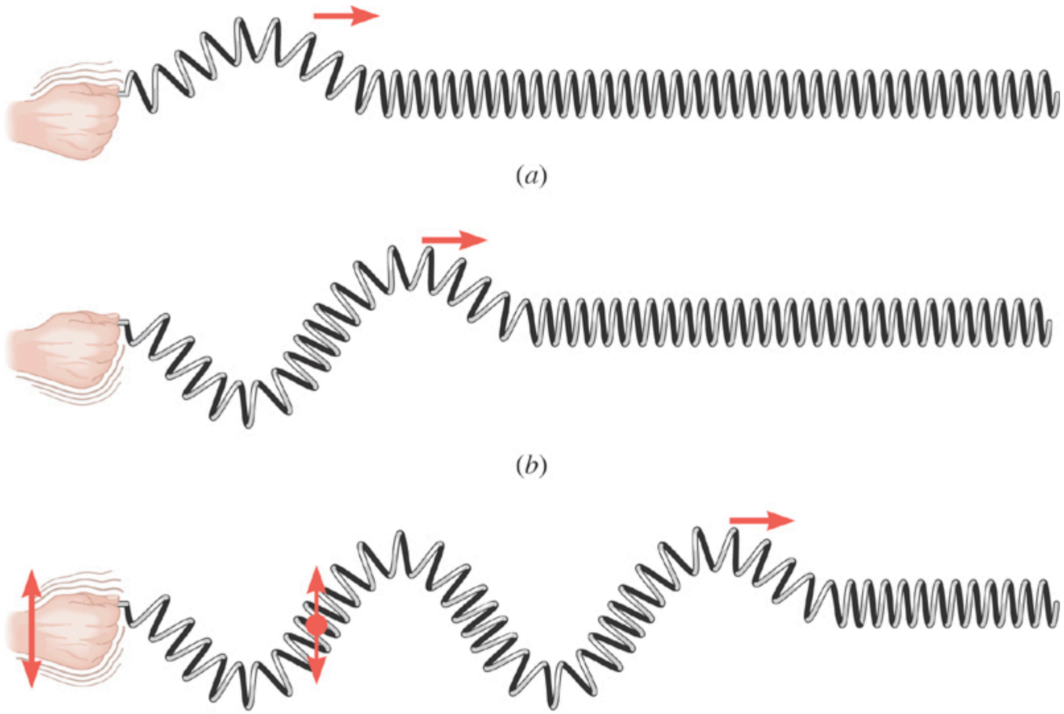
(a)



(b)

(c)

Transverse Wave

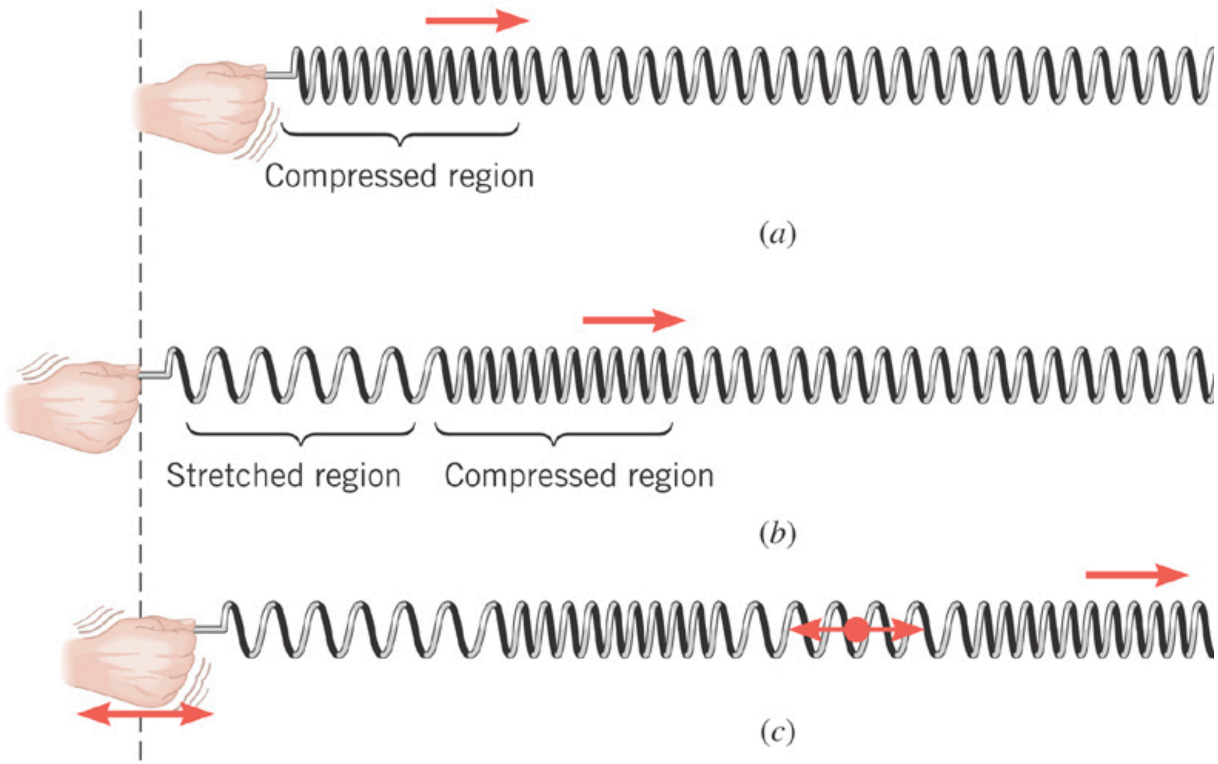


(a)

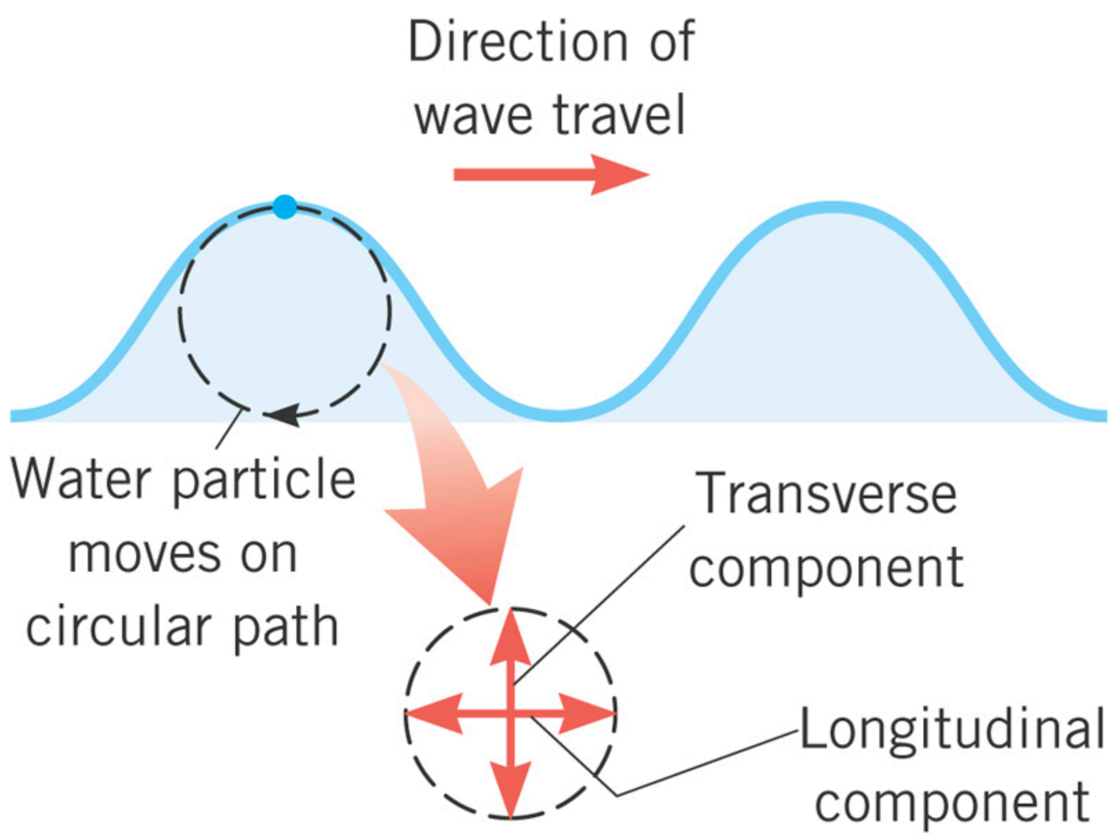
(b)

(c)

Longitudinal Wave



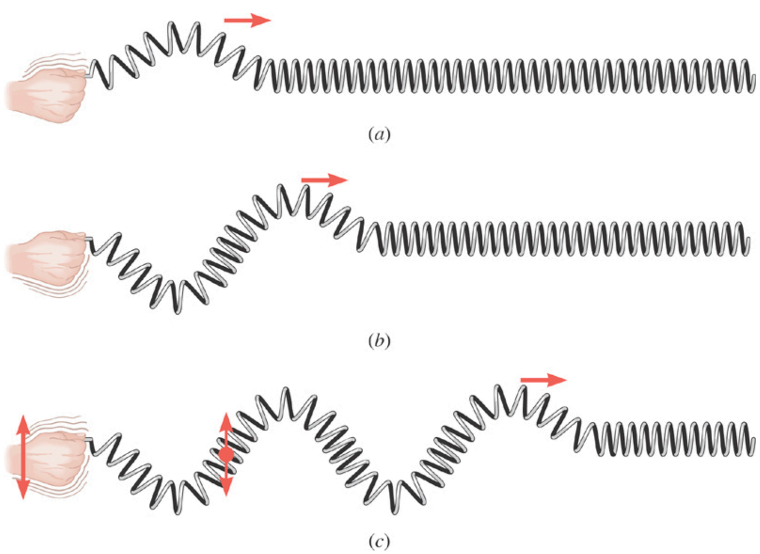
Water waves are partially transverse and partially longitudinal.

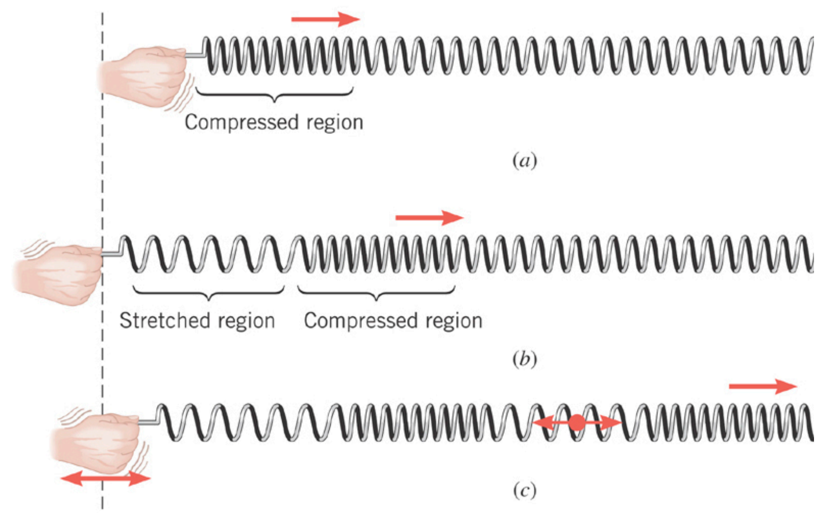


11.2 Periodic Waves

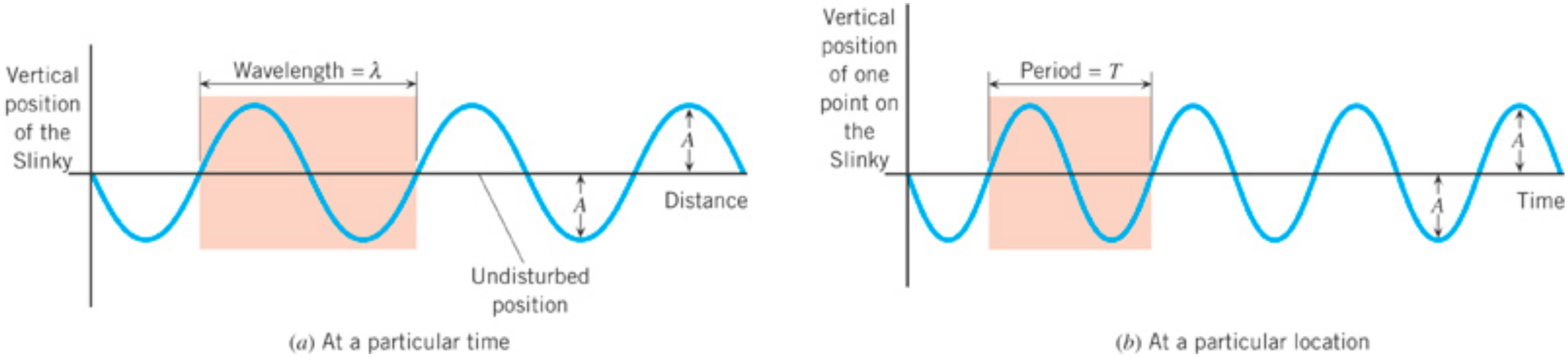
Periodic waves consist of cycles or patterns that are produced over and over again by the source.

In the figures, every segment of the slinky vibrates in simple harmonic motion, provided the end of the slinky is moved in simple harmonic motion.





11.2 Periodic Waves



In the drawing, one ***cycle*** is shaded in color.

The ***amplitude*** A is the maximum excursion of a particle of the medium from the particles undisturbed position.

The ***wavelength*** is the horizontal length of one cycle of the wave.

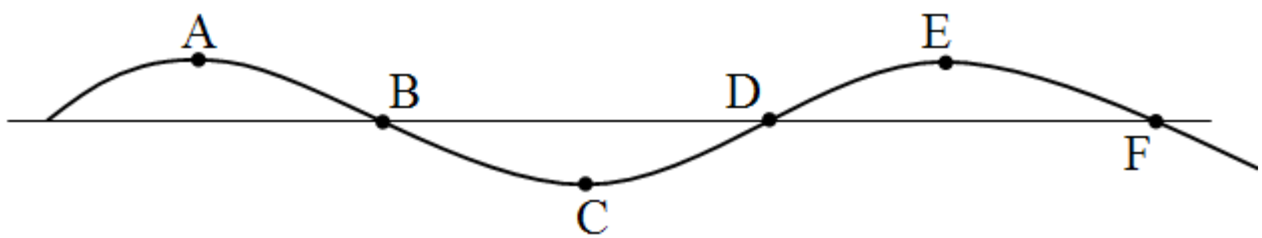
The ***period*** is the time required for one complete cycle.

The ***frequency*** is related to the period and has units of Hz, or s^{-1}.

$$f = \frac{1}{T}$$

Clicker Question 11.1

The drawing shows the vertical position of points along a string versus distance as a wave travels along the string. Six points on the wave are labeled A, B, C, D, E, and F. Between which two points is the length of the segment equal to one wavelength



a) A to E

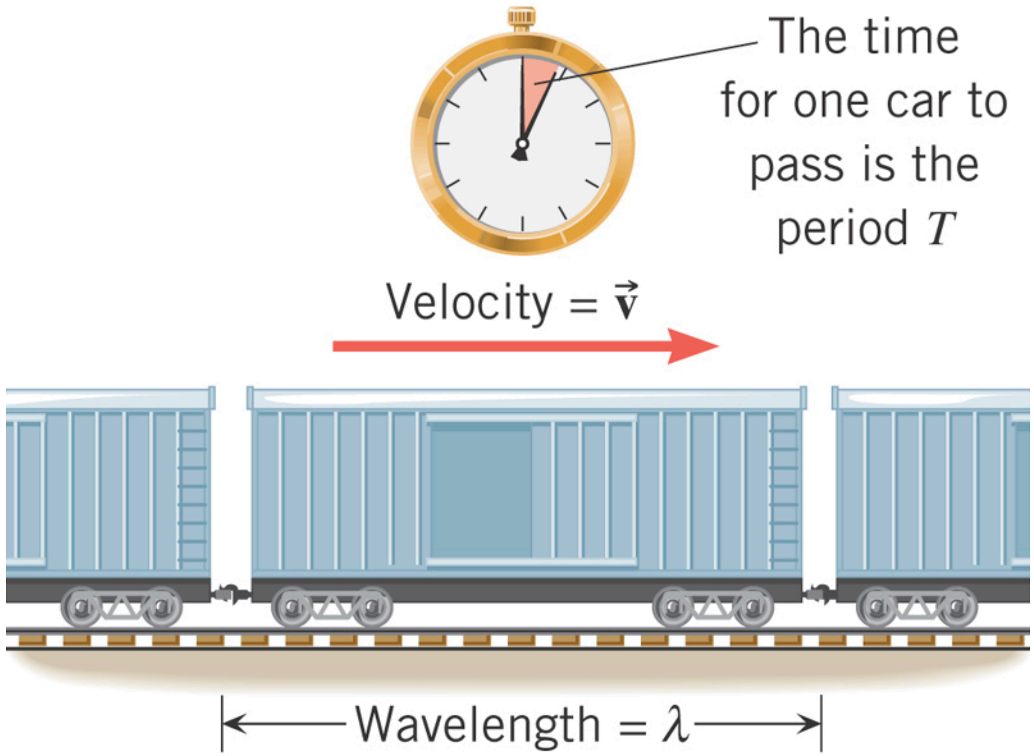
b) B to D

c) A to C

d) A to F

e) C to F

11.2 Periodic Waves



$$\boxed{vT = \lambda; \quad f = \frac{1}{T}}$$

$$\boxed{v = \frac{\lambda}{T} = f\lambda \quad \Rightarrow \quad \lambda = \frac{v}{f}}$$

Example: **The Wavelengths of Radio Waves**

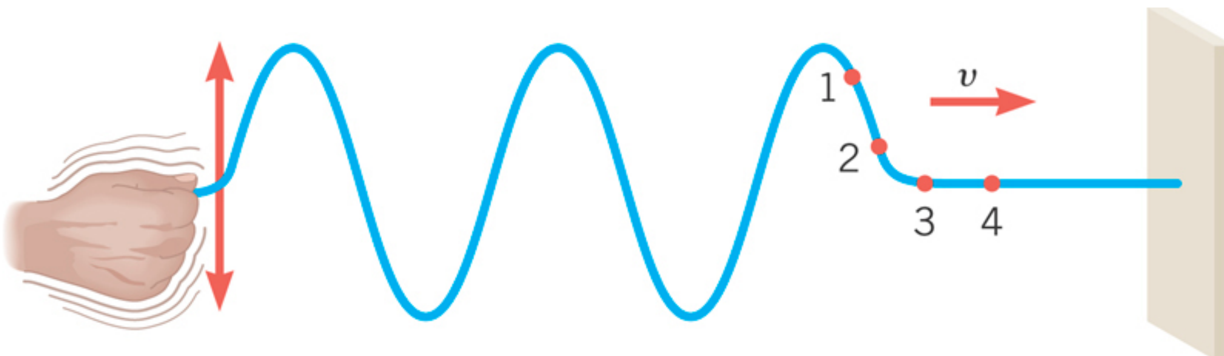
AM and FM radio waves are transverse waves consisting of electric and magnetic field disturbances traveling at a speed of 3.00×10^8 m/s. A station broadcasts AM radio waves whose frequency is 1230×10^3 Hz and an FM radio wave whose frequency is 91.9×10^6 Hz. Find the distance between adjacent crests in each wave.

$$\lambda_{\mathrm{AM}} = \frac{v}{f} = \frac{3.00 \times 10^8 \ \mathrm{m/s}}{1230 \times 10^3 \mathrm{Hz}} = 244 \ \mathrm{m}$$

$$\lambda_{\mathrm{FM}} = \frac{v}{f} = \frac{3.00 \times 10^8 \ \mathrm{m/s}}{91.9 \times 10^6 \mathrm{Hz}} = 3.26 \ \mathrm{m}$$

11.3 The Speed of a Wave on a String

The speed at which the wave moves to the right depends on how quickly one particle of the string is accelerated upward in response to the net pulling force.



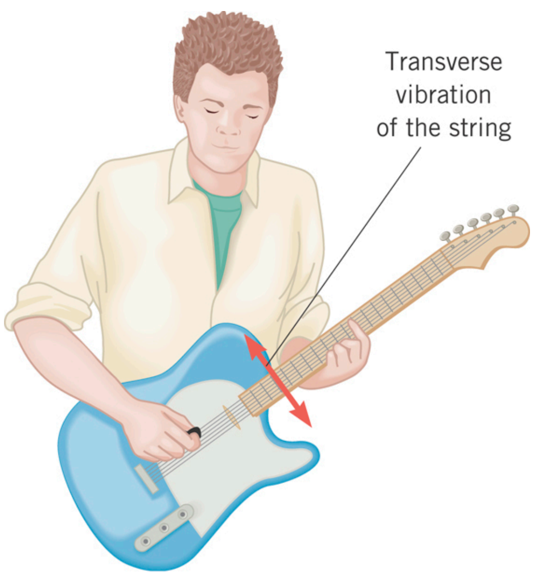
$$\boxed{v = \sqrt{\frac{F}{m/L}}}$$

Tension: F

Linear mass density: m/L

Example: Waves Traveling on Guitar Strings

Transverse waves travel on each string of an electric guitar after the string is plucked. The length of each string between its two fixed ends is 0.628 m, and the mass is 0.208 g for the highest pitched E string and 3.32 g for the lowest pitched E string. Each string is under a tension of 226 N. Find the speeds of the waves on the two strings.



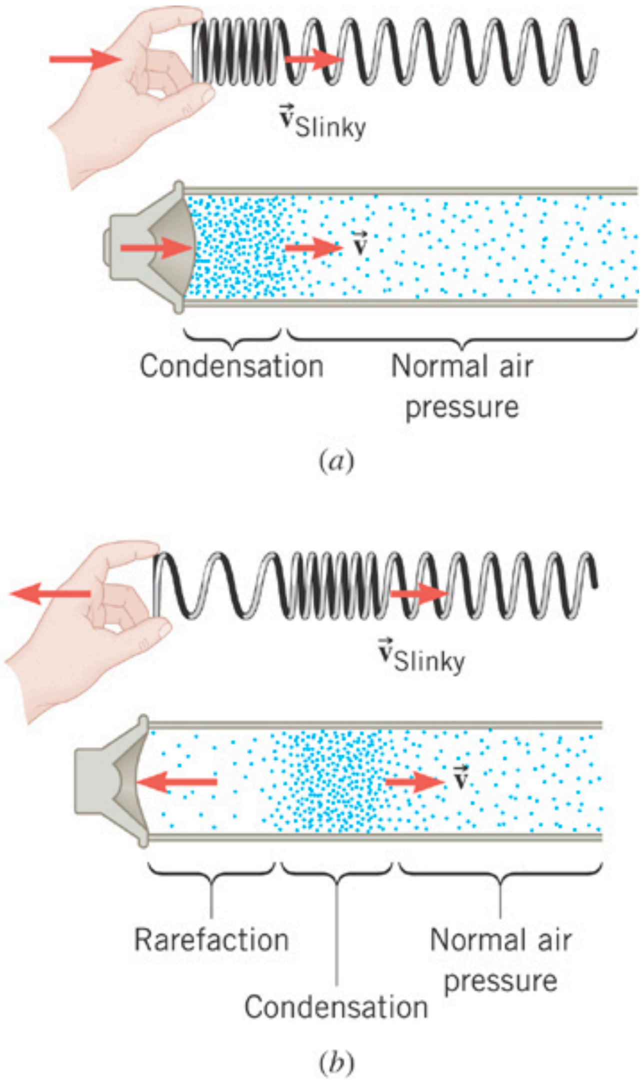
High E

$$v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{226\text{ N}}{\left(0.208\times10^{-3}\text{kg}\right)/\left(0.628\text{ m}\right)}} = 826\,\text{m/s}$$

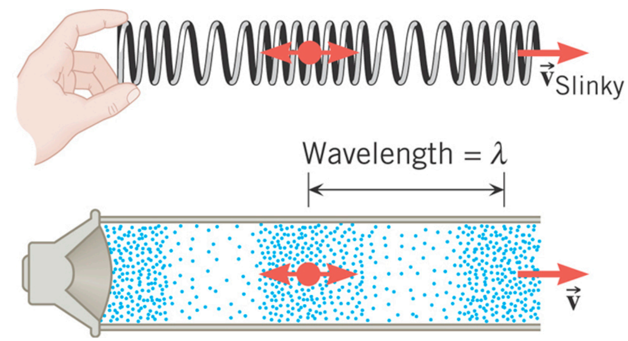
Low E

$$v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{226\text{ N}}{\left(3.32\times10^{-3}\text{kg}\right)/\left(0.628\text{ m}\right)}} = 207\,\text{m/s}$$

LONGITUDINAL SOUND WAVES



The distance between adjacent condensations is equal to the wavelength of the sound wave.



THE FREQUENCY OF A SOUND WAVE

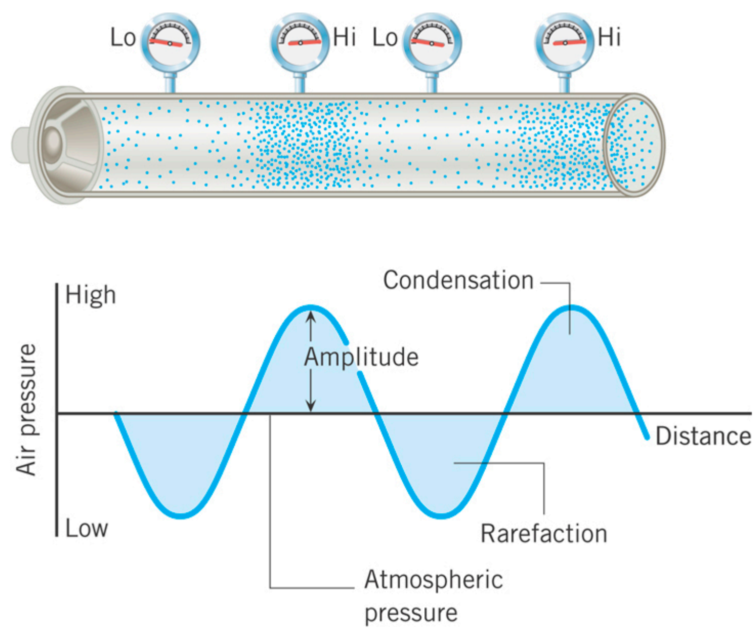
The ***frequency*** is the number of cycles per second.

A sound with a single frequency is called a ***pure tone***.

The brain interprets the frequency in terms of the subjective quality called ***pitch***.

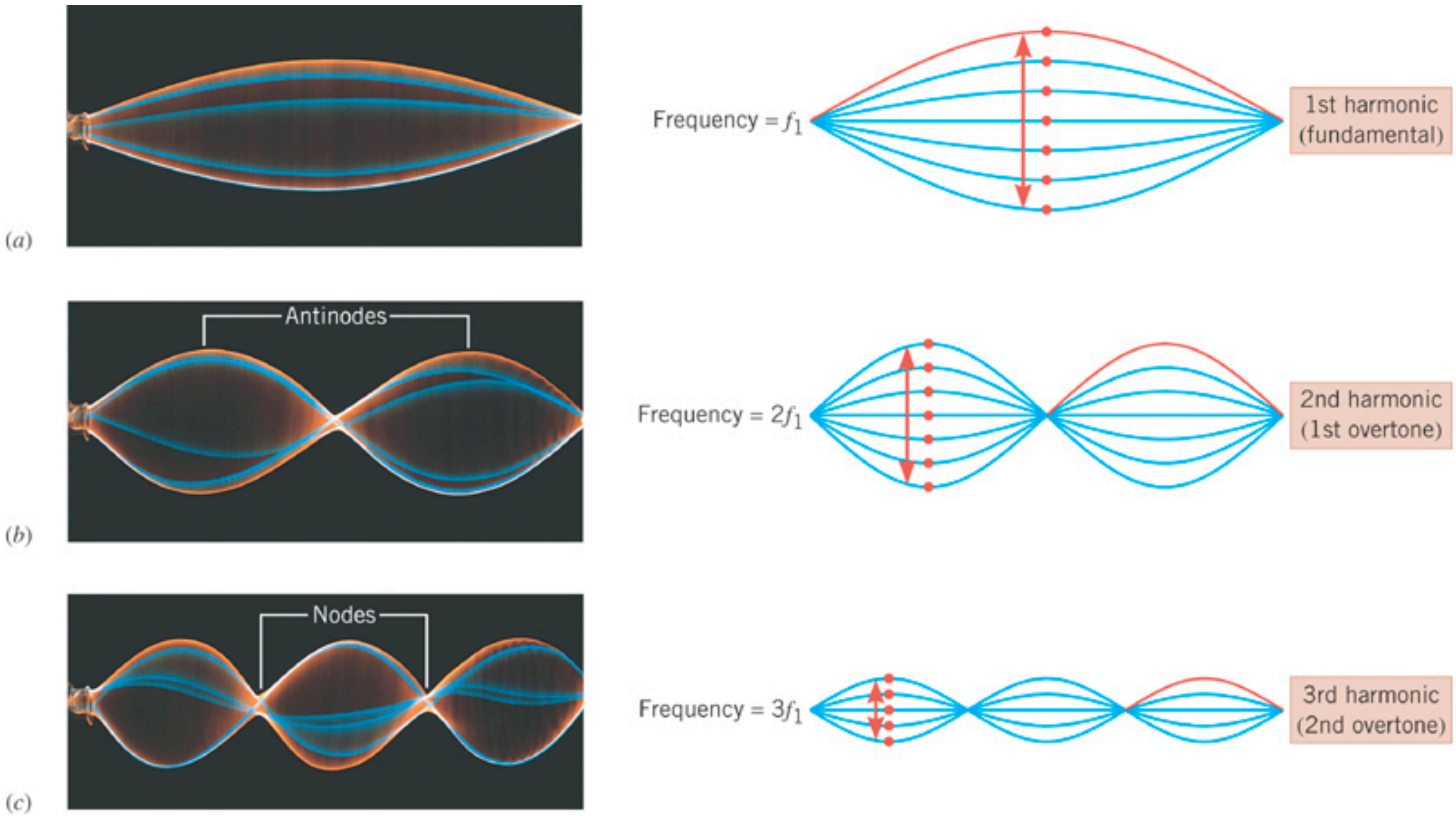
THE AMPLITUDE OF A SOUND WAVE

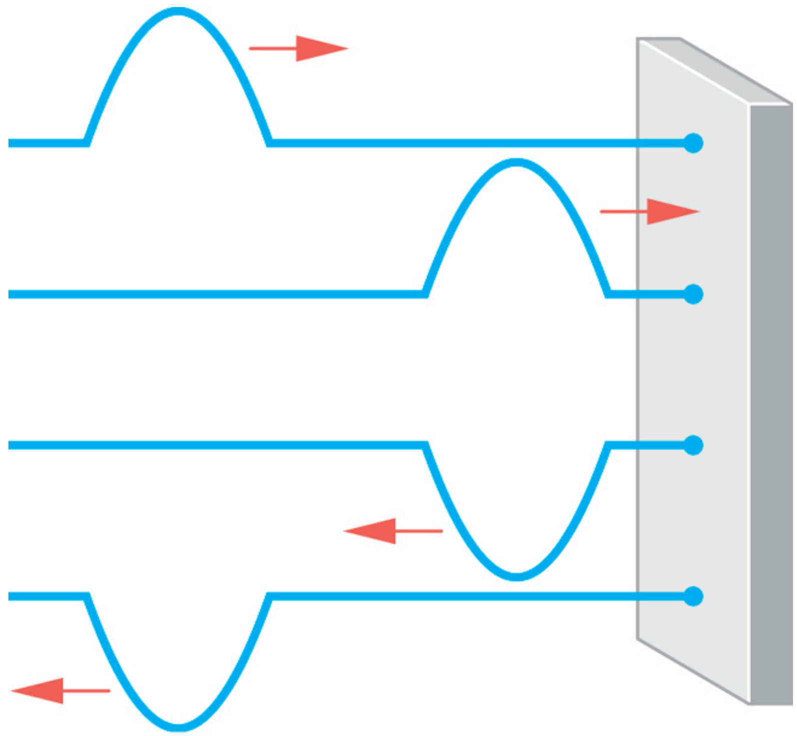
Loudness is an attribute of a sound that depends primarily on the pressure amplitude of the wave.



11.3 Transverse Standing Waves

Transverse standing wave patters.



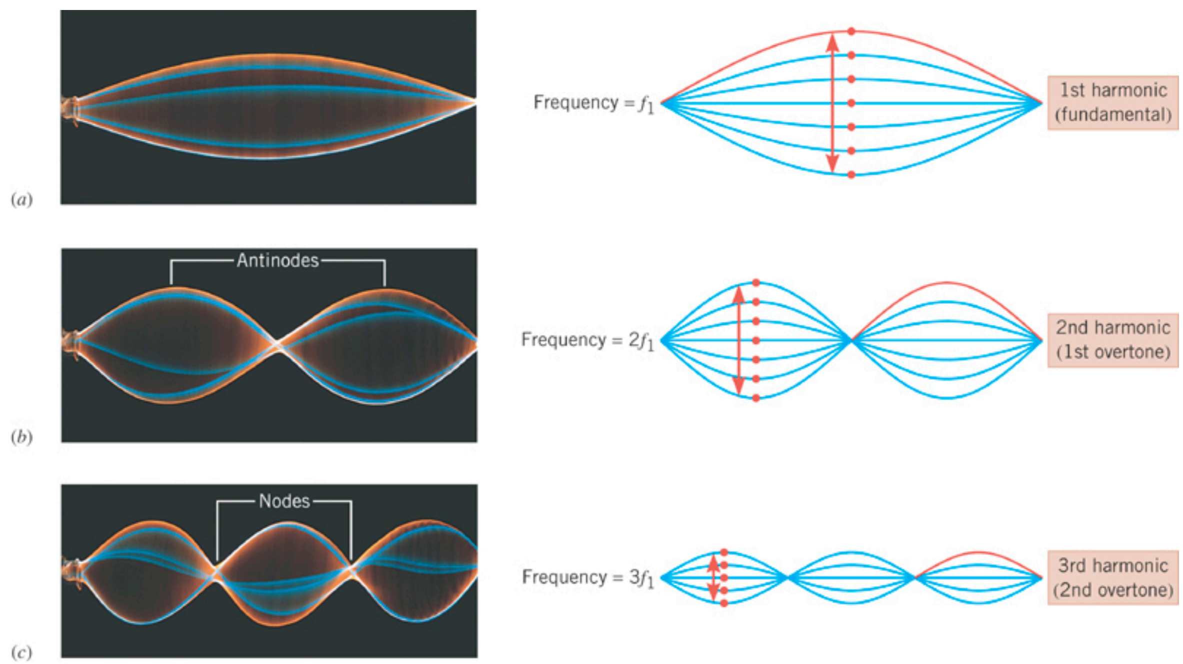


In reflecting from the wall, a forward-traveling half-cycle becomes a backward-traveling half-cycle that is inverted.

Unless the timing is right, the newly formed and reflected cycles tend to offset one another.

Repeated reinforcement between newly created and reflected cycles causes a large amplitude standing wave to develop.

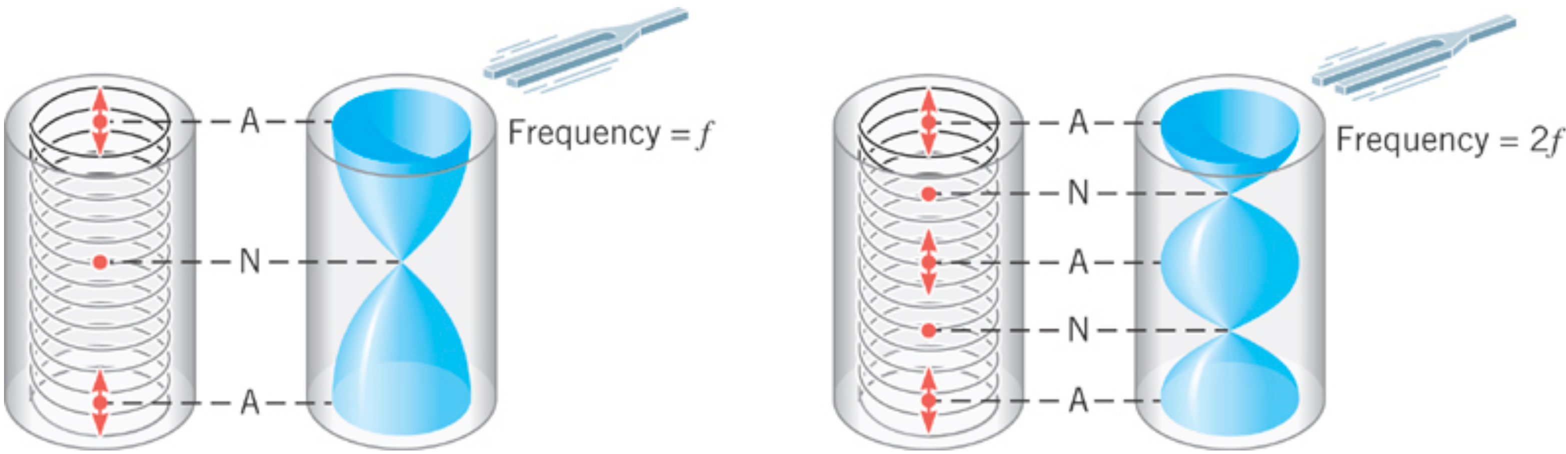
11.3 Transverse Standing Waves



String fixed at both ends

$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

11.3 Longitudinal Standing Waves



Tube open at both ends

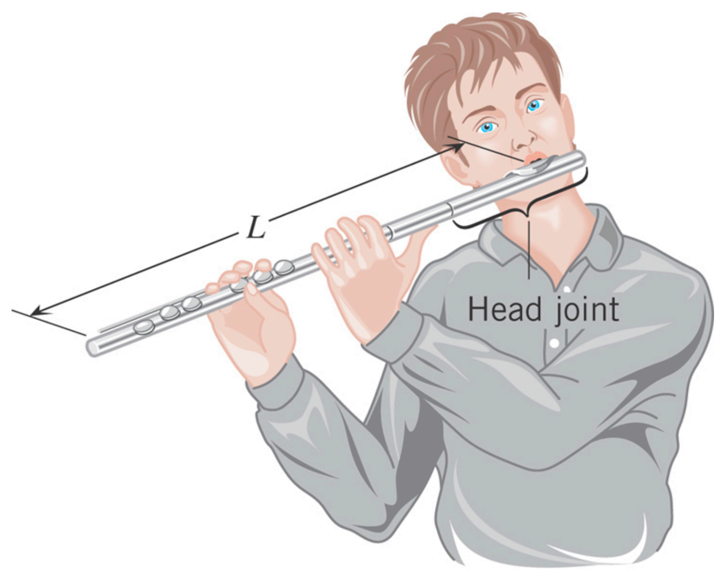
$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

Example 6 Playing a Flute

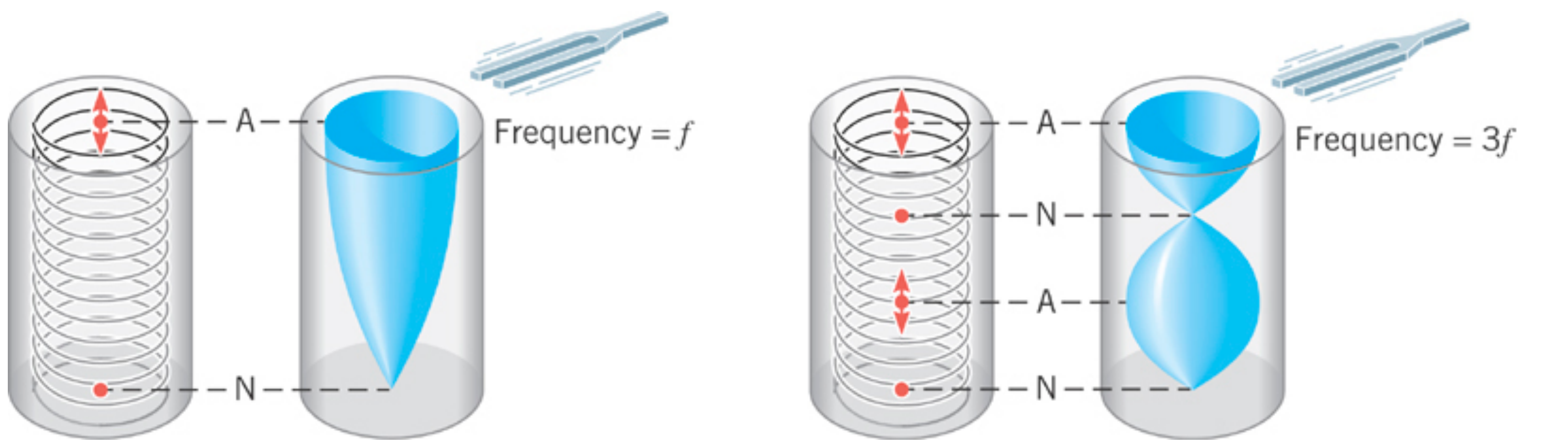
When all the holes are closed on one type of flute, the lowest note it can sound is middle C (261.6 Hz). If the speed of sound is 343 m/s, and the flute is assumed to be a cylinder open at both ends, determine the distance L.

$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

$$L = \frac{nv}{2f_n} = \frac{1(343\,\mathrm{m/s})}{2(261.6\,\mathrm{Hz})} = 0.656\,\mathrm{m}$$



11.3 Longitudinal Standing Waves



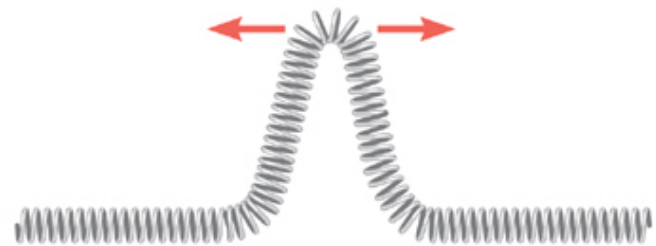
Tube open at one end

$$f_n = n\left(\frac{v}{4L}\right) \qquad n = 1, 3, 5, \ldots$$

11.3 The Principle of Linear Superposition

When the pulses merge, the Slinky assumes a shape that is the sum of the shapes of the individual pulses.

(*a*) Overlap begins

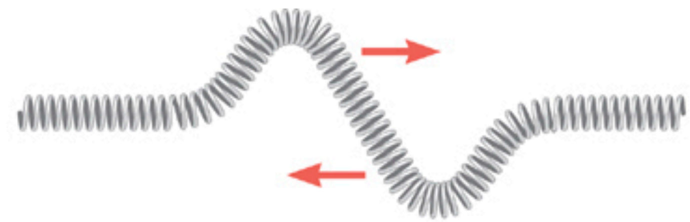


(*b*) Total overlap; the Slinky has twice the height of either pulse

(*c*) The receding pulses

11.3 The Principle of Linear Superposition

When the pulses merge, the Slinky assumes a shape that is the sum of the shapes of the individual pulses.



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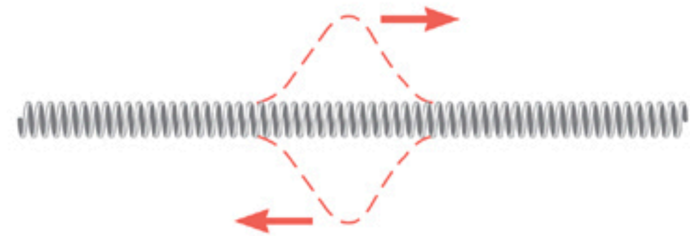
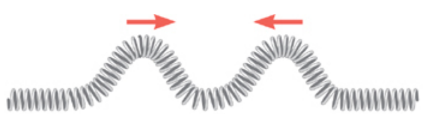


(*b*) Total overlap

(*c*) The receding pulses

THE PRINCIPLE OF LINEAR SUPERPOSITION

When two or more waves are present simultaneously at the same place, the resultant disturbance is the sum of the disturbances from the individual waves.



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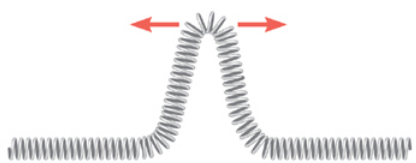
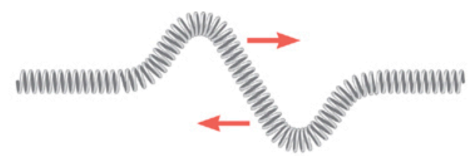
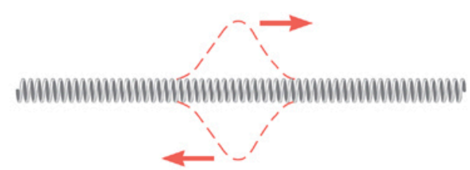


(*b*) Total overlap; the Slinky has twice the height of either pulse

(*c*) The receding pulses



(*a*) Overlap begins

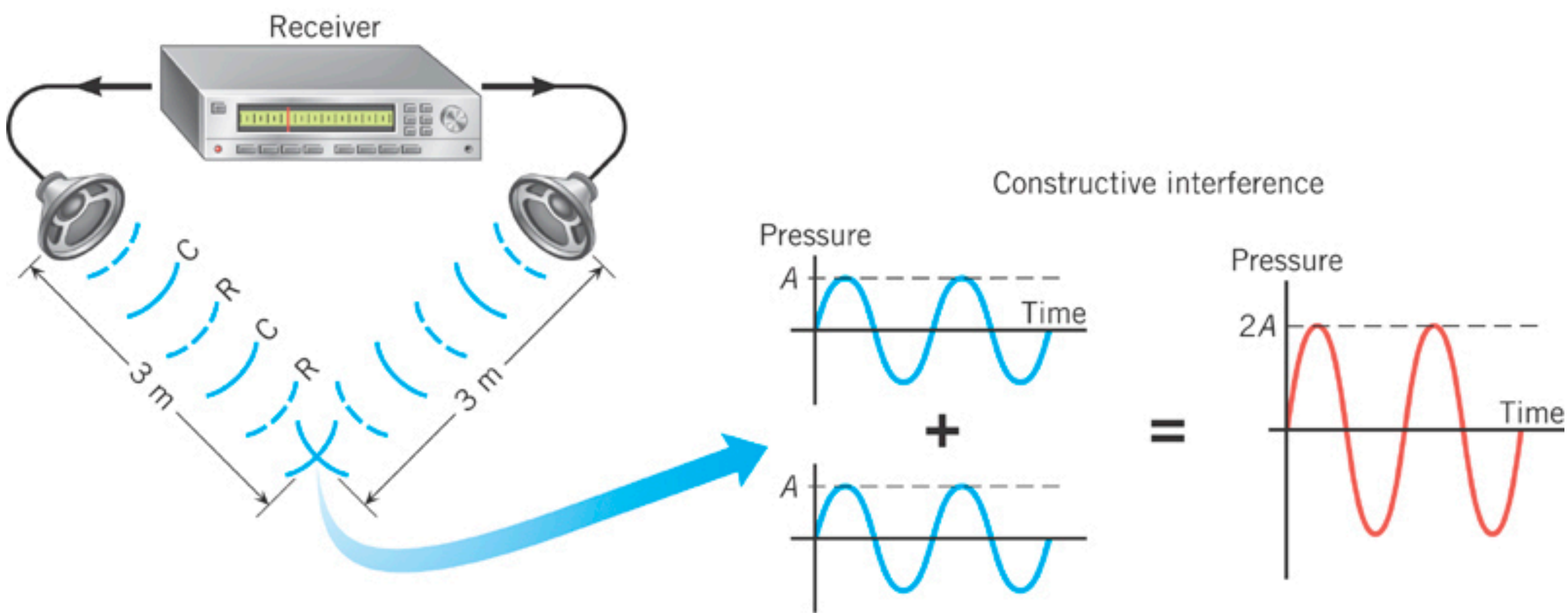


(*b*) Total overlap

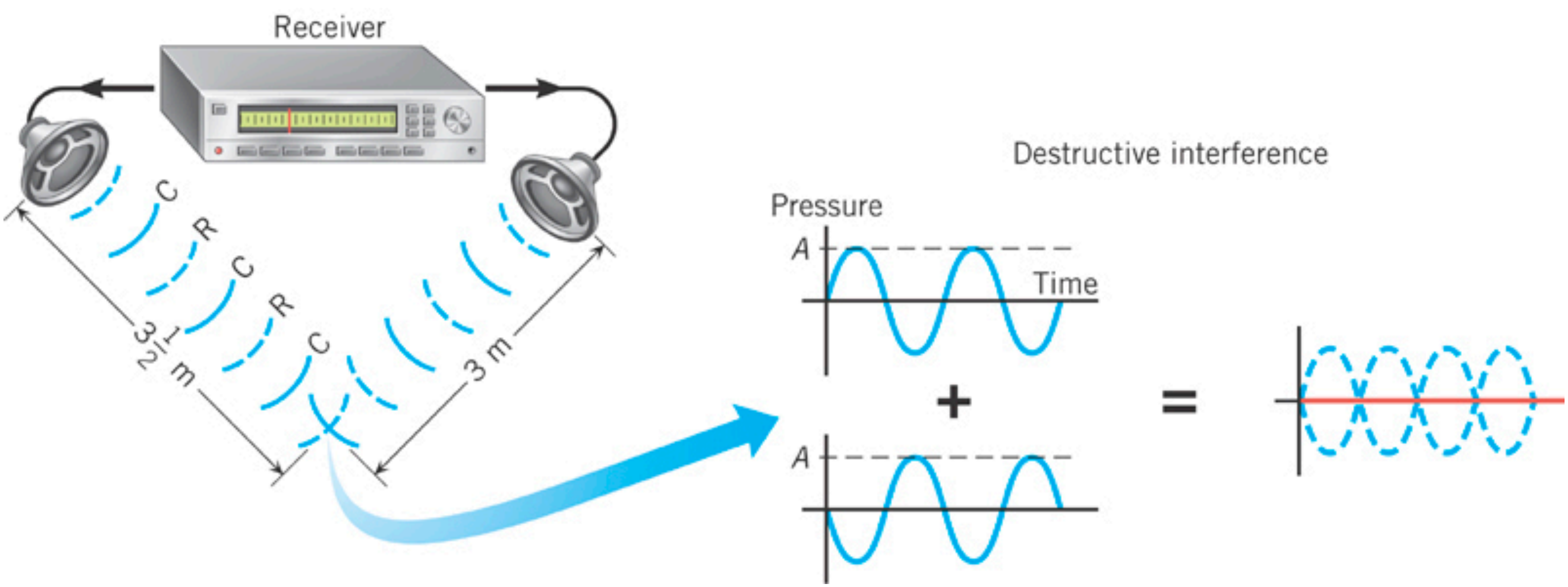
(*c*) The receding pulses

11.3 Constructive and Destructive Interference of Sound Waves

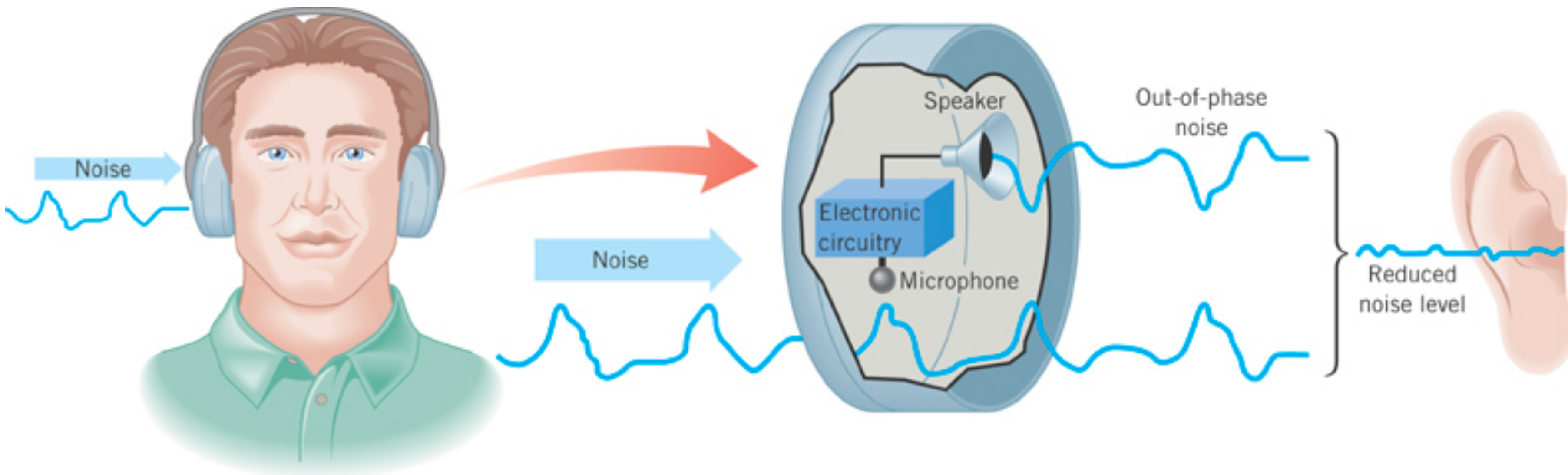
When two waves always meet condensation-to-condensation and rarefaction-to-rarefaction, they are said to be ***exactly in phase*** and to exhibit ***constructive interference.***



When two waves always meet condensation-to-rarefaction, they are said to be ***exactly out of phase*** and to exhibit ***destructive interference.***



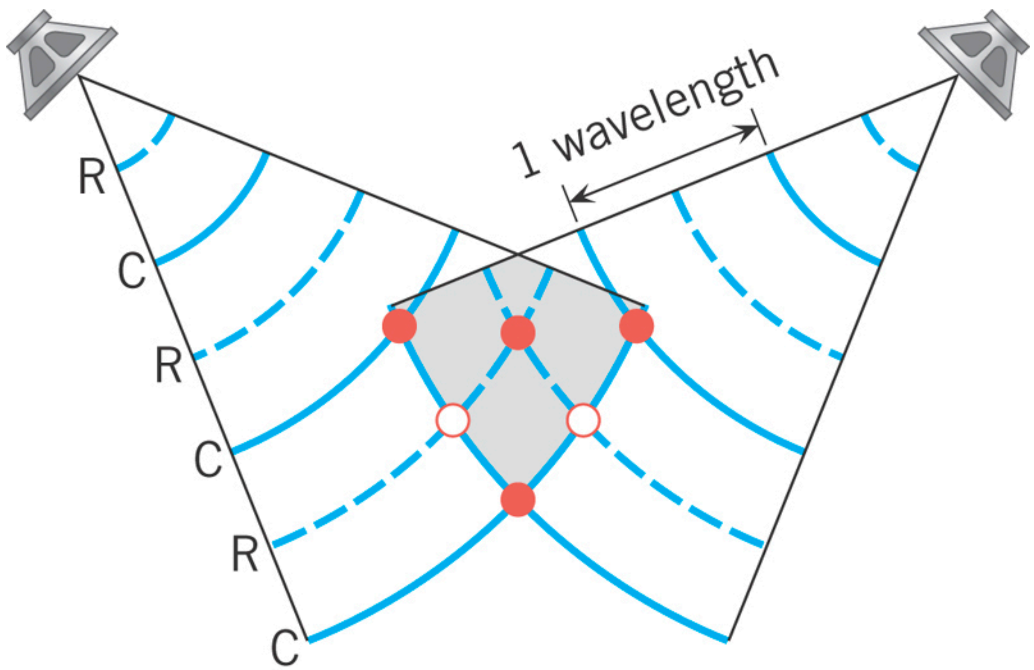
11.3 Constructive and Destructive Interference of Sound Waves



11.3 Constructive and Destructive Interference of Sound Waves

If the wave patters do not shift relative to one another as time passes, the sources are said to be ***coherent***.

For two wave sources vibrating in phase, a difference in path lengths that is zero or an integer number (1, 2, 3, . .) of wavelengths leads to constructive interference; a difference in path lengths that is a half-integer number (½ , 1 ½, 2 ½, . .) of wavelengths leads to destructive interference.



11.3 Sound Intensity

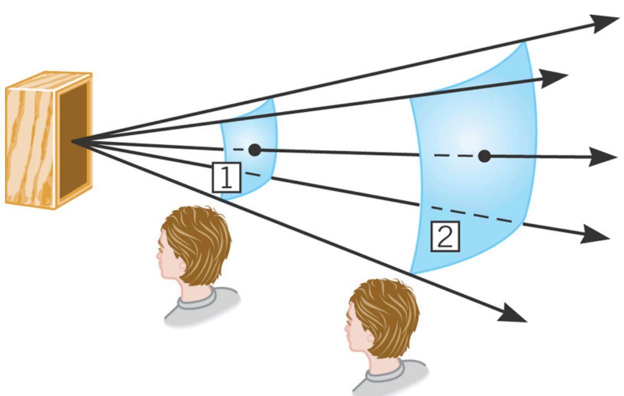
The amount of energy transported per second is called the ***power*** of the wave.

The ***sound intensity*** is defined as the power that passes perpendicularly through a surface divided by the area of that surface.

$$\boxed{I = P/A; \quad \text{power: } P \quad \text{(watts)}}$$

Example 6 Sound Intensities

12x10^{-5} W of sound power passed through the surfaces labeled 1 and 2. The areas of these surfaces are 4.0m^2 and 12m^2. Determine the sound intensity at each surface.



$$I_1 = \frac{P}{A_1} = \frac{12\times 10^{-5}\,\text{W}}{4.0\text{m}^2} = 3.0\times 10^{-5}\,\text{W/m}^2$$

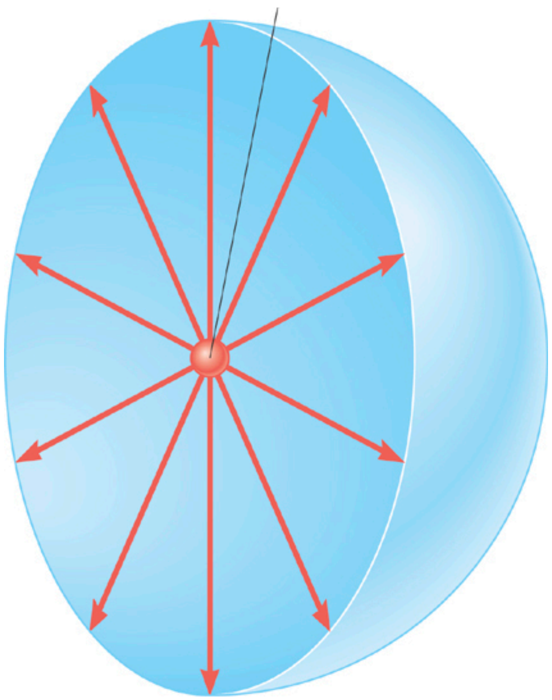
$$I_2 = \frac{P}{A_2} = \frac{12\times 10^{-5}\,\text{W}}{12\text{m}^2} = 1.0\times 10^{-5}\,\text{W/m}^2$$

11.3 Sound Intensity

For a 1000 Hz tone, the smallest sound intensity that the human ear can detect is about $1 \times 10^{-12}\ W/m^2$. This intensity is called the ***threshold of hearing***.

On the other extreme, continuous exposure to intensities greater than $1 W/m^2$ can be painful.

If the source emits sound *uniformly in all directions,* the intensity depends on the distance from the source in a simple way.



$$\boxed{I = \frac{P}{4\pi r^2}}$$

Intensity depends inversely on the square of the distance from the source.

11.3 Decibels

The ***decibel*** (dB) is a measurement unit used when comparing two sound Intensities.

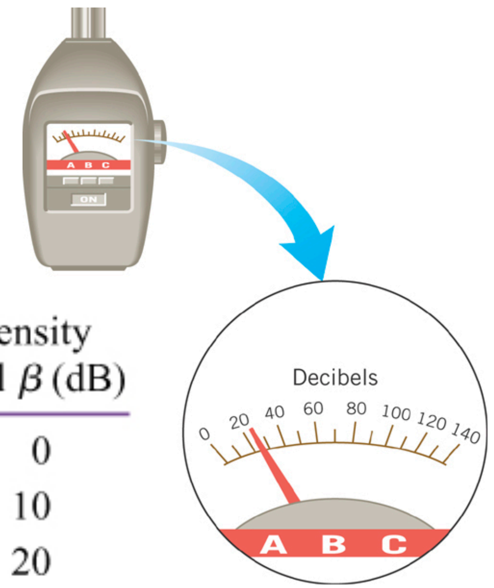
Human hearing mechanism responds to sound ***intensity level*** , logarithmically.

$$\boxed{\beta = (10\text{ dB})\log\left(\frac{I}{I_o}\right)}$$

$$\boxed{\text{dB (decibel)}}$$

$$\boxed{I_o = 1.00\times 10^{-12}\ \text{W/m}^2}$$

Note that log(1) = 0



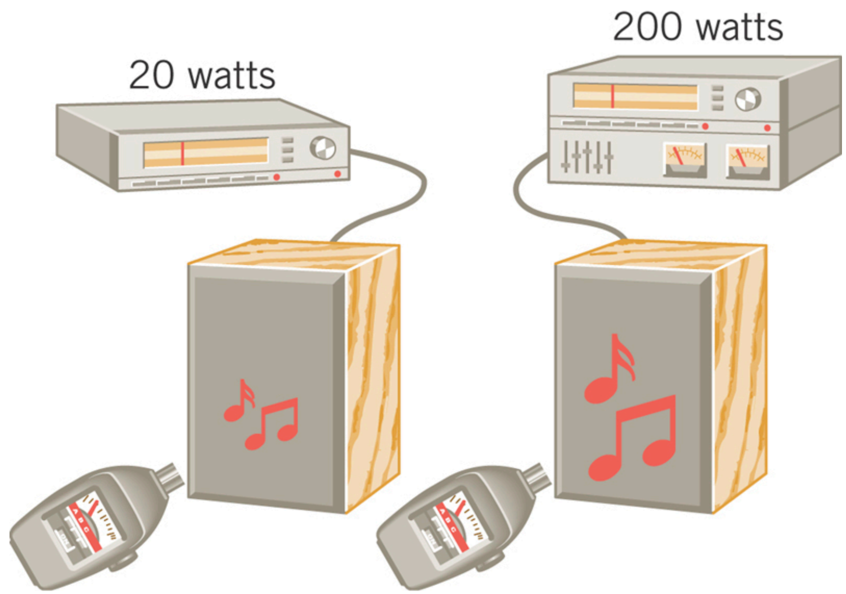
	Intensity I (W/m^2)	Intensity Level β (dB)
Threshold of hearing	1.0×10^{-12}	0
Rustling leaves	1.0×10^{-11}	10
Whisper	1.0×10^{-10}	20
Normal conversation (1 meter)	3.2×10^{-6}	65
Inside car in city traffic	1.0×10^{-4}	80
Car without muffler	1.0×10^{-2}	100
Live rock concert	1.0	120
Threshold of pain	10	130

Example: **Comparing Sound Intensities**

Audio system 1 produces a sound intensity level of 90.0 dB, and system 2 produces an intensity level of 93.0 dB. Determine the ratio of intensities.

$$\beta = (10\text{ dB})\log\left(\frac{I}{I_o}\right)$$

$$90\text{ dB} = (10\text{ dB})\log(I/I_o)$$
$$\log(I/I_o) = 9;$$
$$I = I_o \times 10^9 = (1\times10^{-12}\text{ W/m}^2)\times10^9$$
$$= 1\times10^{-3}\text{ W/m}^2$$



$$93\text{ dB} = (10\text{ dB})\log(I/I_o)$$
$$\log(I/I_o) = 9.3;$$
$$I = I_o \times 10^{9.3} = (1\times10^{-12}\text{ W/m}^2)\times10^{9.3}$$
$$= 1\times10^{-3.3}\text{ W/m}^2 = 1\times10^{-3}(10^{0.3})\text{W/m}^2$$
$$= 1\times10^{-3}(2)\text{W/m}^2 = 2\times10^{-3}\text{W/m}^2$$

$$93\text{dB} = 90\text{dB} + 3\text{dB}$$

Adding 3dB results in a factor of 2

$$3\text{ dB} = (10\text{dB})\log(I_2/I_1)$$
$$0.3 = \log(I_2/I_1);$$
$$I_2 = 10^{0.3} I_1 = 2I_1$$

Clicker Question 11.2

Software is used to amplify a digital sound file on a computer by 20 dB. By what factor has the intensity of the sound been increased as compared to the original sound file?

$$\boxed{\beta = (10\ \mathrm{dB})\log\left(\frac{I}{I_o}\right)}$$

a) 2

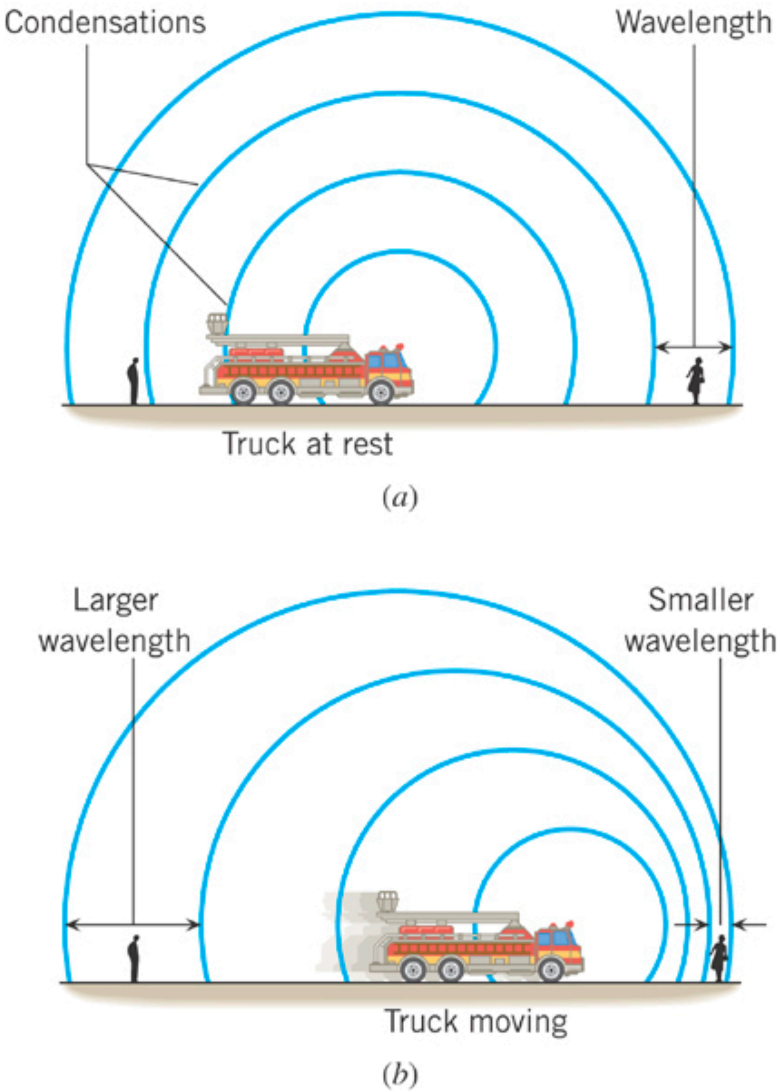
b) 5

c) 10

d) 20

e) 100

11.5 The Doppler Effect



The ***Doppler effect*** is the change in frequency or pitch of the sound detected by an observer because the sound source and the observer have different velocities with respect to the medium of sound propagation.

SOURCE (s) MOVING AT v_s TOWARD OBSERVER (o)

$$f_o = f_s\left(\frac{1}{1 - v_s/v}\right)$$

SOURCE (s) MOVING AT v_s AWAY FROM OBSERVER (o)

$$f_o = f_s\left(\frac{1}{1 + v_s/v}\right)$$