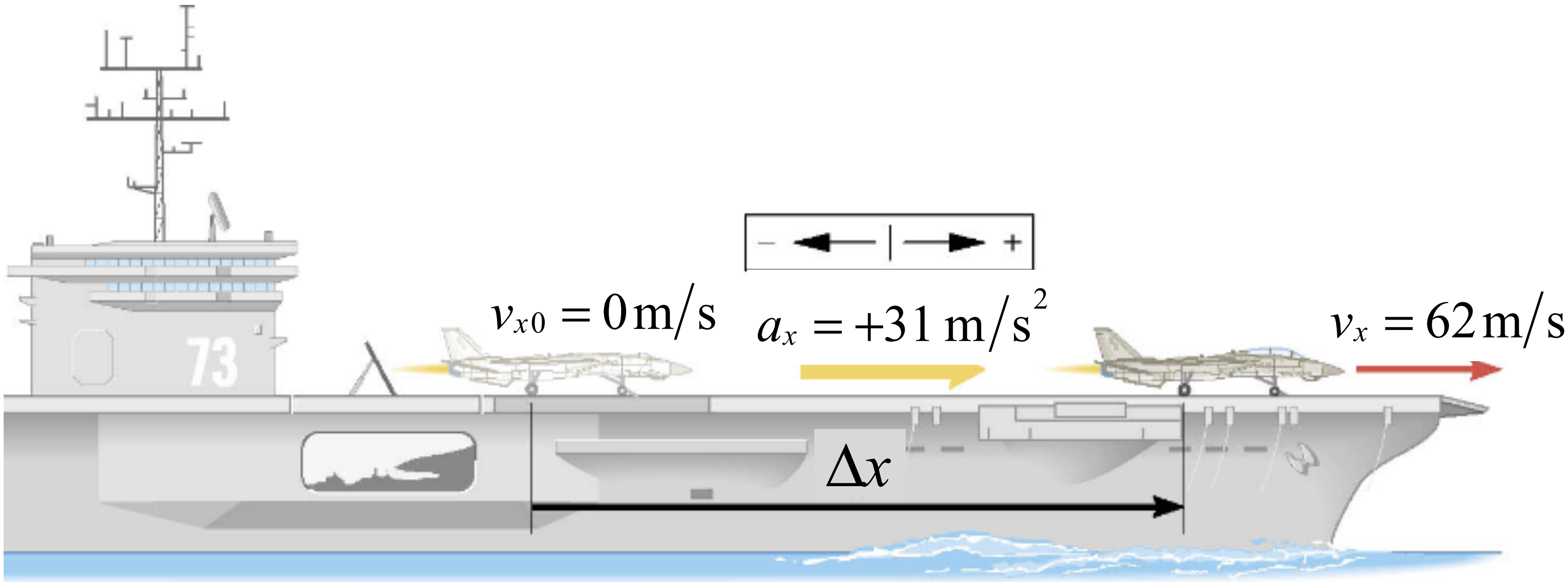
Chapter 2

Kinematics in One Dimension

continued

2.4 Equations of Kinematics for Constant Acceleration



(b)

Example: Catapulting a Jet

Find its displacement.

$$v_{x0} = 0\,\mathrm{m/s} \qquad v_x = +62\,\mathrm{m/s} \qquad a_x = +31\,\mathrm{m/s^2}$$

$$\Delta x = ??$$

2.4 *Equations of Kinematics for Constant Acceleration*

definition of acceleration

$$\boxed{a_x = \frac{v_x - v_{x0}}{t}} \Rightarrow t = \frac{v_x - v_{x0}}{a_x}$$

time velocity is changing

$$\boxed{\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right)t} = \tfrac{1}{2}\left(v_{x0} + v_x\right)\frac{\left(v_x - v_{x0}\right)}{a_x}$$

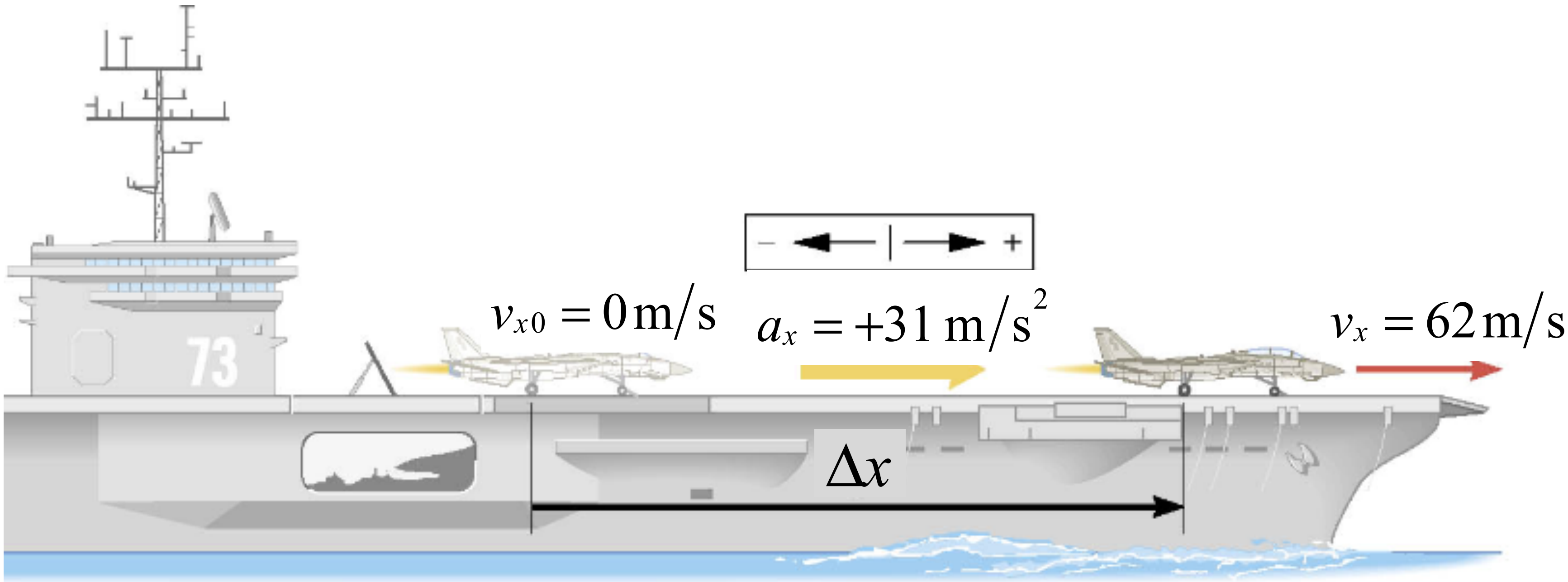
displacement = average velocity × time

$$\Delta x = \frac{v_x^2 - v_{x0}^2}{2a_x}$$

Solve for final velocity

$$\boxed{v_x^2 = v_{x0}^2 + 2a_x\Delta x}$$

2.4 Equations of Kinematics for Constant Acceleration



(b)

$$\Delta x = \frac{v_x^2 - v_{x0}^2}{2a_x} = \frac{(62\,\mathrm{m/s})^2 - (0\,\mathrm{m/s})^2}{2(31\,\mathrm{m/s^2})} = +62\,\mathrm{m}$$

Clicker Question 2.3

A car accelerates from rest to 100 m/s in 200 meters.
What was the acceleration of the car in this motion?

Hint: use $v_x^2 = v_{x0}^2 + 2a_x\Delta x$

a) 12.5 m/s^2

b) 4.00 m/s^2

c) 0.25 m/s^2

d) 400 m/s^2

e) 25.0 m/s^2

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a) 12.5 m/s^2

b) 4.00 m/s^2

c) 0.25 m/s^2

d) 400 m/s^2

e) 25.0 m/s^2

$$v_{x0} = 0; \quad v_x = 100 \text{ m/s}; \quad \Delta x = 200 \text{ m}$$

$$a_x = \frac{v_x^2}{2\Delta x} = \frac{10{,}000 \text{ m}^2/\text{s}^2}{2(200 \text{ m})} = 25.0 \text{ m/s}^2$$

2.4 Equations of Kinematics for Constant Acceleration

Equations of Kinematics for Constant Acceleration

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right) t$$

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

$$\Delta x = v_{x0} t + \tfrac{1}{2} a_x t^2$$

Except for *t*, every variable has a direction and thus can have a positive or negative value.

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (–).

3. Write down the values that are given for any of the five kinematic variables.

4. Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

2.5 *Applications of the Equations of Kinematics*

Example: An Accelerating Spacecraft

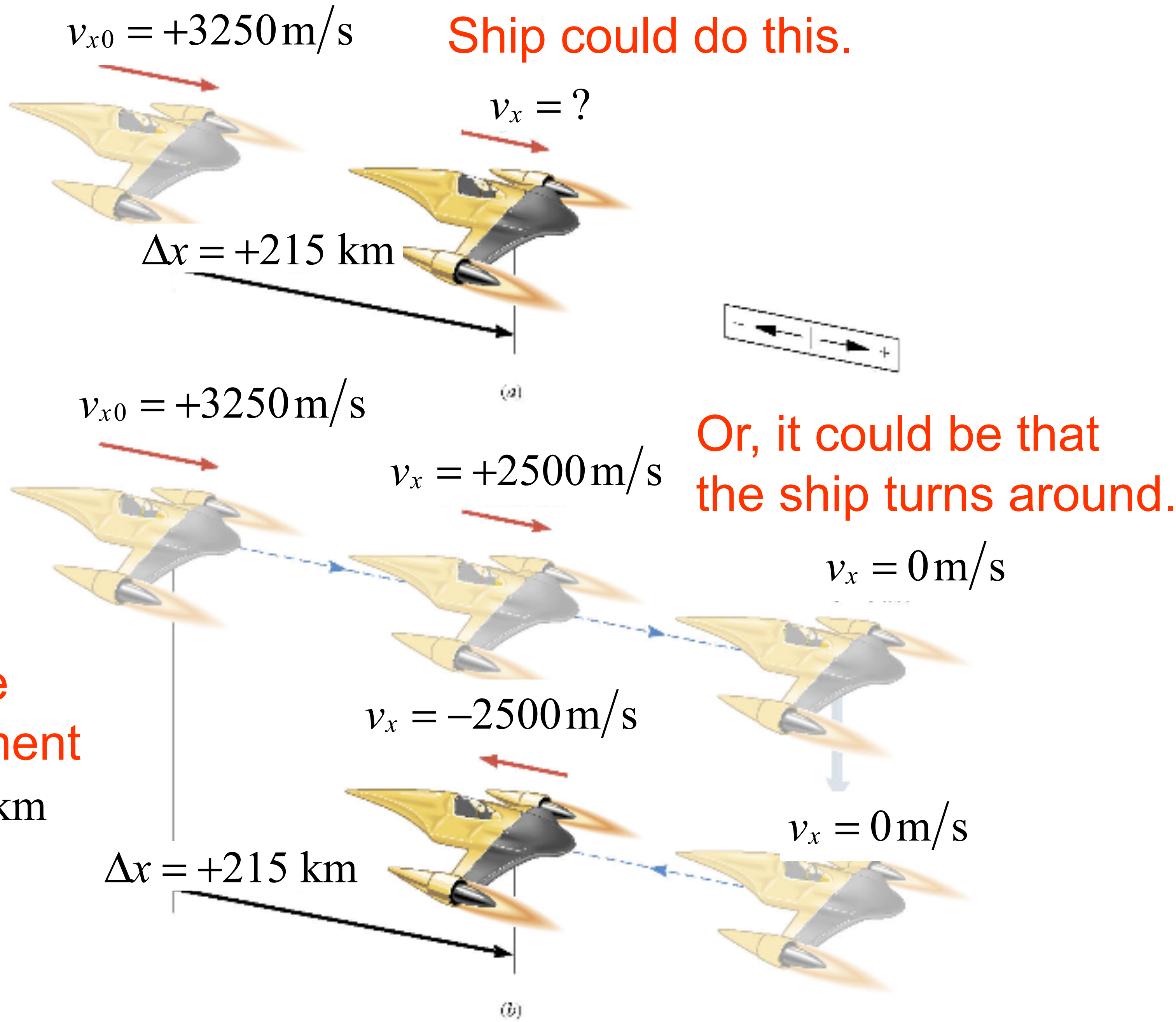
initial velocity

A spacecraft is traveling with a velocity of +3250 m/s. Suddenly the retrorockets are fired, and the spacecraft begins to slow down with an acceleration whose magnitude is 10.0 m/s^2. What is the velocity of the spacecraft when the displacement of the craft is +215 km, relative to the point where the retrorockets began firing?

final velocity

Δx	a_x	v_x	v_{x0}	t
+215000 m	–10.0 m/s^2 acceleration is negative	?	+3250 m/s	

2.5 *Applications of the Equations of Kinematics*



The same displacement

$\Delta x = +215\ \mathrm{km}$

2.5 Applications of the Equations of Kinematics

Δx	a_x	v_x	v_{x0}	t
+215000 m	-10.0 m/s^2	?	+3250 m/s	

$$v_x^2 = v_{x0}^2 + 2a_x\Delta x \longrightarrow v_x = \sqrt{v_{x0}^2 + 2a_x\Delta x}$$

$$v_x = \pm\sqrt{\left(3250\,\mathrm{m/s}\right)^2 + 2\left(-10.0\,\mathrm{m/s^2}\right)\left(215000\ \mathrm{m}\right)}$$

$$= \pm 2500\,\mathrm{m/s}$$

Clicker Question 2.4

A car starting at rest maintains a <u>constant acceleration a_x</u>.

After a time t, its displacement and velocity are Δx and v_x.

What are the displacement and velocity at time $2t$?

a) $2\Delta x$ and $2v_x$

b) $2\Delta x$ and $4v_x$

c) $4\Delta x$ and $2v_x$

d) $4\Delta x$ and $4v_x$

e) $8\Delta x$ and $4v_x$

Clicker Question 2.4

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b) $2\Delta x$ and $4v_x$

c) $\boxed{4\Delta x \text{ and } 2v_x}$

d) $4\Delta x$ and $4v_x$

e) $8\Delta x$ and $4v_x$

acceleration, a_x, is the same at all times.

$v_{x0} = 0$, x, v_x at time t, x', v'_x at time $\underline{t' = 2t}$

$$\Delta x = \tfrac{1}{2} a_x t^2 \quad \Big| \quad \Delta x' = \tfrac{1}{2} a_x t'^2$$

$$= \tfrac{1}{2} a_x (2t)^2 = \tfrac{1}{2} a_x (4) t^2$$

$$= (4)(\tfrac{1}{2} a_x t^2) = (4)\Delta x$$

$$v_x = a_x t \quad \Big| \quad v'_x = a_x t'$$

$$= a_x (2t) = (2)(a_x t)$$

$$= (2) v_x$$

2.5 Freely Falling Bodies

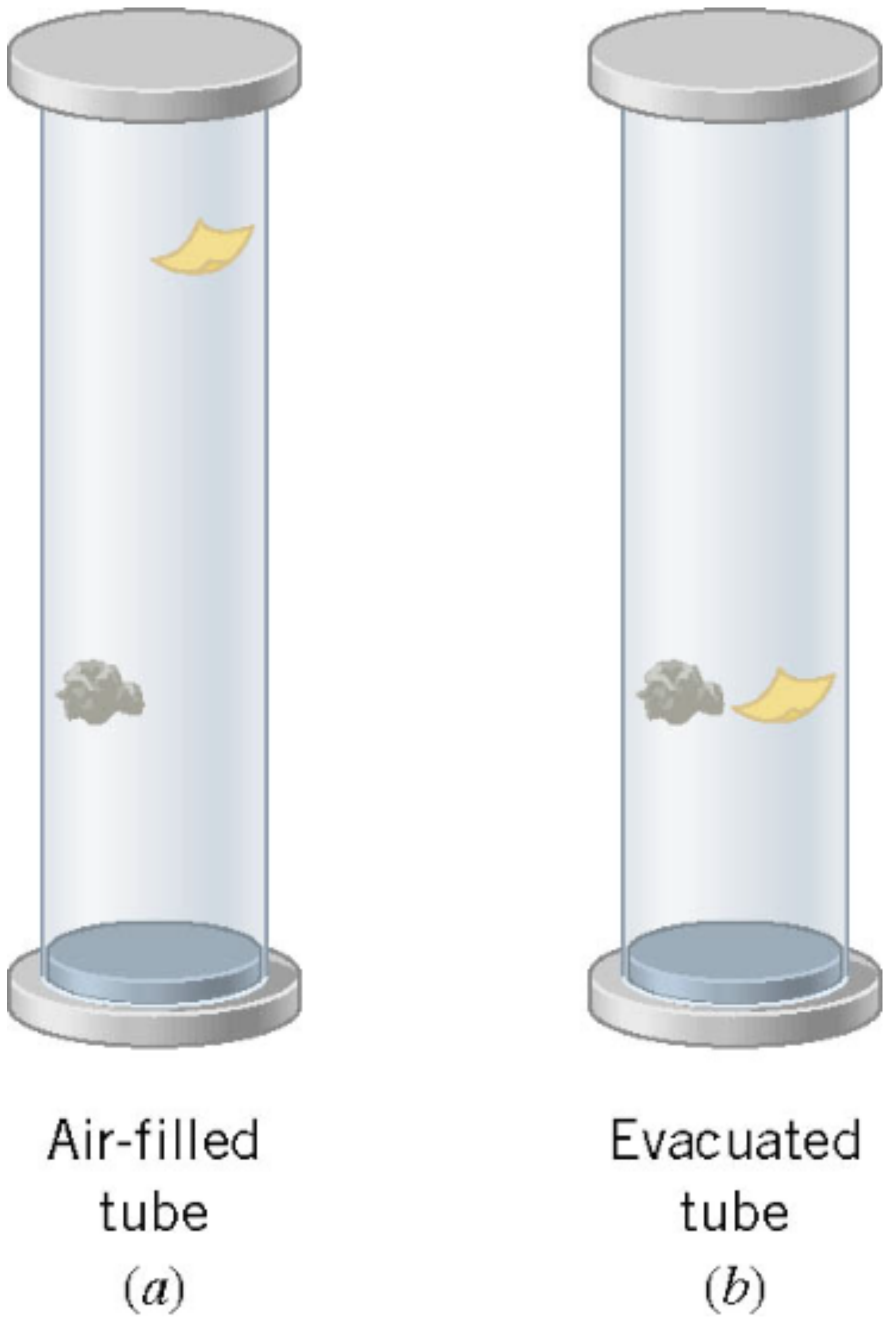
For vertical motion, we will replace the *x* label with *y* in all kinematic equations, and use upward as positive.

In the absence of air resistance, it is found that all bodies at the same location above the Earth fall vertically with the same acceleration. If the distance of the fall is small compared to the radius of the Earth, then the acceleration remains essentially constant throughout the descent.

This idealized motion is called *free-fall* and the acceleration of a freely falling body is called the *acceleration due to gravity*, and the acceleration is downward or negative.

$$a_y = -g = -9.81\,\mathrm{m/s^2} \quad \text{or} \quad -32.2\,\mathrm{ft/s^2}$$

2.5 *Freely Falling Bodies*

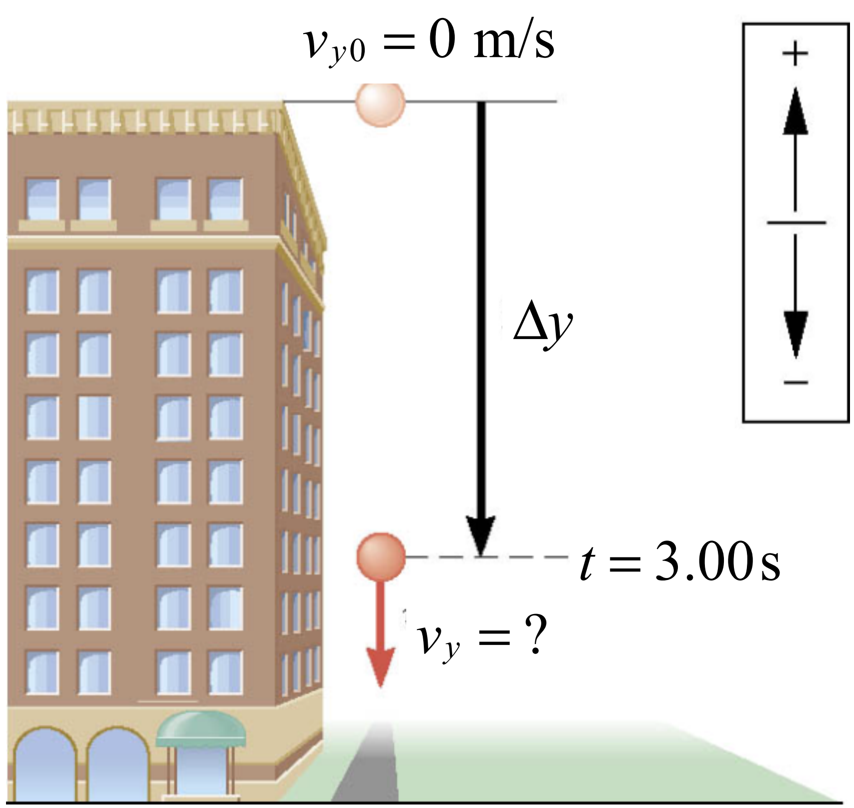


acceleration due to gravity.

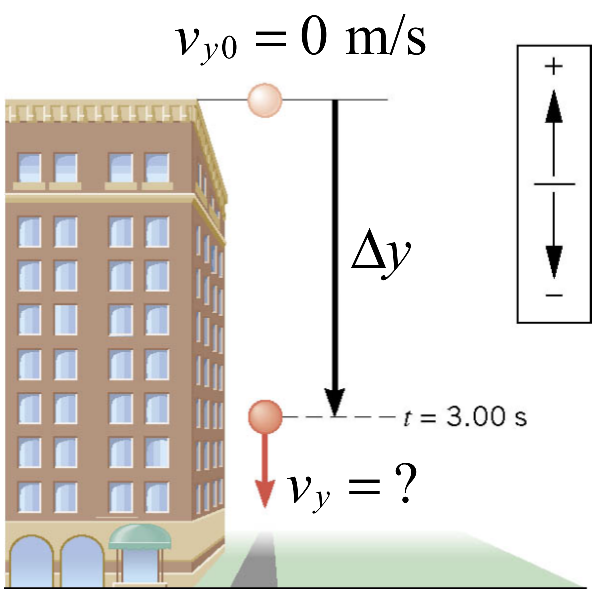
$$a_y = -g = -9.80\,\mathrm{m/s^2}$$

Example: A Falling Stone

A stone is dropped from the top of a tall building. After 3.00s of free fall, what is the displacement, Δy of the stone?



2.5 Freely Falling Bodies



Δy	a_y	v_y	v_{y0}	t
?	–9.80 m/s^2		0 m/s	3.00 s

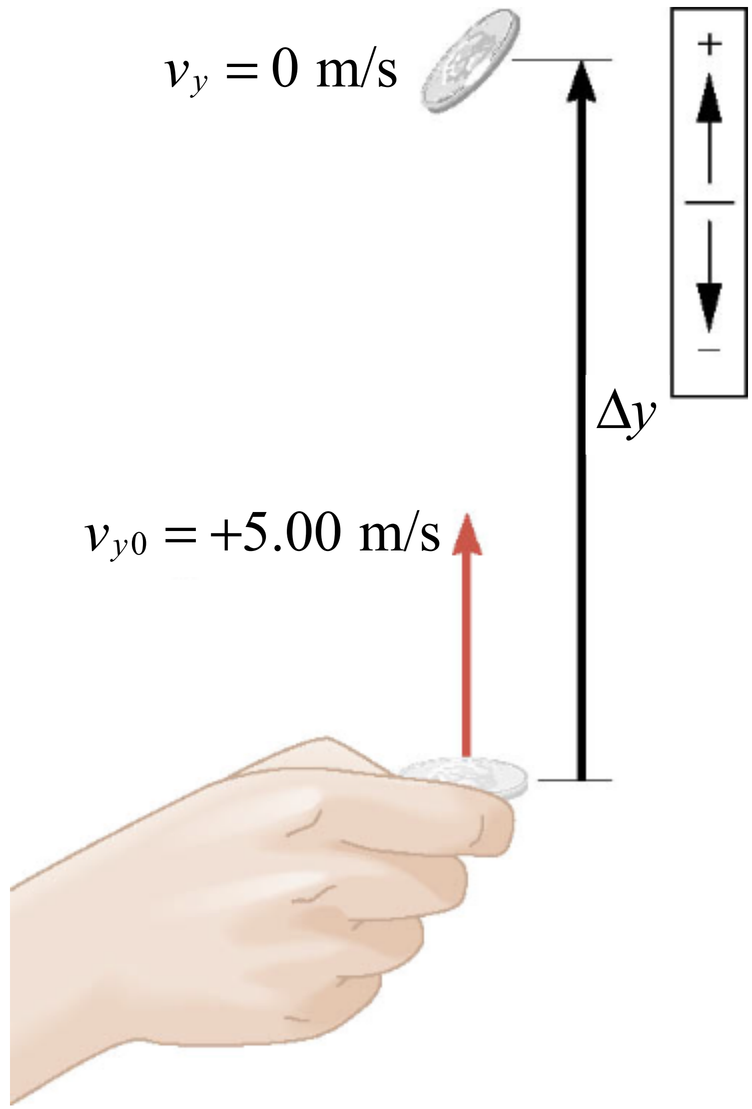
$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

$$= (0\,\text{m/s})(3.00\ \text{s}) + \tfrac{1}{2}(-9.80\,\text{m/s}^2)(3.00\ \text{s})^2$$

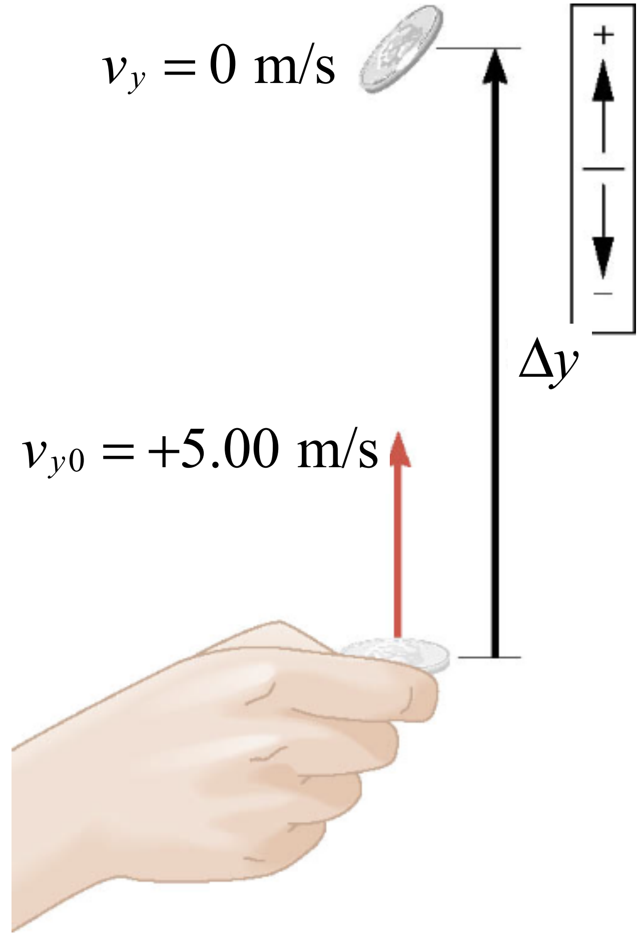
$$= -44.1\ \text{m}$$

Example: How High Does it Go?

The referee tosses the coin up with an initial speed of 5.00m/s. In the absence if air resistance, how high does the coin go above its point of release?



2.5 Freely Falling Bodies



Δy	a_y	v_y	y_{y0}	t
?	-9.80 m/s^2	0 m/s	+5.00 m/s	

$$v_y^2 = v_{y0}^2 + 2a_y\Delta y \quad \Longrightarrow \quad \Delta y = \frac{v_y^2 - v_{y0}^2}{2a_y}$$

$$\Delta y = \frac{v_y^2 - v_{y0}^2}{2a_y} = \frac{(0\,\mathrm{m/s})^2 - (5.00\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 1.28\ \mathrm{m}$$

2.5 *Freely Falling Bodies*

Conceptual Example 14 Acceleration Versus Velocity

There are three parts to the motion of the coin.

1) On the way up, the coin has an upward-pointing velocity with a decreasing magnitude.
2) At that time the coin reaches the top of its path, the coin has an instantaneously zero velocity.
3) On the way down, the coin has a downward-pointing velocity with an increasing magnitude.

In the absence of air resistance, does the acceleration vector of the coin, like the velocity, change from one part to another?

Clicker Question 2.5

During the free flight (no air resistance) of a coin thrown upward (+), what are the values of the acceleration at these times during the motion ?

	On the way up	At the top	On the way down
	$a_y =$	$a_y =$	$a_y =$
a)	$-9.80\mathrm{m/s^2}$	$-9.80\mathrm{m/s^2}$	$-9.80\mathrm{m/s^2}$
b)	$+9.80\mathrm{m/s^2}$	$0.0\ \mathrm{m/s^2}$	$+9.80\mathrm{m/s^2}$
c)	$+9.80\mathrm{m/s^2}$	$0.0\ \mathrm{m/s^2}$	$-9.80\mathrm{m/s^2}$
d)	$-9.80\mathrm{m/s^2}$	$0.0\ \mathrm{m/s^2}$	$-9.80\mathrm{m/s^2}$
e)	$+9.80\mathrm{m/s^2}$	$+9.80\mathrm{m/s^2}$	$+9.80\mathrm{m/s^2}$

Clicker Question 2.5

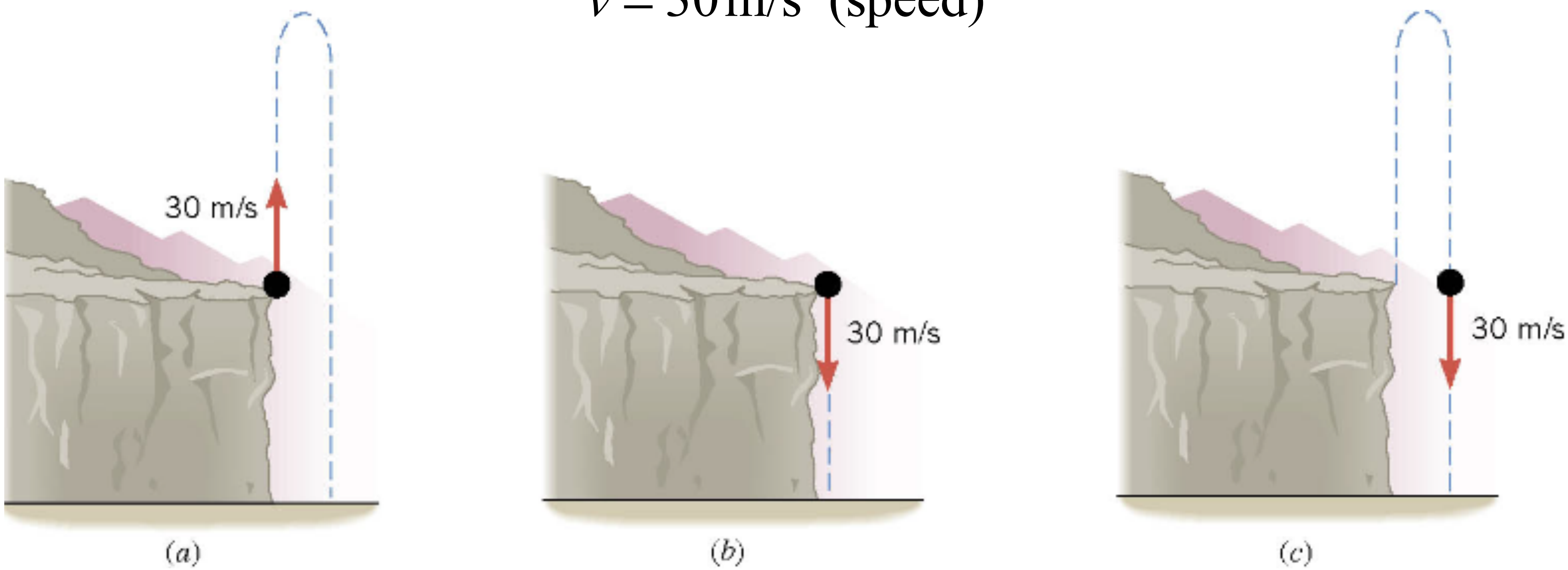
During the free flight (no air resistance) of a coin thrown upward (+), what are the values of the acceleration at these times during the motion ?

	On the way up $a_y =$	At the top $a_y =$	On the way down $a_y =$
a)	$-9.80 \mathrm{m/s^2}$	$-9.80 \mathrm{m/s^2}$	$-9.80 \mathrm{m/s^2}$
b)	$+9.80 \mathrm{m/s^2}$	$0.0 \mathrm{\ m/s^2}$	$+9.80 \mathrm{m/s^2}$
c)	$+9.80 \mathrm{m/s^2}$	$0.0 \mathrm{\ m/s^2}$	$-9.80 \mathrm{m/s^2}$
d)	$-9.80 \mathrm{m/s^2}$	$0.0 \mathrm{\ m/s^2}$	$-9.80 \mathrm{m/s^2}$
e)	$+9.80 \mathrm{m/s^2}$	$+9.80 \mathrm{m/s^2}$	$+9.80 \mathrm{m/s^2}$

Conceptual Example: Taking Advantage of Symmetry

Does the pellet in part *b* strike the ground beneath the cliff with a smaller, greater, or the same speed as the pellet in part *a*?

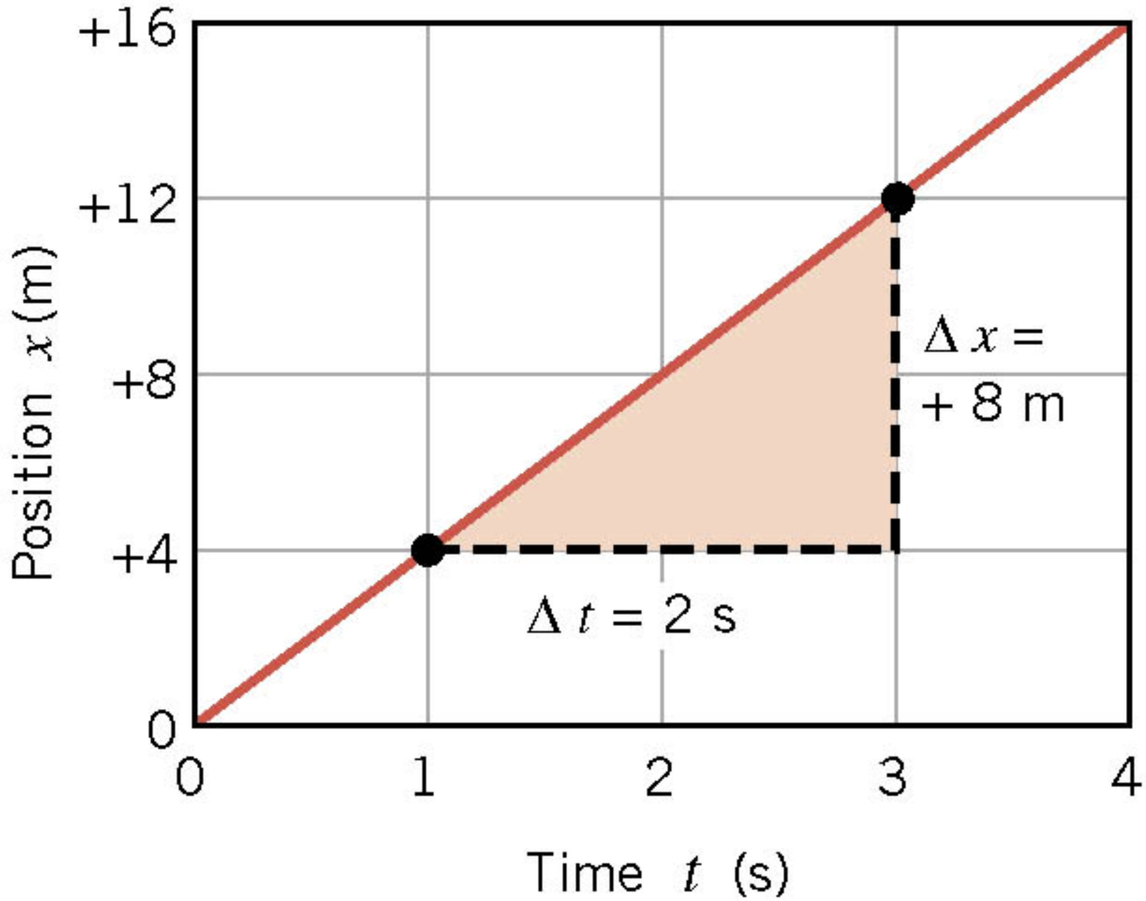
$$v = 30\,\text{m/s} \quad \text{(speed)}$$



$v_{y0} = +30\,\text{m/s}$ $\qquad$ $v_{y0} = -30\,\text{m/s}$ $\qquad$ $v_{y} = -30\,\text{m/s}$

2.5 Graphical Analysis of Velocity and Acceleration

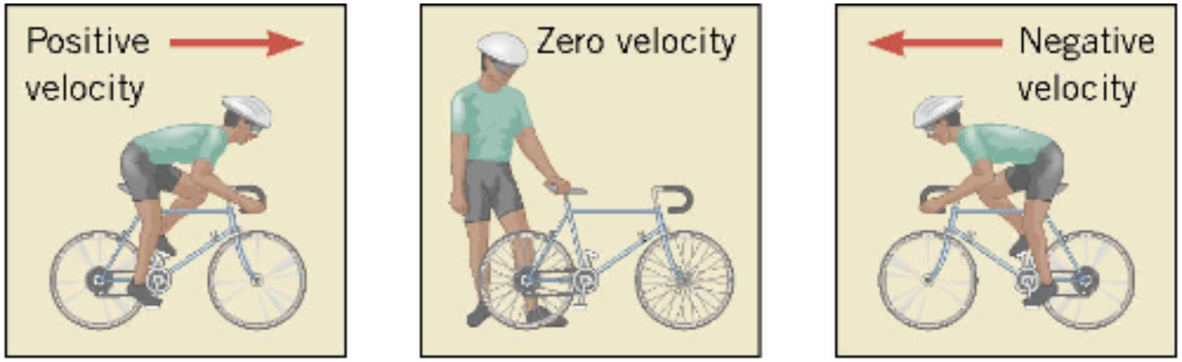
Graph of position vs. time.

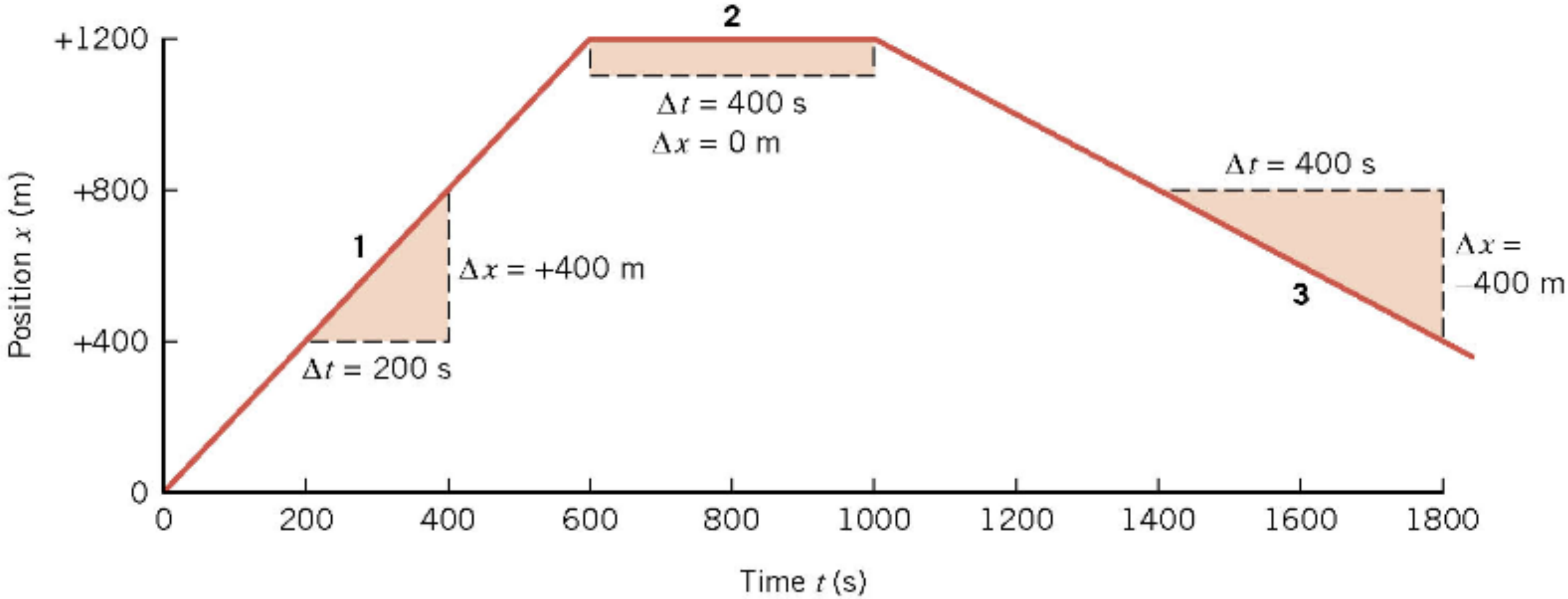


$$\text{Slope} = \frac{\Delta x}{\Delta t} = \frac{+8\text{ m}}{2\text{ s}} = +4\,\text{m/s}$$

The same slope at all times.
This means constant velocity!

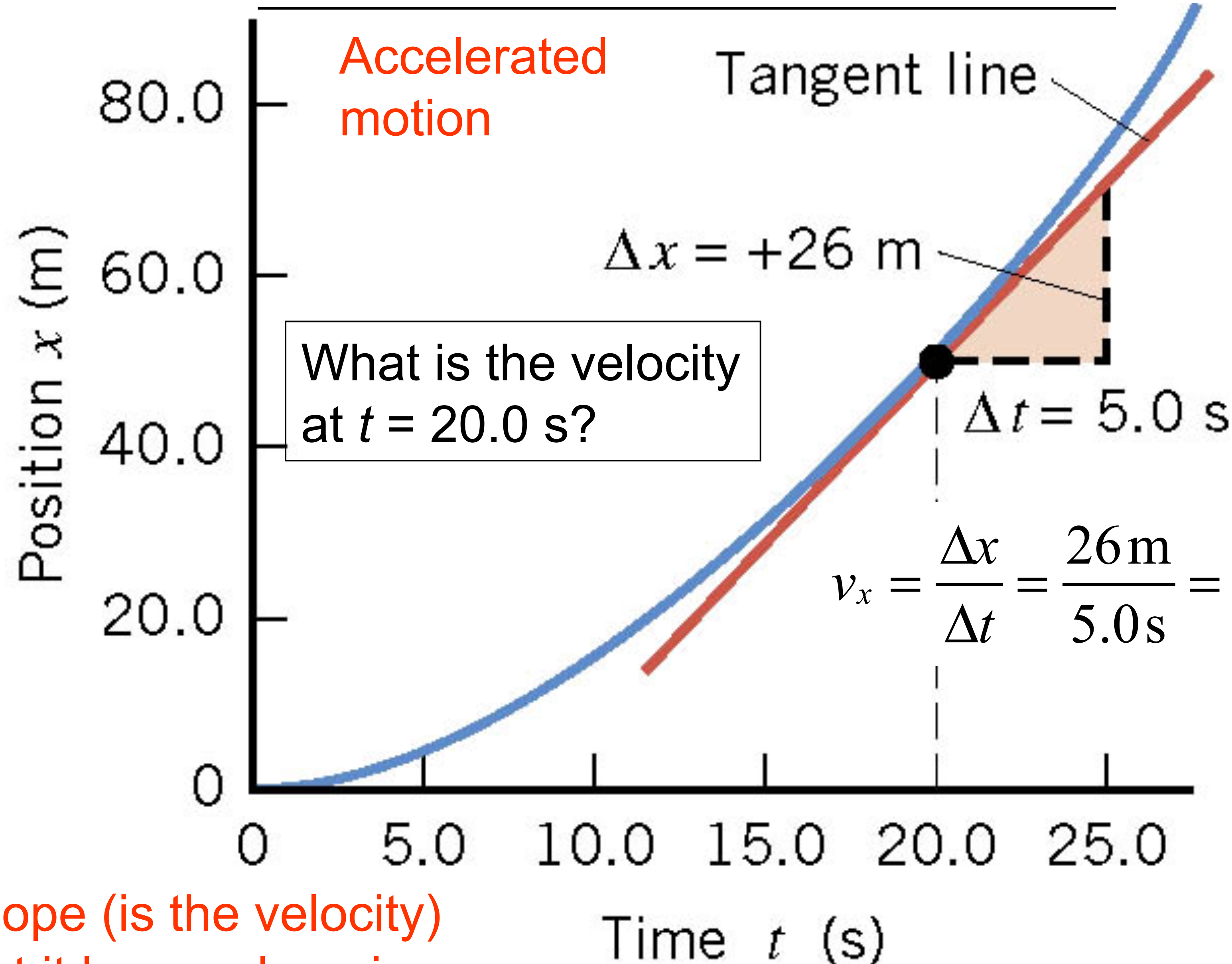
2.5 Graphical Analysis of Velocity and Acceleration





2.5 *Graphical Analysis of Velocity and Acceleration*

Graph of position vs. time (blue curve)



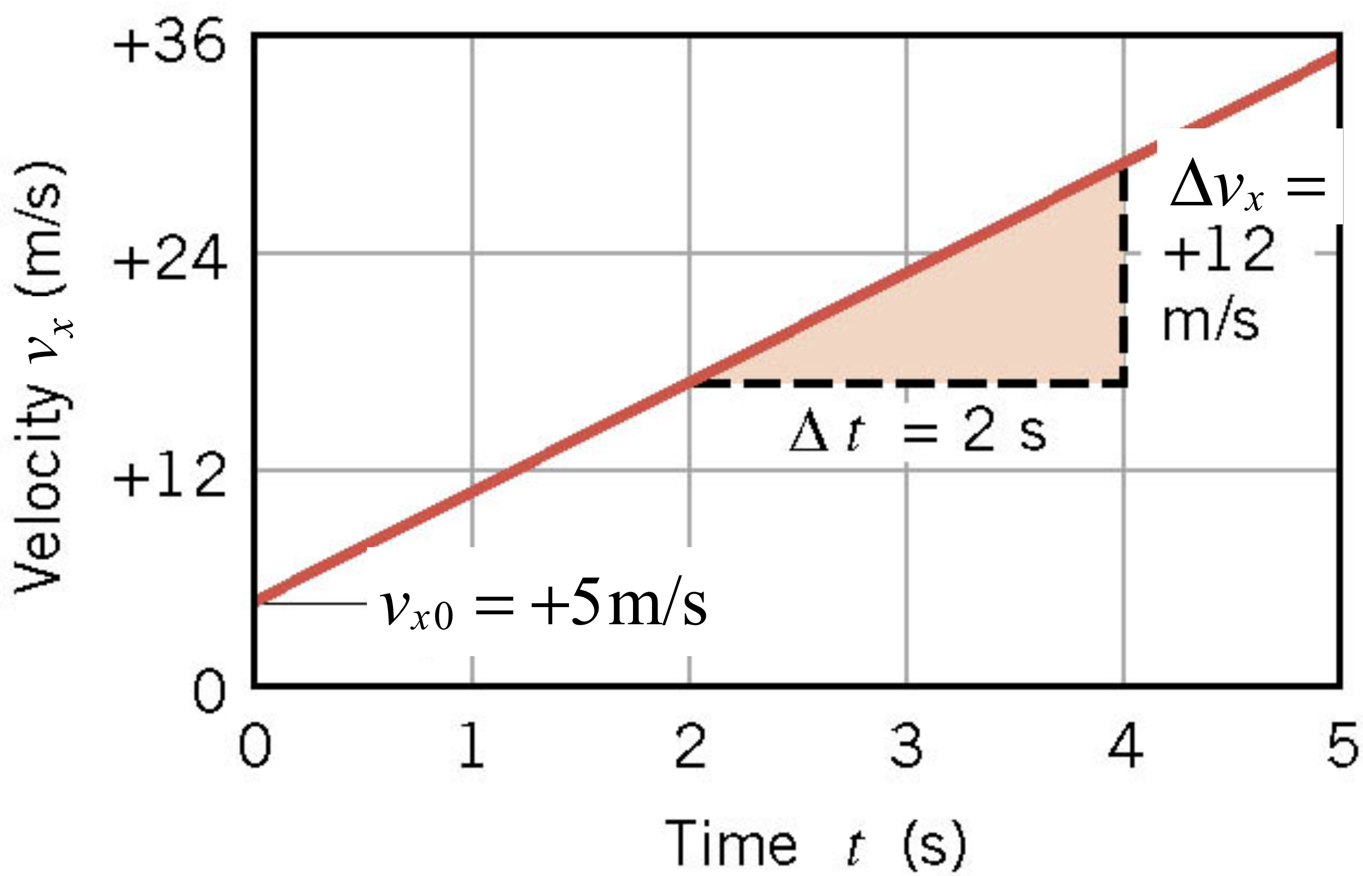
What is the velocity at t = 20.0 s?

$$v_x = \frac{\Delta x}{\Delta t} = \frac{26\,\text{m}}{5.0\,\text{s}} = 5.2\,\text{m/s}$$

Slope (is the velocity) but it keeps changing.

2.5 Graphical Analysis of Velocity and Acceleration

Graph of velocity vs. time (red curve)



$$\text{Slope} = \frac{\Delta v_x}{\Delta t} = \frac{+12\ \mathrm{m/s}}{2\ \mathrm{s}} = +6\,\mathrm{m/s^2}$$

$$a_x = +6\,\mathrm{m/s^2}$$

The same slope at all times. This means a constant acceleration!

2.5 *Summary equations of kinematics in one dimension*

Equations of Kinematics for <u>Constant Acceleration</u>

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right) t$$

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

$$\Delta x = v_{x0} t + \tfrac{1}{2} a_x t^2$$

Except for *t*, every variable has a direction and thus can have a positive or negative value.

For vertical motion replace x with y