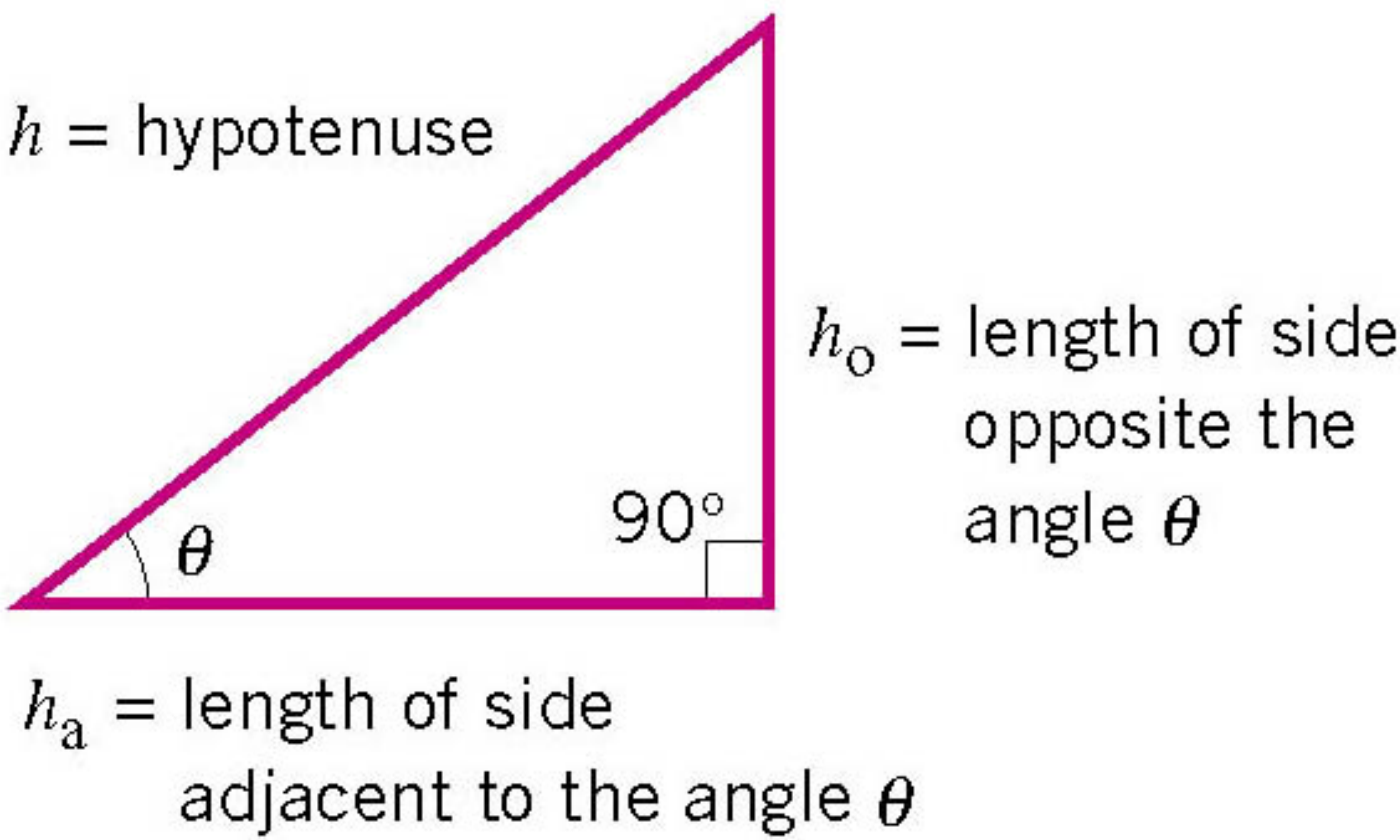
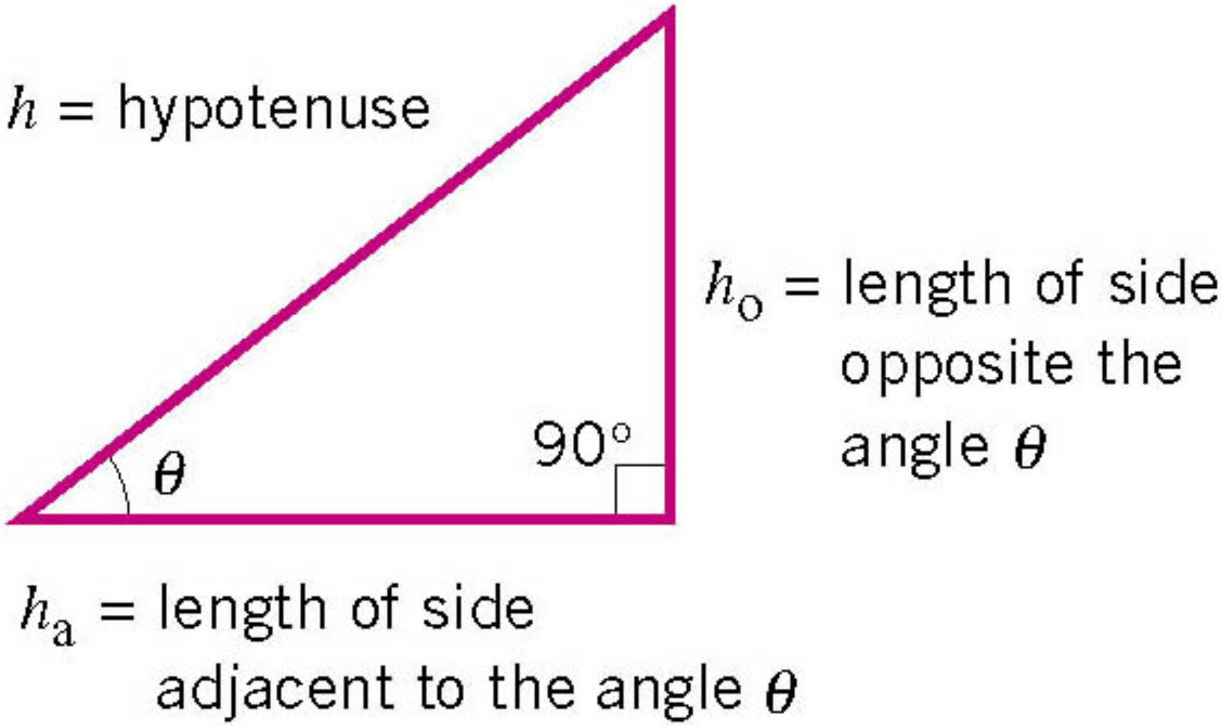
Chapter 3

Kinematics in Two Dimensions

3.1 Trigonometry



3.1 Trigonometry

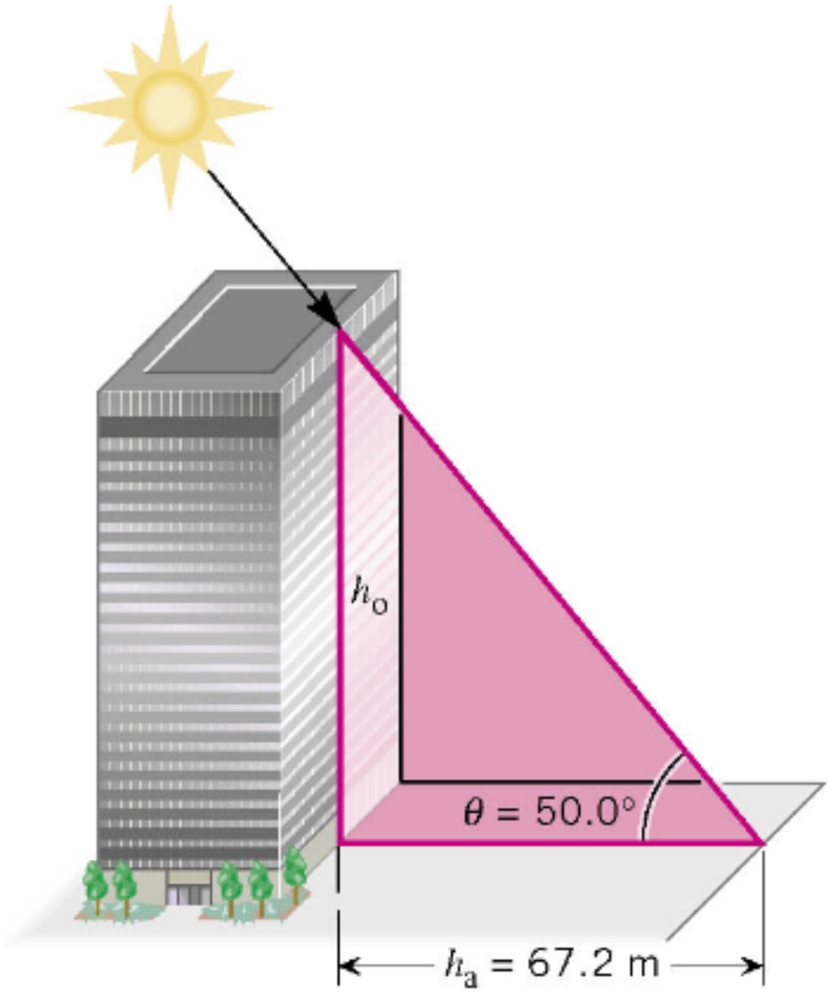


$$\sin\theta = \frac{h_o}{h}$$

$$\cos\theta = \frac{h_a}{h}$$

$$\tan\theta = \frac{h_o}{h_a}$$

3.1 Trigonometry

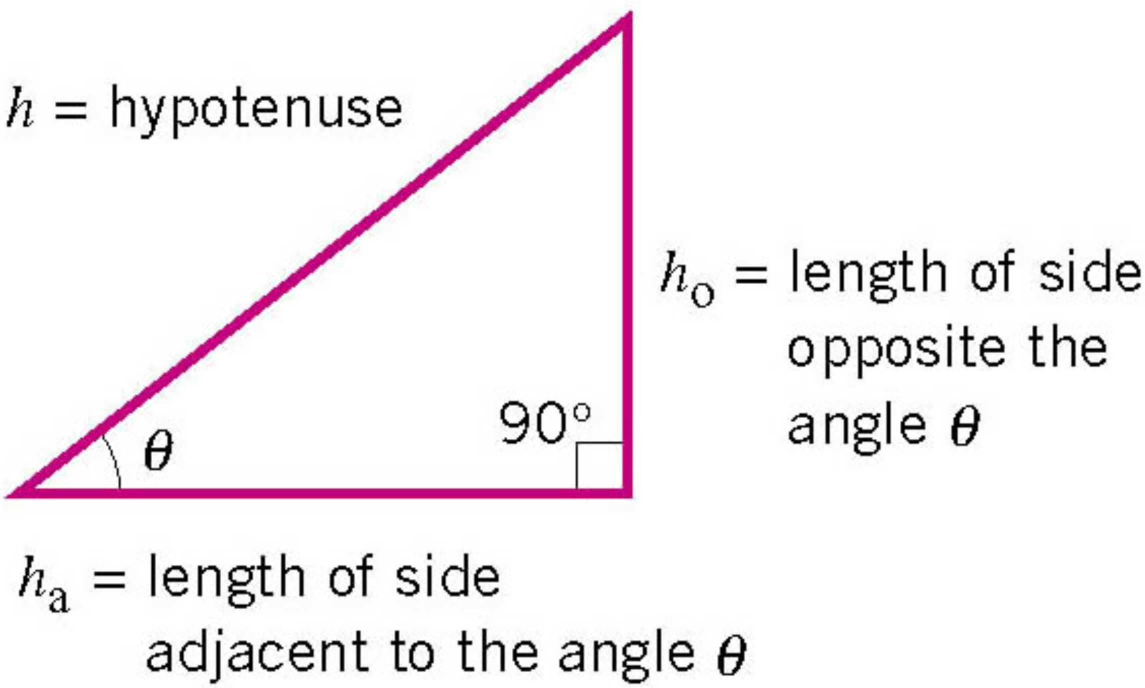


$$\tan\theta = \frac{h_o}{h_a}$$

$$\tan 50^\circ = \frac{h_o}{67.2\text{m}}$$

$$h_o = \tan 50^\circ (67.2\text{m}) = 80.0\text{m}$$

3.1 Trigonometry

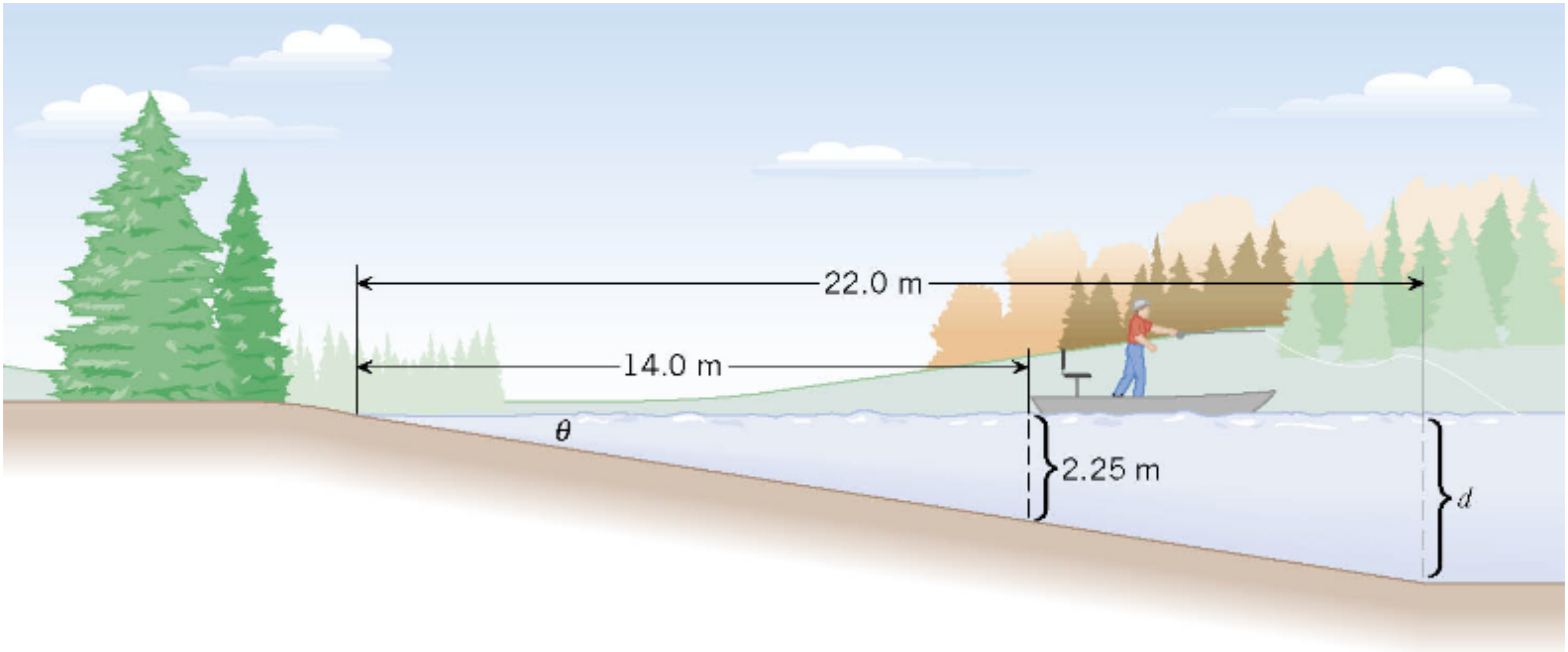


$$\theta = \sin^{-1}\left(\frac{h_o}{h}\right)$$

$$\theta = \cos^{-1}\left(\frac{h_a}{h}\right)$$

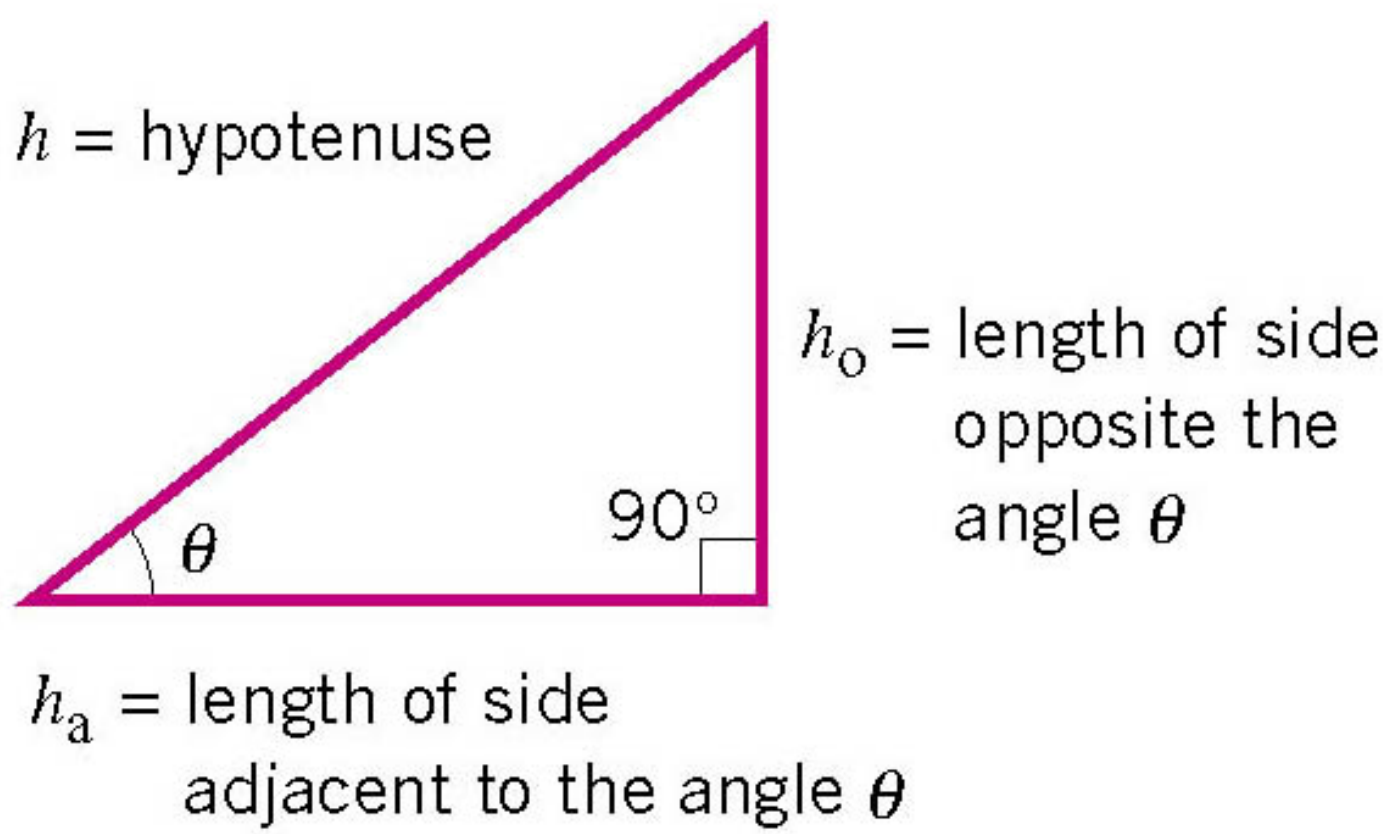
$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right)$$

3.1 Trigonometry



$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) \qquad \theta = \tan^{-1}\left(\frac{2.25\text{m}}{14.0\text{m}}\right) = 9.13^\circ$$

Pythagorean theorem: $$h^2 = h_o^2 + h_a^2$$

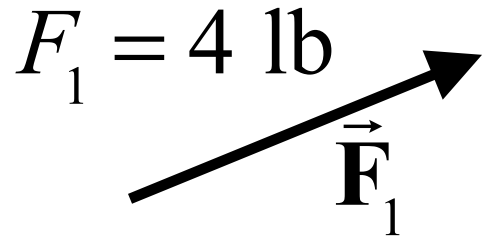


3.2 Scalars and Vectors

Directions of vectors $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ appear to be the same.

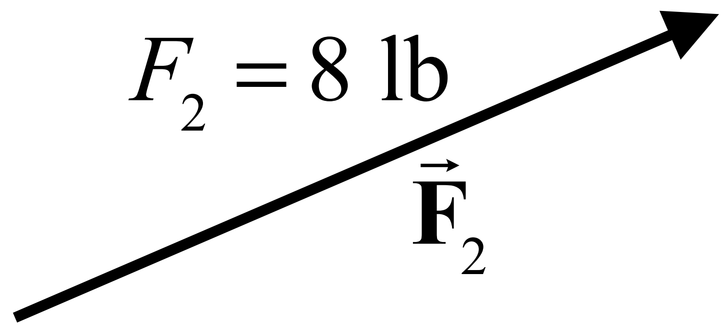
Vector $\vec{\mathbf{F}}_1$, (bold + arrow over it)

has 2 parts:
- magnitude = F_1 *(italics)*
- direction = up & to the right



Vector $\vec{\mathbf{F}}_2$, (bold + arrow over it)

has 2 parts:
- magnitude = F_2 *(italics)*
- direction = up & to the right

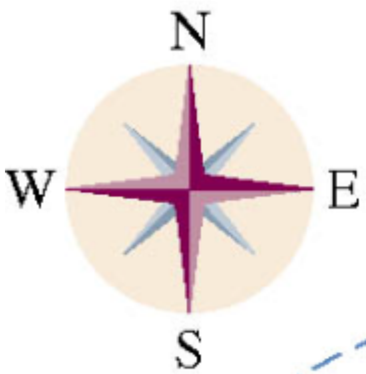


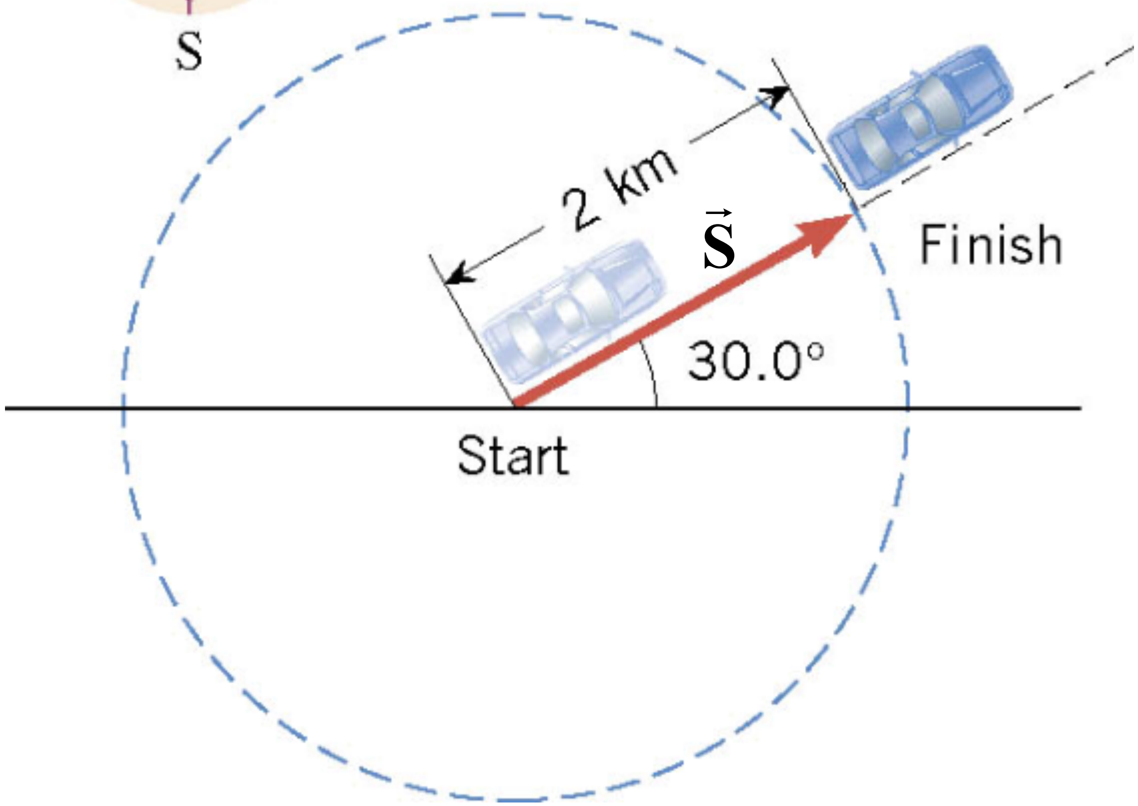
Directions of vectors $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ appear to be the same.

3.2 Scalars and Vectors

Displacement vector $\vec{\mathbf{S}}$

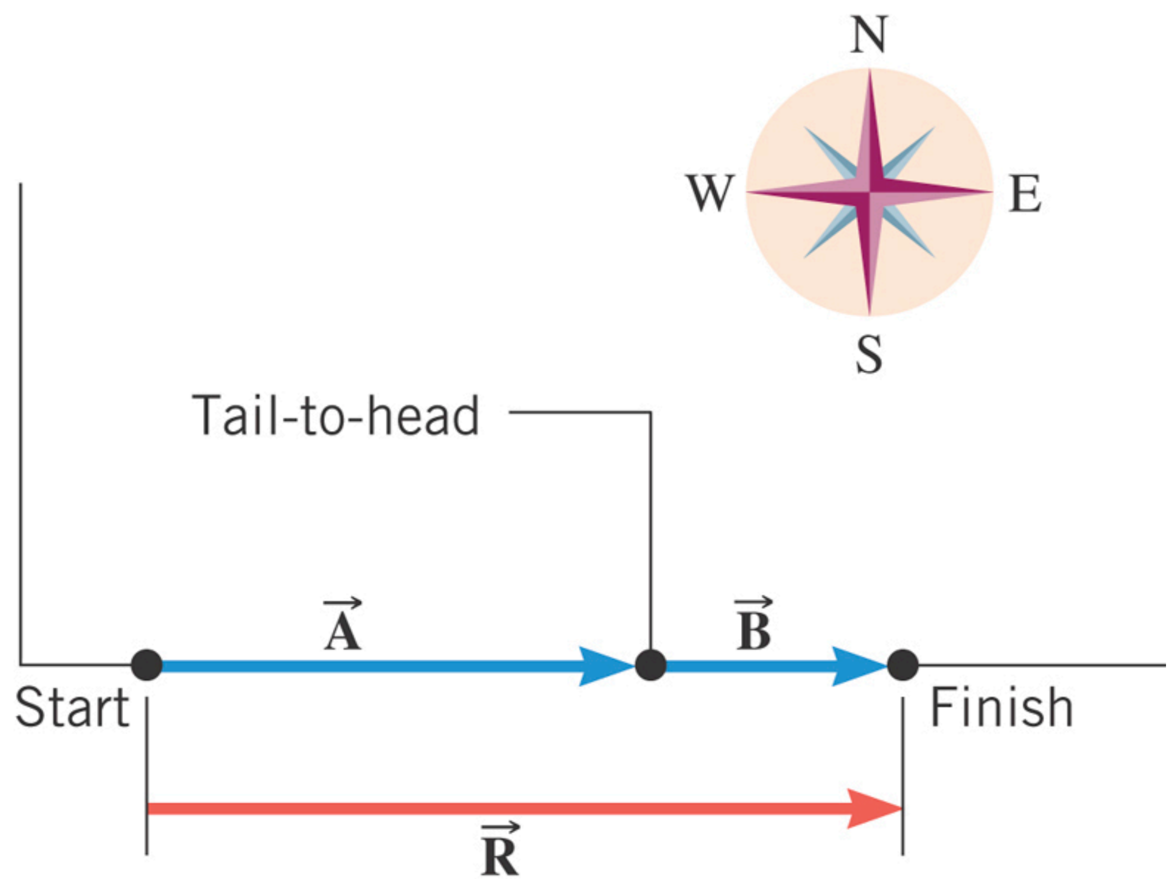
has 2 parts: $\begin{cases} S = 2 \text{ km (magnitude) , and} \\ 30^\circ \text{ North of East (direction)} \end{cases}$



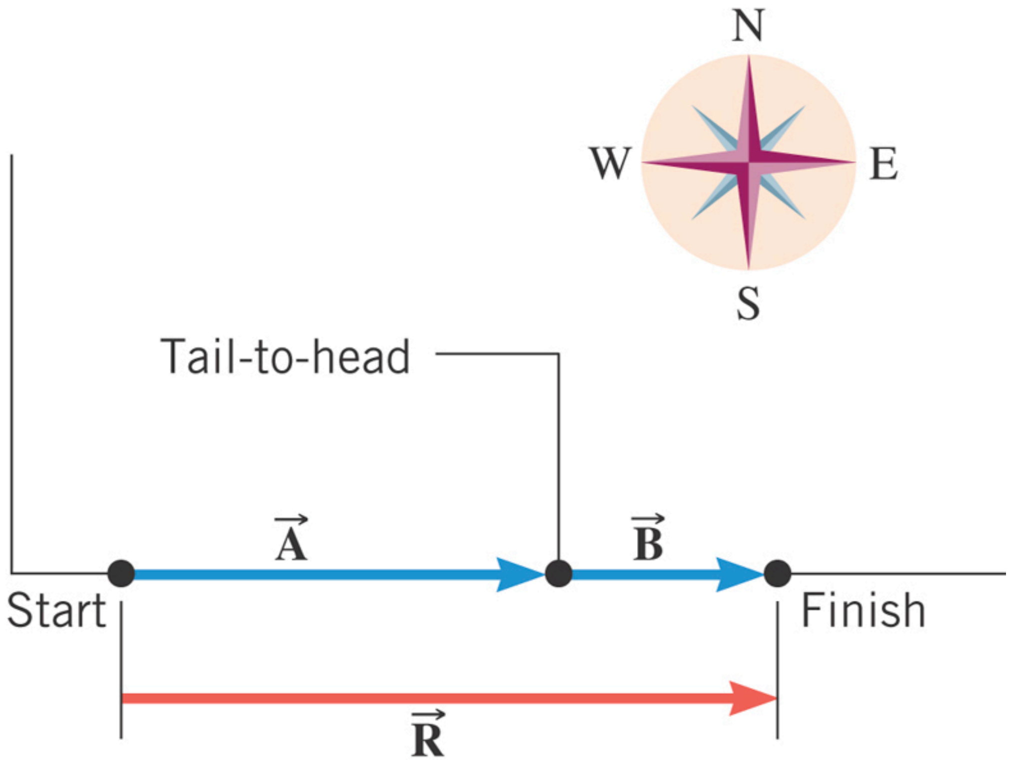


3.2 Scalars and Vectors (Vector Addition and Subtraction)

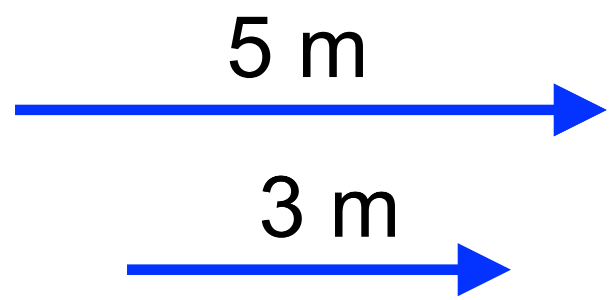
Often it is necessary to add one vector to another.



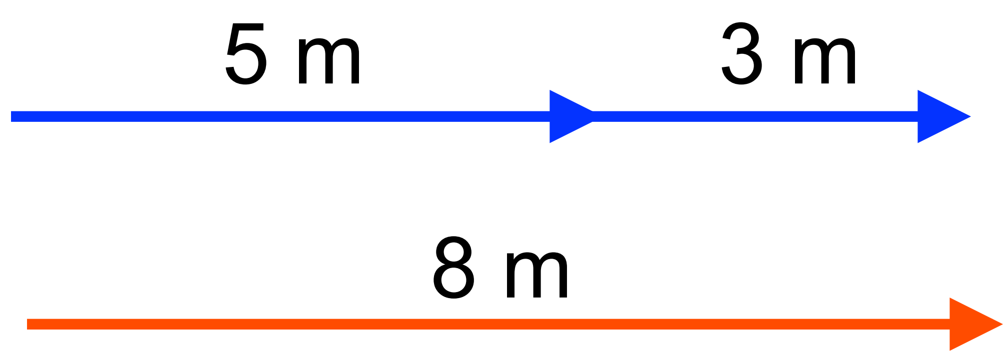
3.2 Scalars and Vectors (Vector Addition and Subtraction)



Two displacement vectors



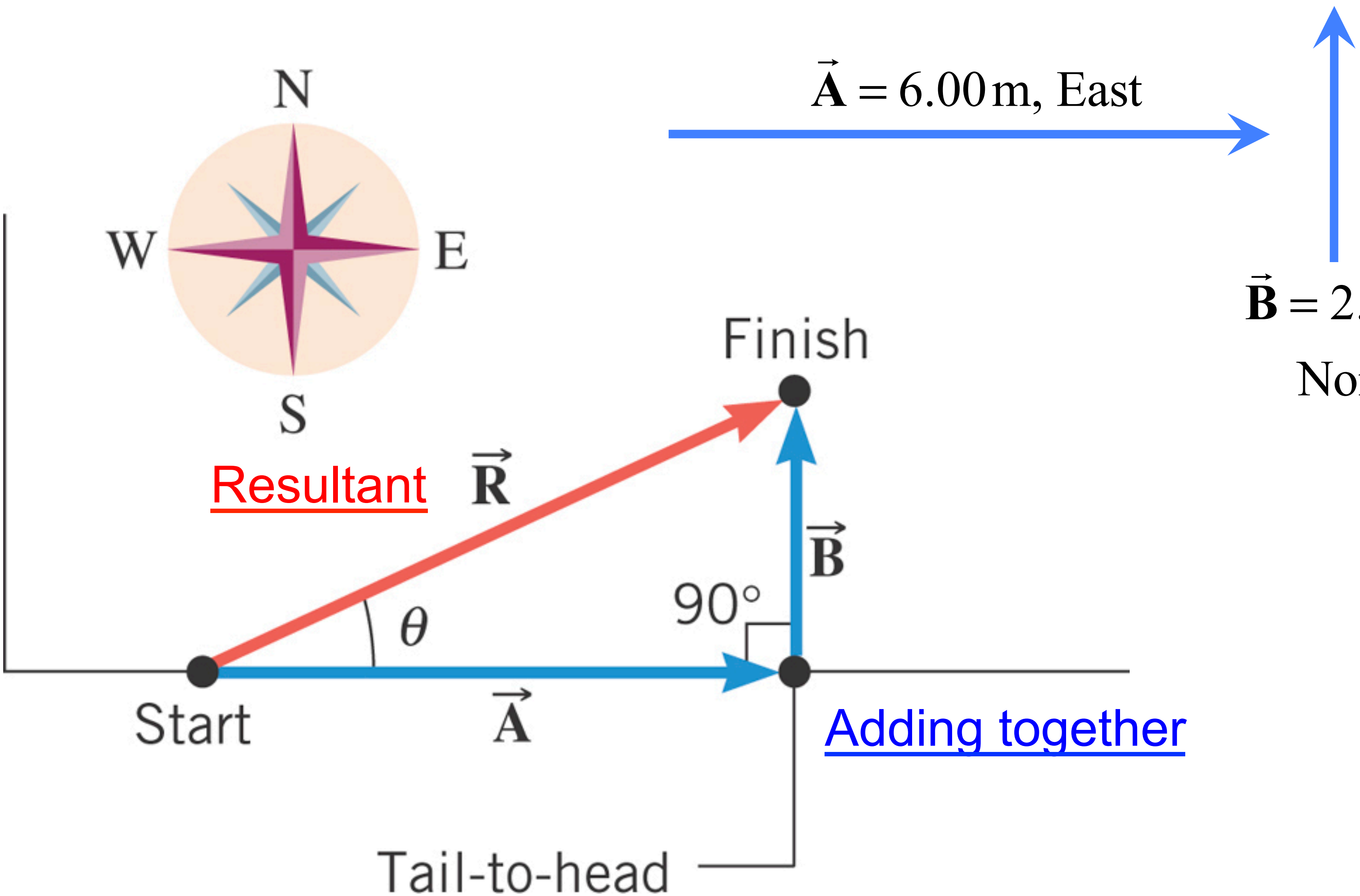
Adding together



Resultant

3.2 Scalars and Vectors (Vector Addition and Subtraction)

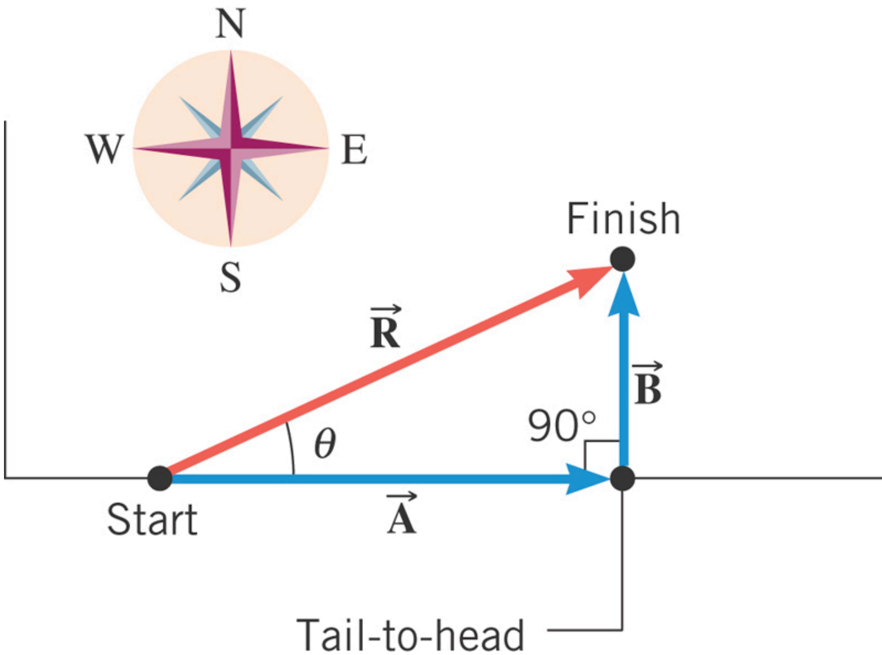
Two displacement vectors



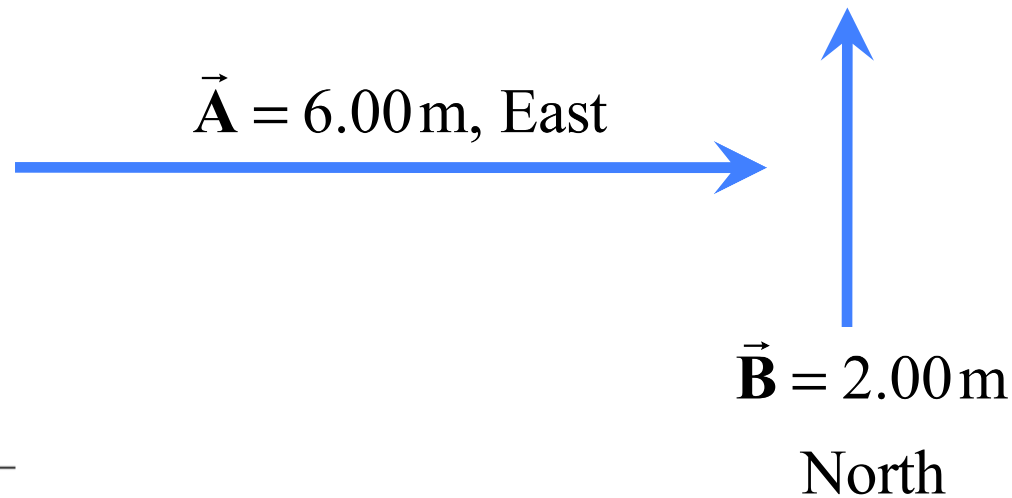
Adding together

3.2 Scalars and Vectors (Vector Addition and Subtraction)

Graphical Addition

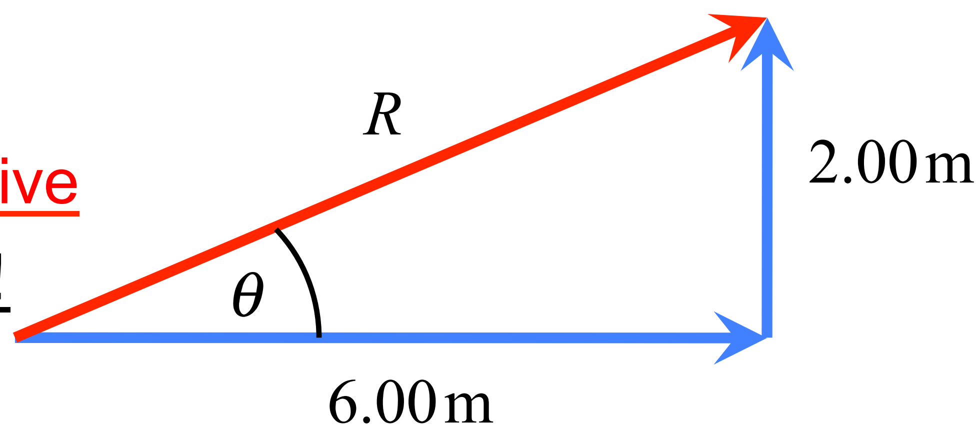


Two displacement vectors



Graphical addition only qualitative

YOU need quatitative addition!

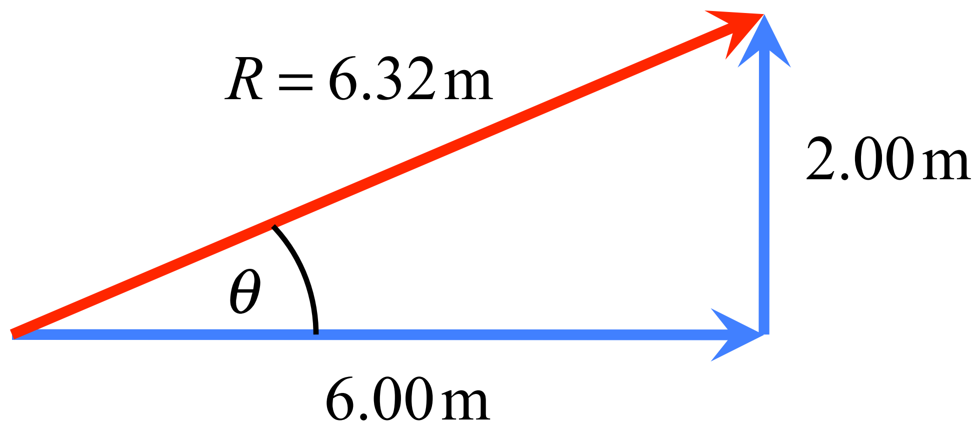


To do this addition of vectors requires trigonometry

Apply Pythagorean Theroem

$$R^2 = (2.00\ \mathrm{m})^2 + (6.00\ \mathrm{m})^2$$

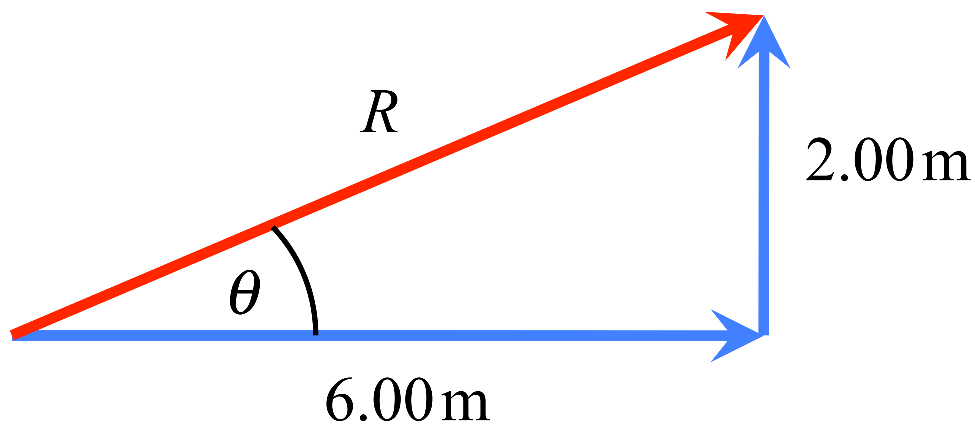
$$R = \sqrt{(2.00\ \mathrm{m})^2 + (6.00\ \mathrm{m})^2} = 6.32\mathrm{m}$$



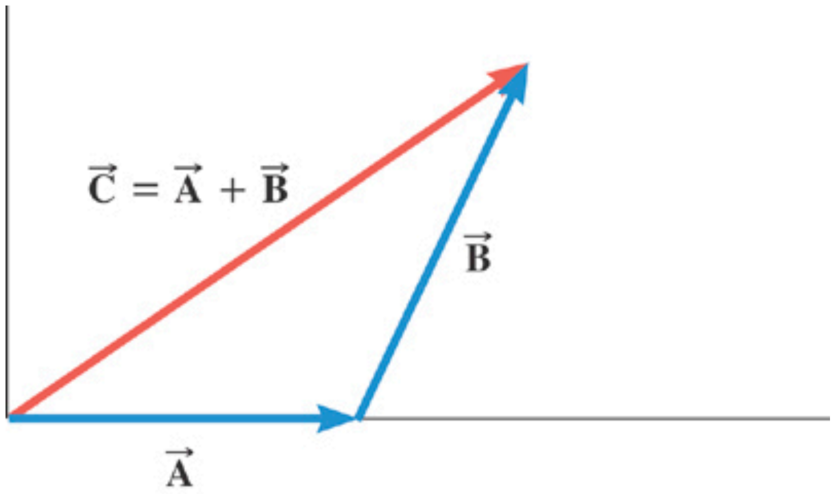
Use trigonometry to determine the angle

$$\tan\theta = 2.00/6.00 \qquad \text{tangent (angle)} = \frac{\text{opposite side}}{\text{adjacent side}}$$

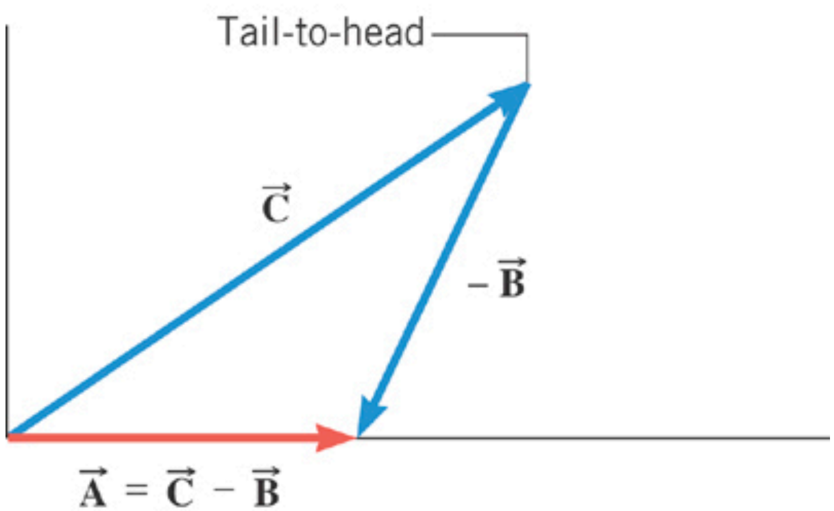
$$\theta = \tan^{-1}\left(2.00/6.00\right) = 18.4^{\circ}$$



3.2 Scalars and Vectors (Vector Addition and Subtraction)



(a)

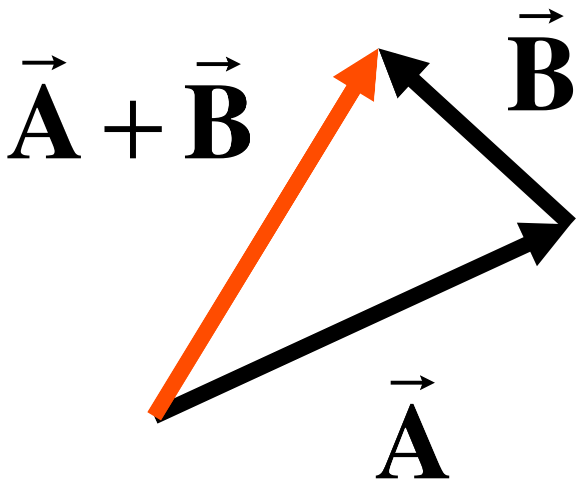


(b)

When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed.

3.2 Scalars and Vectors (Vector Addition and Subtraction)

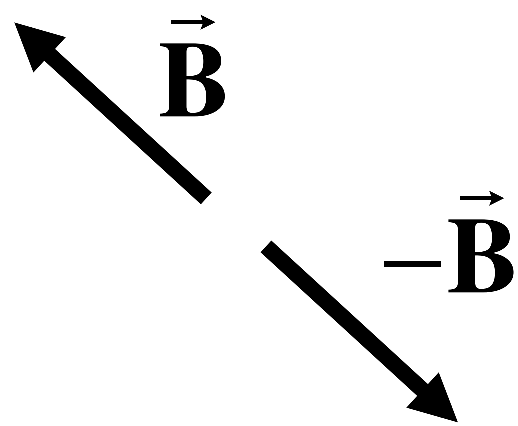
Add vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$



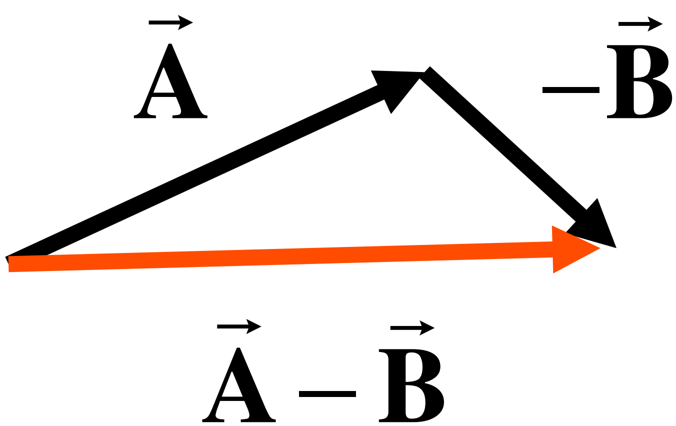
Now you are asked to find $\vec{\mathbf{A}}-\vec{\mathbf{B}}$

Instead of trying to do Vector Subtraction

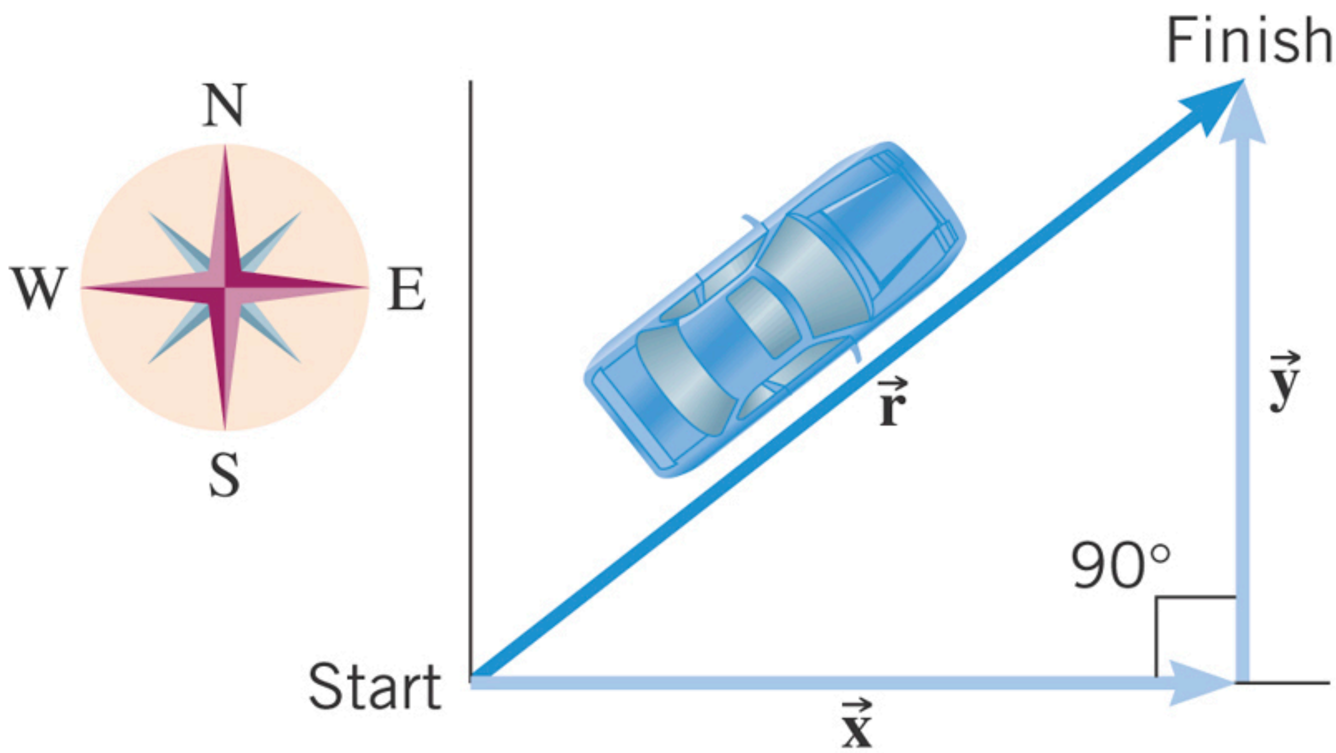
add to vector $\vec{\mathbf{A}}$ the negative of the vector $\vec{\mathbf{B}}$



Subtracting vector $\vec{\mathbf{B}}$ from vector $\vec{\mathbf{A}}$

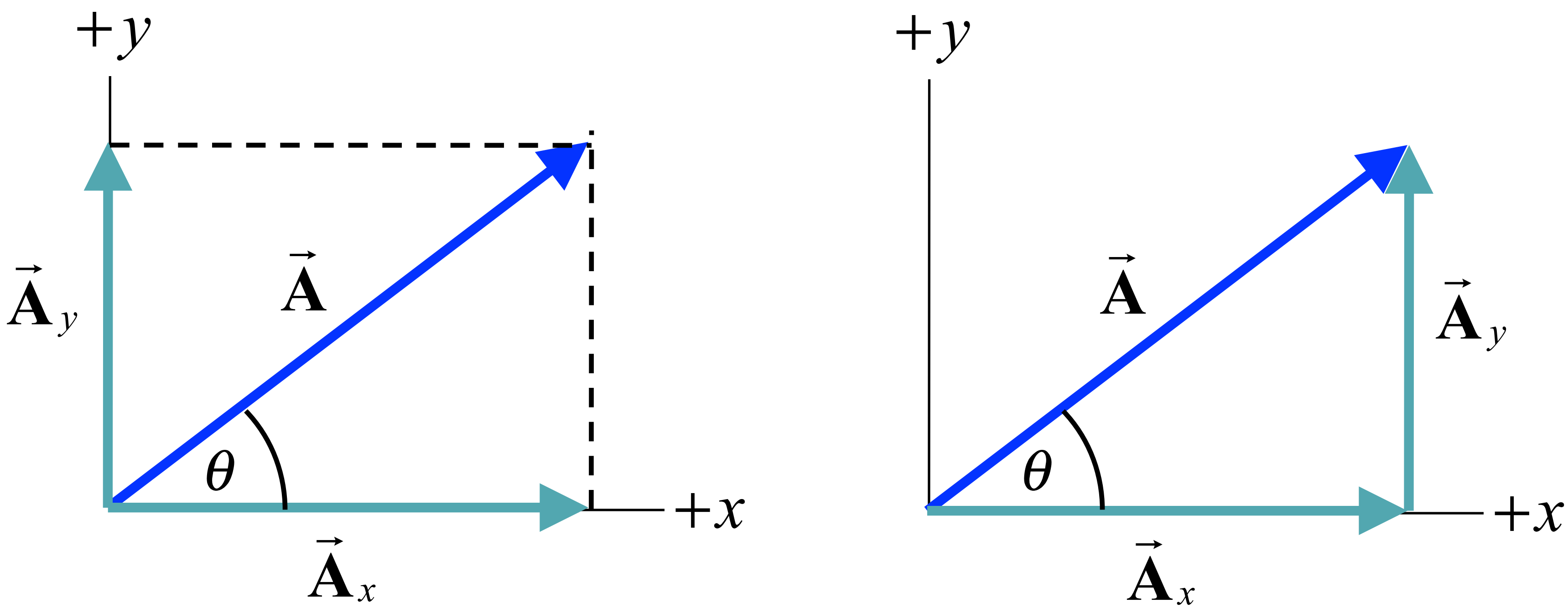


3.2 Vector Addition and Subtraction (The Components of a Vector)



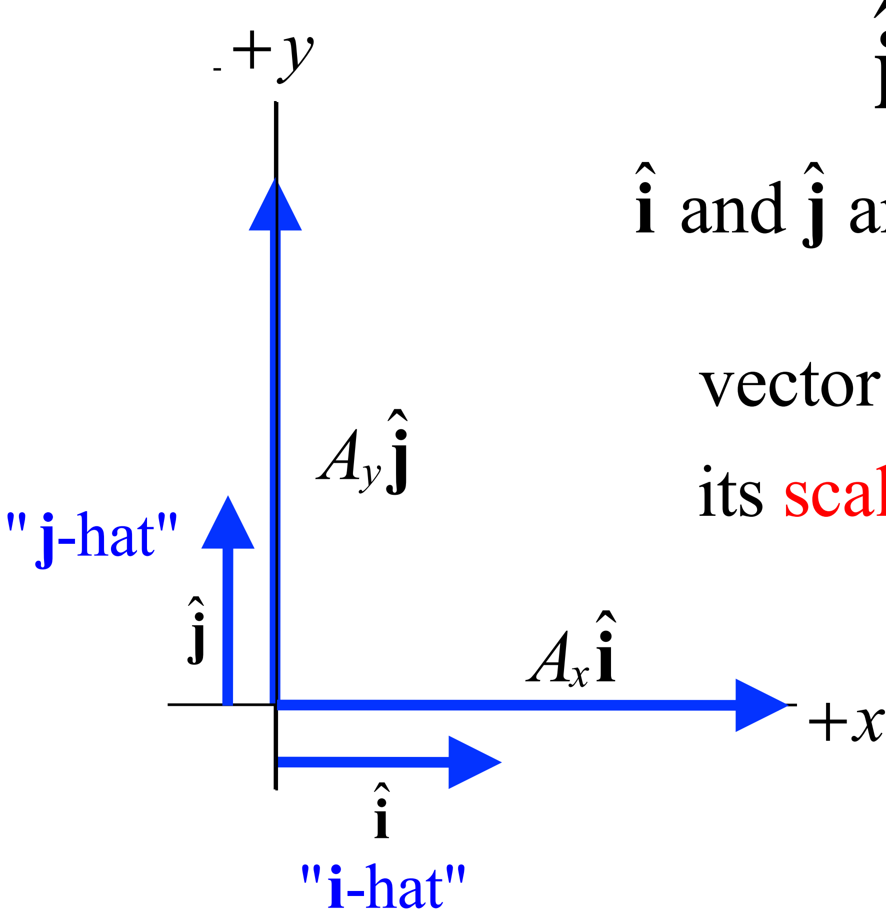
$\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ are called the $x-$component vector and the $y-$component vector of $\vec{\mathbf{r}}$.

3.2 Vector Addition and Subtraction (The Components of a Vector)



The vector components of $\vec{\mathbf{A}}$ are two perpendicular vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ that are parallel to the x and y axes, and add together vectorially so that $\vec{\mathbf{A}} = \vec{\mathbf{A}}_\mathbf{x} + \vec{\mathbf{A}}_\mathbf{y}$.

It is often easier to work with the **scalar components** of the vectors, rather than the component vectors themselves.



"**i**-hat" "**j**-hat"

$\hat{\mathbf{i}}$ $\hat{\mathbf{j}}$

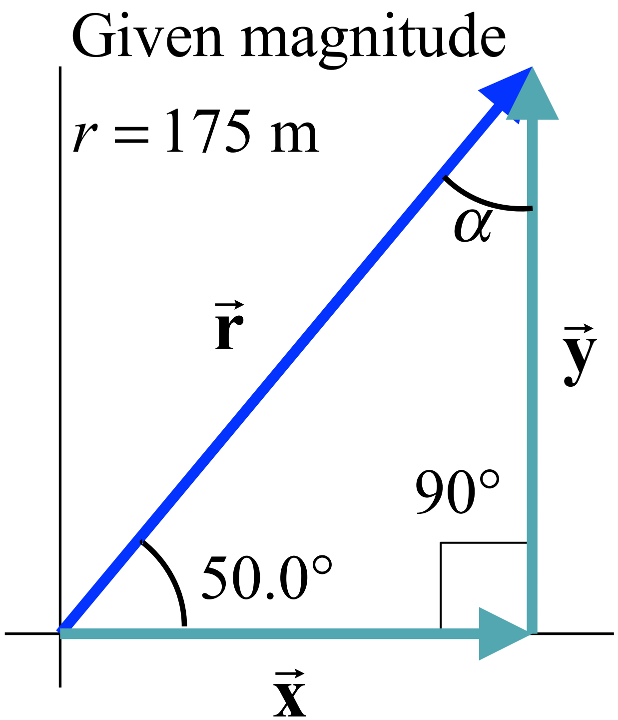
$\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are "unit vectors", magnitude is 1.

vector $\vec{\mathbf{A}}$ expressed in terms of its scalar components: $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$

A_x and A_y are the scalar components of the vector, $\vec{\mathbf{A}}$.

Example

A displacement vector has a magnitude of 175 m and points at an angle of 50.0 degrees relative to the *x* axis. Find the *x* and *y* components of this vector.



vector $\vec{\mathbf{x}}$ has magnitude x

vector $\vec{\mathbf{y}}$ has magnitude y

$$\sin\theta = y/r$$

y-component of the vector $\vec{\mathbf{r}}$

$$y = r\sin\theta = (175\text{ m})(\sin 50.0^\circ) = 134\text{ m}$$

$$\cos\theta = x/r$$

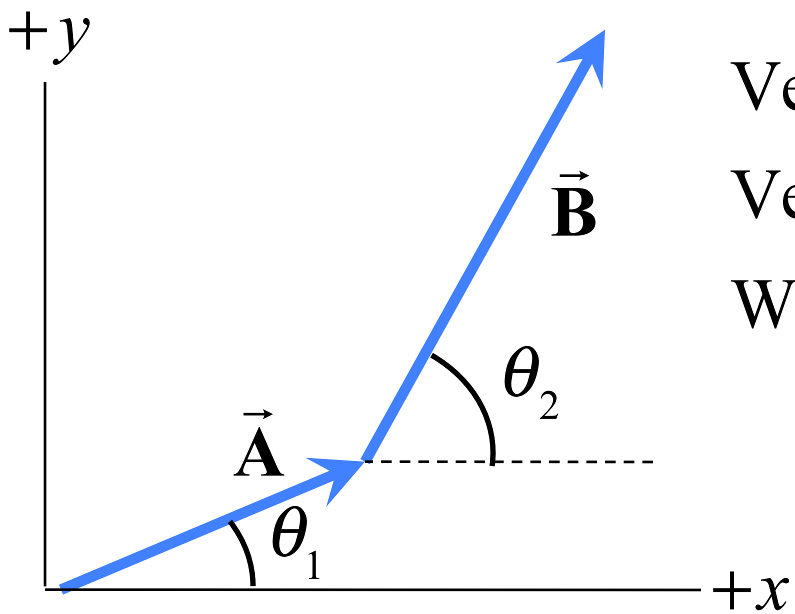
x-component of the vector $\vec{\mathbf{r}}$

$$x = r\cos\theta = (175\text{ m})(\cos 50.0^\circ) = 112\text{ m}$$

$$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$$

$$= (112\text{ m})\hat{\mathbf{i}} + (134\text{ m})\hat{\mathbf{j}}$$

3.2 Vector Addition and Subtraction (using Components)

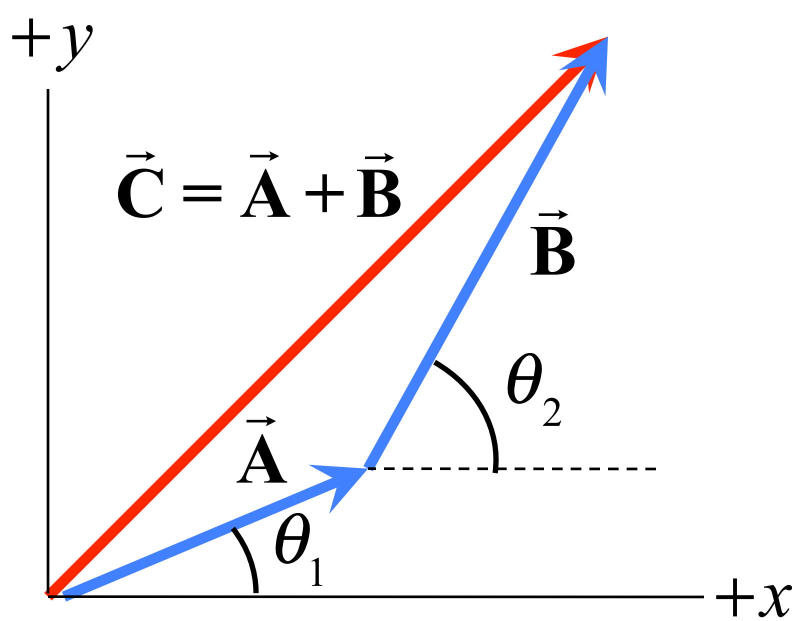


Vector $\vec{\mathbf{A}}$ has magnitude A and angle θ_1

Vector $\vec{\mathbf{B}}$ has magnitude B and angle θ_2

What is the vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$?

Graphically no PROBLEM



THIS IS A BIG PROBLEM

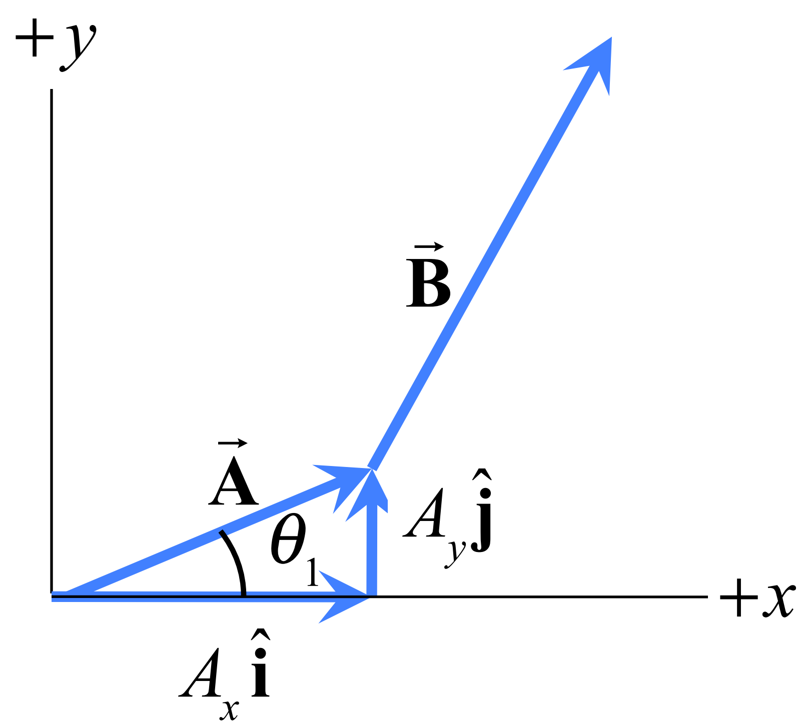
What is the magnitude of $\vec{\mathbf{C}}$, and what angle θ does it make relative to x-axis ?

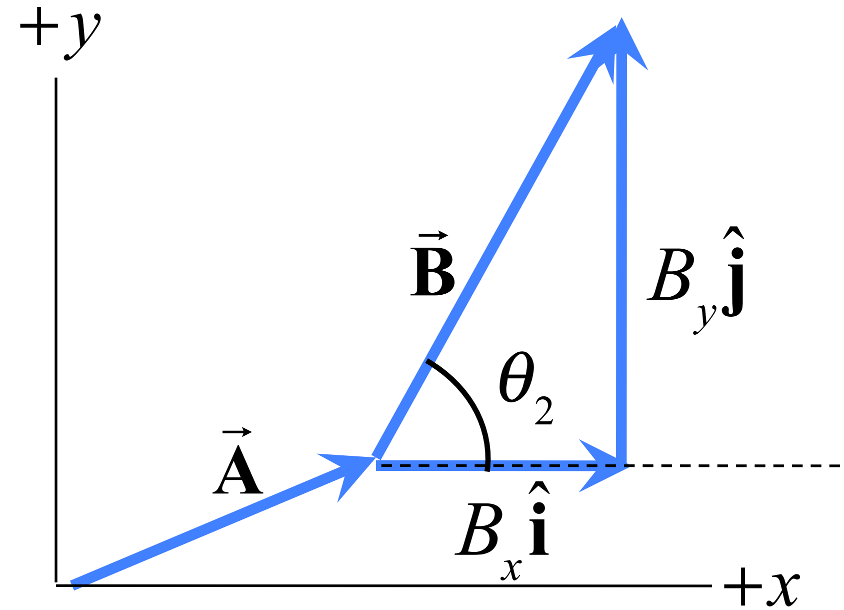
3.2 Vector Addition and Subtraction (using Components)

THIS IS A BIG PROBLEM

What is the magnitude of $\vec{\mathbf{C}}$, and what angle θ does it make relative to x-axis ?

The only way to solve this problem is to use vector components!





3.2 Vector Addition and Subtraction (using Components)

Get the components of the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$\vec{\mathbf{A}}$: magnitude A and angle θ_1

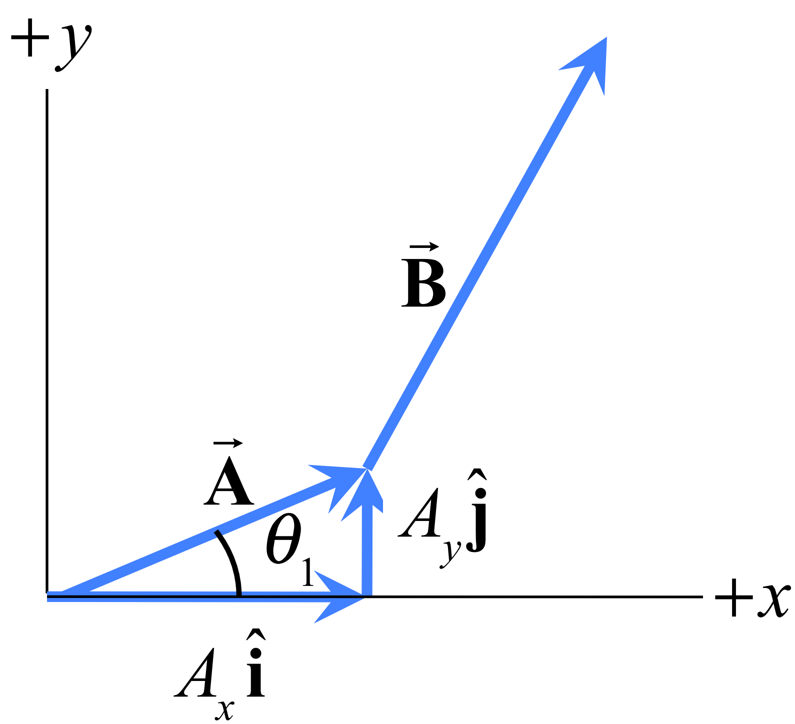
$\vec{\mathbf{B}}$: magnitude B and angle θ_2

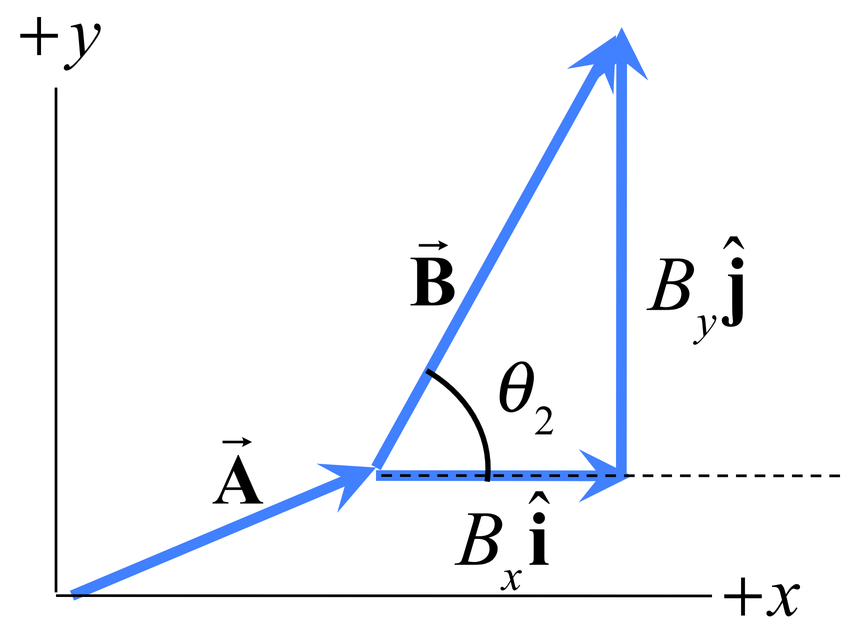
$$A_x = A\cos\theta_1$$

$$A_y = A\sin\theta_1$$

$$B_x = B\cos\theta_2$$

$$B_y = B\sin\theta_2$$



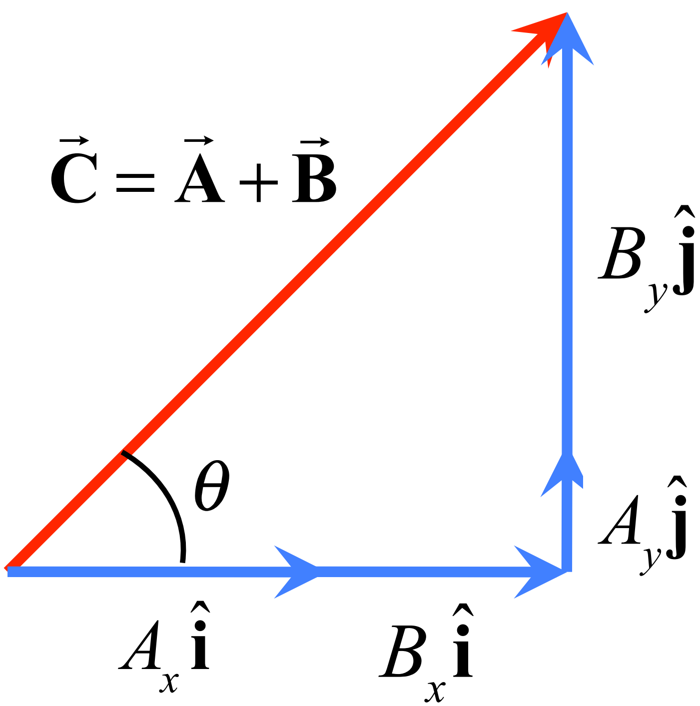


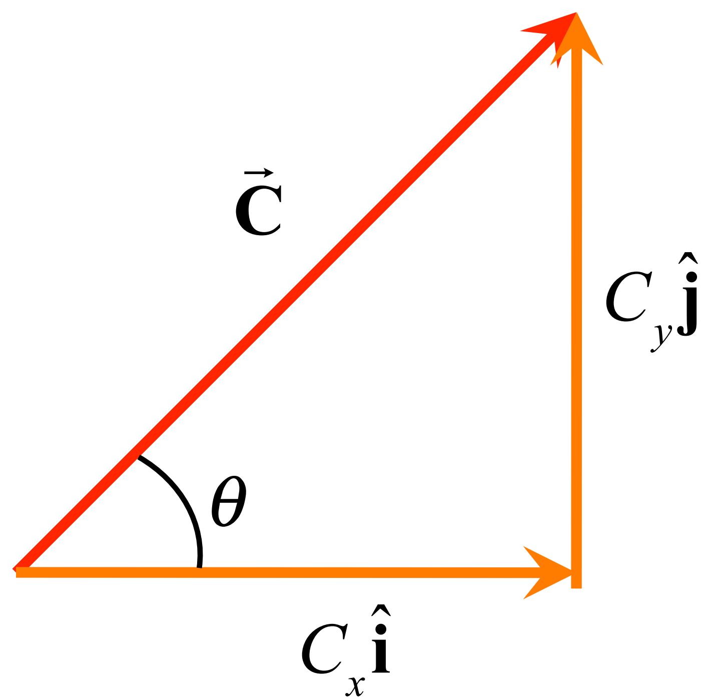
3.2 Vector Addition and Subtraction (using Components)

Get components of vector $\vec{\mathbf{C}}$ from components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$$A_x = A\cos\theta_1 \qquad B_x = B\cos\theta_2$$
$$A_y = A\sin\theta_1 \qquad B_y = B\sin\theta_2$$

$$\Rightarrow \quad C_x = A_x + B_x$$
$$C_y = A_y + B_y$$

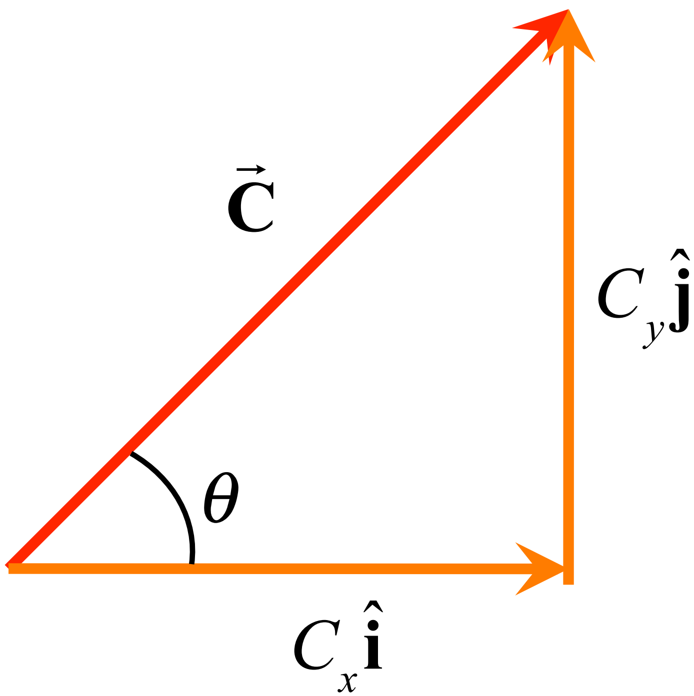




3.2 Vector Addition and Subtraction (using Components)

What is the magnitude of $\vec{\mathbf{C}}$, and what angle θ does it make relative to x-axis ?

$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$



PROBLEM IS SOLVED

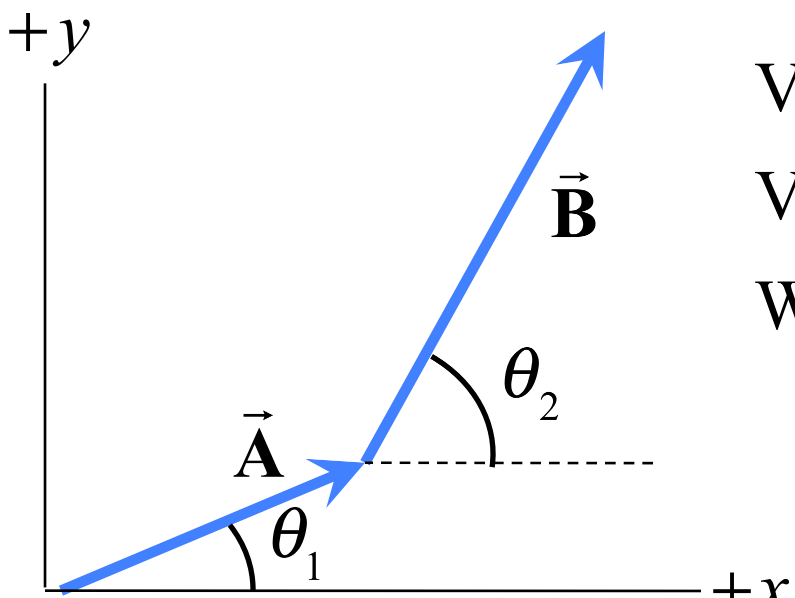
magnitude of $\vec{\mathbf{C}}$: $C = \sqrt{C_x^2 + C_y^2}$

Angle θ: $\tan\theta = \dfrac{C_y}{C_x}$;

$$\theta = \tan^{-1}(C_y / C_x)$$

3.2 Addition of Vectors by Means of Components

Summary of adding two vectors together



Vector $\vec{\mathbf{A}}$ has magnitude A and angle θ_1

Vector $\vec{\mathbf{B}}$ has magnitude B and angle θ_2

What is the vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$?

1) Determine components of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$: A_x, A_y and B_x, B_y

2) Add x-components to find $C_x = A_x + B_x$

3) Add y-components to find $C_y = A_y + B_y$

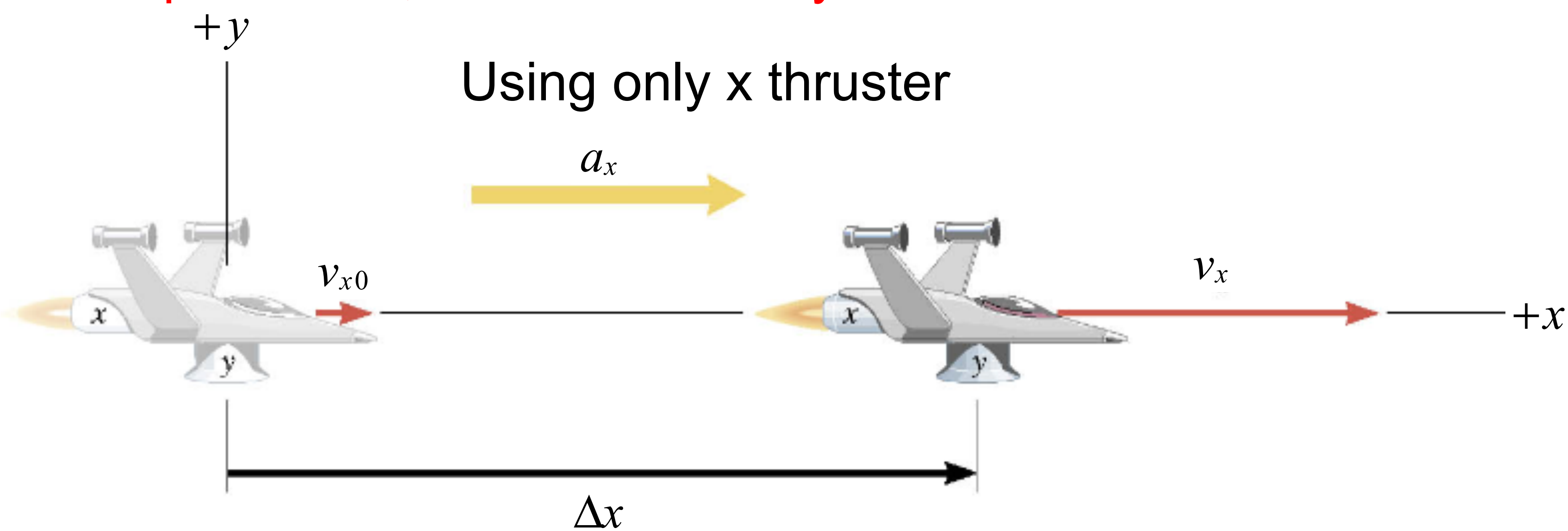
4) Determine the magnitude and angle of vector $\vec{\mathbf{C}}$

magnitude $C = \sqrt{C_x^2 + C_y^2}$; $\theta = \tan^{-1}(C_y / C_x)$

3.2 *Equations of Kinematics in Two Dimensions*

Except for time, motion in x and y directions are INDEPENDENT.

Using only x thruster



Motion in x direction.

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right) t$$

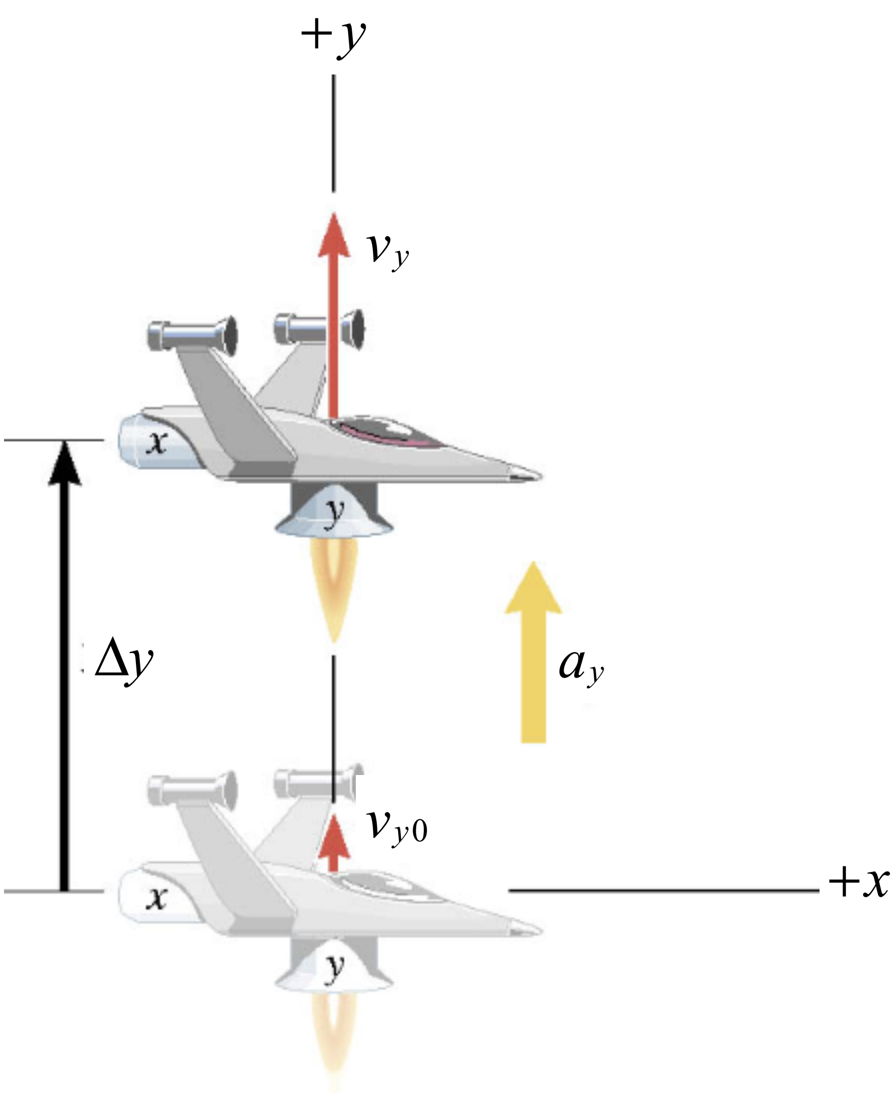
$$\Delta x = v_{x0} t + \tfrac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2 a_x \Delta x$$

3.2 *Equations of Kinematics in Two Dimensions*

Except for time, motion in x and y directions are INDEPENDENT.

Using only y thruster



Motion in y direction.

$$v_y = v_{y0} + a_y t$$

$$\Delta y = v_{y0} t + \tfrac{1}{2} a_y t^2$$

$$\Delta y = \tfrac{1}{2}\left(v_{y0} + v_y\right) t$$

$$v_y^2 = v_{y0}^2 + 2 a_y \Delta y$$

3.2 Equations of Kinematics in Two Dimensions

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables associated with each direction.

3.2 Equations of Kinematics in Two Dimensions

Example: A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s². In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s². Find (a) Δx and v_x, (b) Δy and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

Want final values:

$\Delta x, v_x, \Delta y, v_y$, and v, at time $t = 7\text{s}$.

initial values

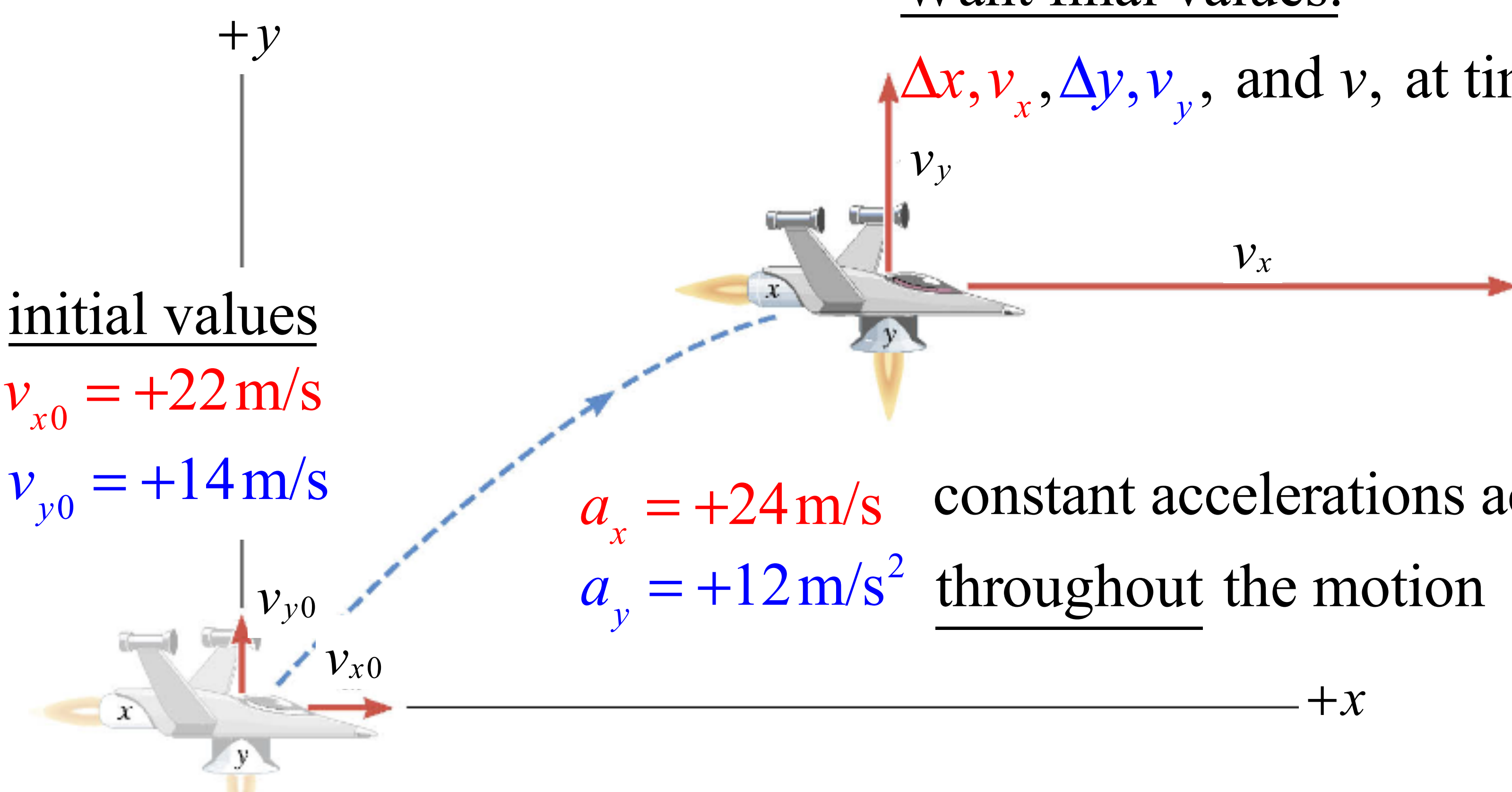
$v_{x0} = +22\,\text{m/s}$

$v_{y0} = +14\,\text{m/s}$

$a_x = +24\,\text{m/s}$

$a_y = +12\,\text{m/s}^2$

constant accelerations act throughout the motion



3.2 *Equations of Kinematics in Two Dimensions*

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables associated with each direction.

4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y*. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

3.2 Equations of Kinematics in Two Dimensions

Example: A Moving Spacecraft:

x direction motion

Δx	a_x	v_x	v_{x0}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{x0}t + \tfrac{1}{2}a_x t^2$$

$$= \left(22\,\text{m/s}\right)\left(7.0\ \text{s}\right) + \tfrac{1}{2}\left(24\,\text{m/s}^2\right)\left(7.0\ \text{s}\right)^2 = +740\ \text{m}$$

$$v_x = v_{x0} + a_x t$$

$$= \left(22\,\text{m/s}\right) + \left(24\,\text{m/s}^2\right)\left(7.0\ \text{s}\right) = +190\,\text{m/s}$$

3.2 *Equations of Kinematics in Two Dimensions*

Example: A Moving Spacecraft:

y direction motion

Δy	a_y	v_y	v_{y0}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

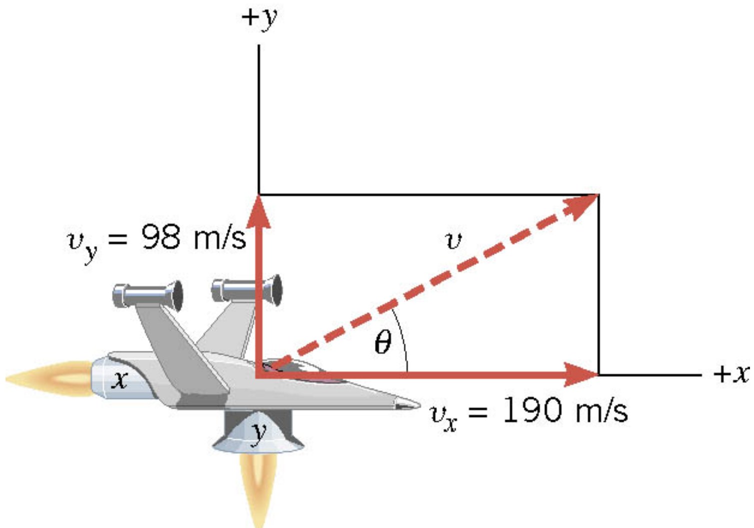
$$= (14\,\text{m/s})(7.0\text{ s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\text{ s})^2 = +390\text{ m}$$

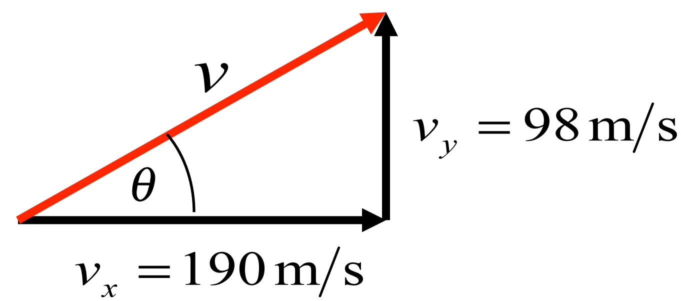
$$v_y = v_{y0} + a_y t$$

$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\text{ s}) = +98\,\text{m/s}$$

3.2 Equations of Kinematics in Two Dimensions

Can also find final speed and direction (angle) at $t = 7\text{s}$.





$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(190\,\text{m/s})^2 + (98\,\text{m/s})^2} = 210\,\text{m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

3.3 Projectile Motion

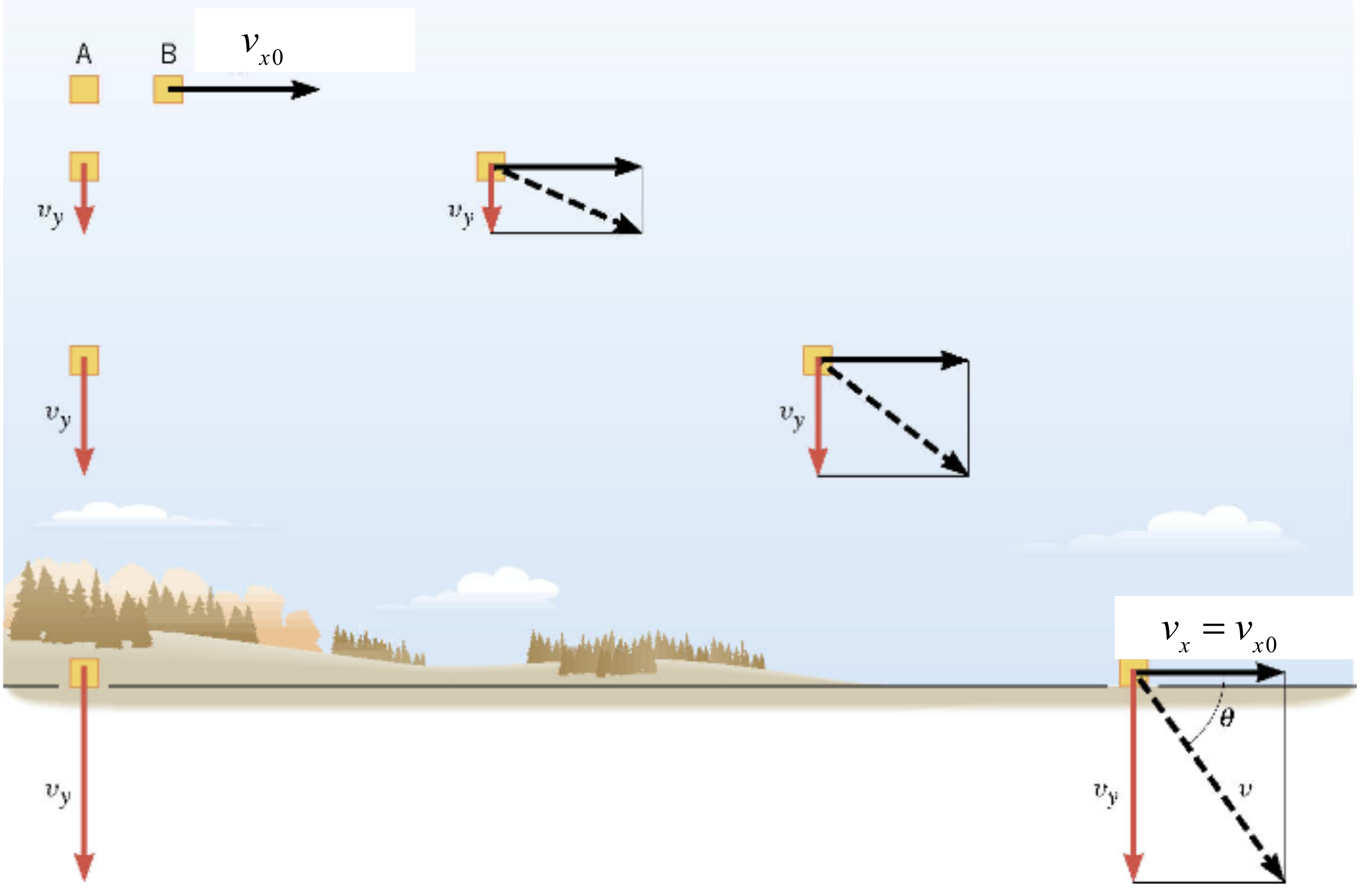
Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at $9.81 m/s^2$.

$$a_y = -9.81\,\mathrm{m/s^2} \qquad a_x = 0$$

Great simplification for projectiles !

$$\Rightarrow \quad v_x = v_{x0} = \text{constant}$$

Pop and Drop Demonstration.

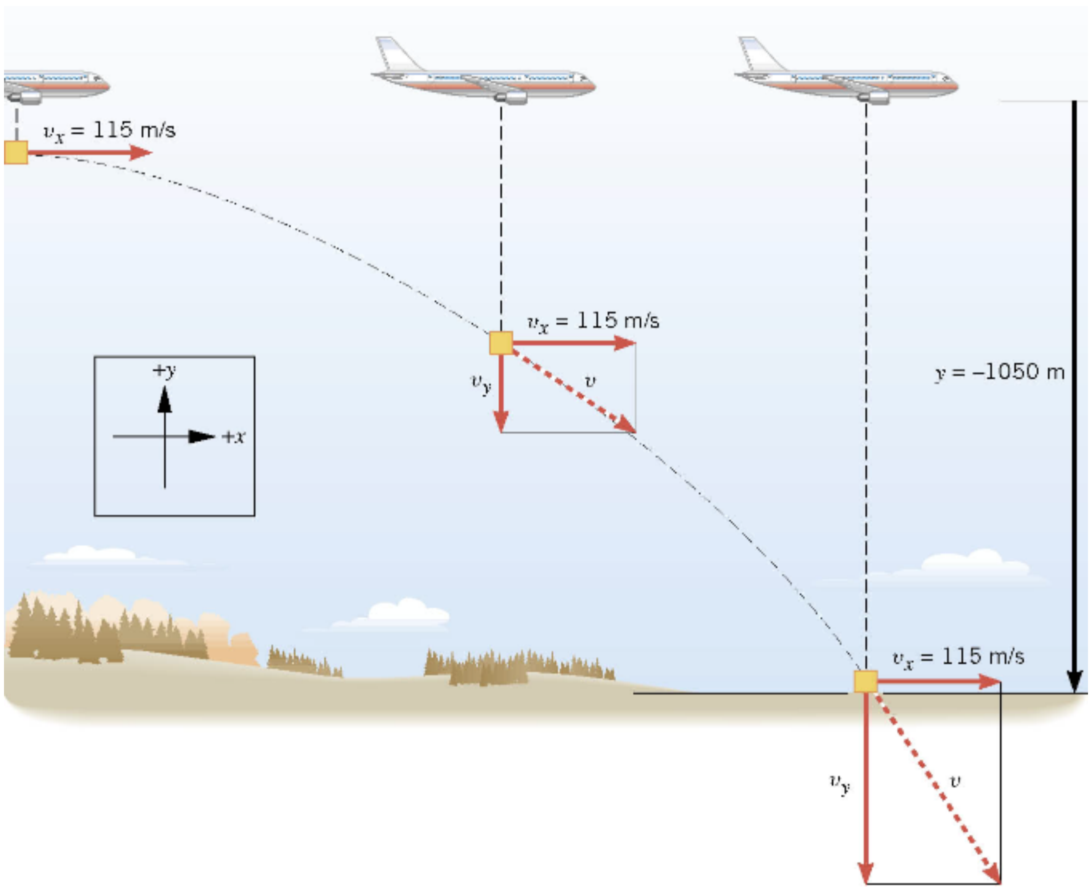


3.3 ***Projectile Motion***

Example: A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.

Time to hit the ground depends ONLY on vertical motion



$$v_{y0} = 0$$

$$a_y = -9.81 \text{ m/s}^2$$

$$y = 1050 \text{ m}$$

3.3 Projectile Motion

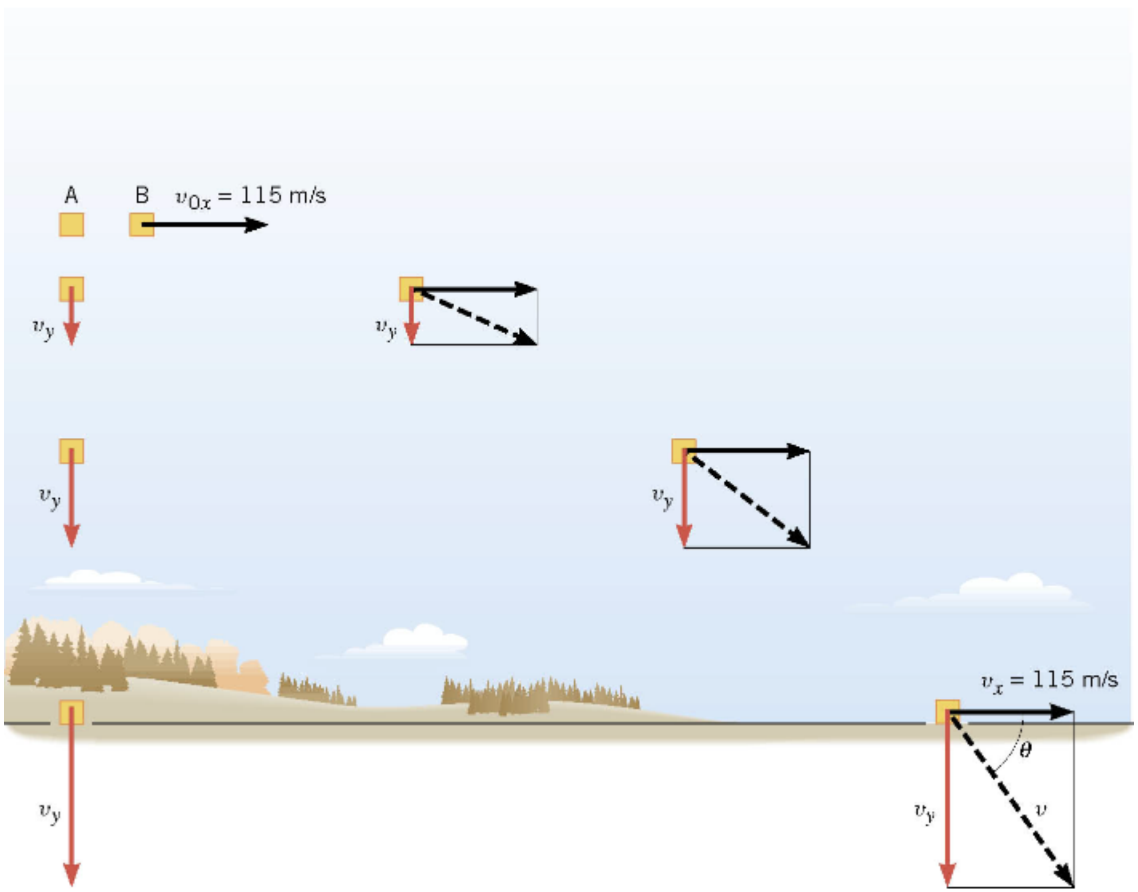
Δy	a_y	v_y	v_{y0}	t
-1050 m	-9.81 m/s^2		0 m/s	?

$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \Rightarrow \quad \Delta y = \tfrac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2\left(-1050\ \text{m}\right)}{-9.81\text{m}/\text{s}^2}} = 14.6\ \text{s}$$

Example: The Velocity of the Care Package

What are the magnitude and direction of the final velocity of the care package?



$$v_{y0} = 0$$

$$a_y = -9.81 \text{ m/s}^2$$

$$\Delta y = 1050 \text{ m}$$

$$t = 14.6 \text{ s}$$

$$v_{x0} = +115 \text{ m/s}$$

$$a_x = 0$$

$$v_x = v_{0x} = +115 \text{ m/s}$$

x-component does not change

Δy	a_y	v_y	v_{y0}	t
–1050 m	–9.81 m/s^2	?	0 m/s	14.6 s

$$v_y = v_{oy} + a_y t = 0 + \left(-9.81\,\text{m/s}^2\right)\left(14.6\ \text{s}\right)$$

$$= -143\,\text{m/s}$$ y-component of final velocity.

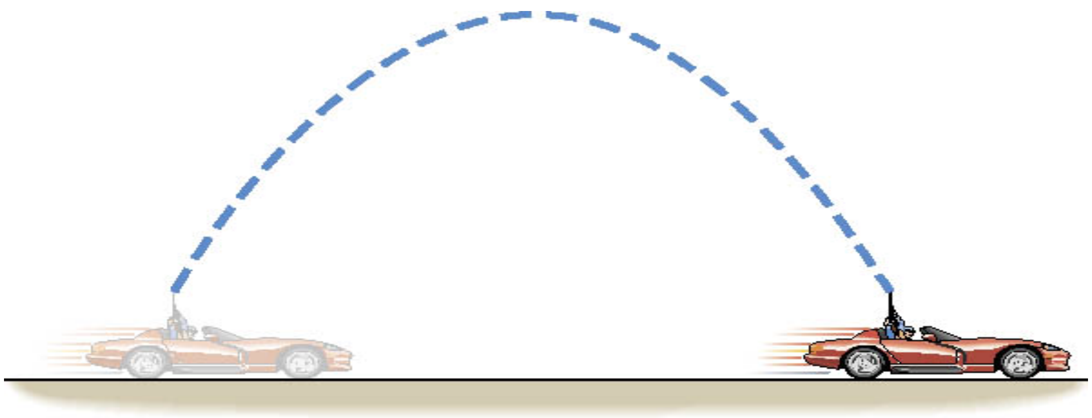
$$v_x = v_{ox} = +115\,\text{m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = 184\ \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-143}{+115}\right) = -51^\circ$$

Conceptual Example: I Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land – behind you, ahead of you, or in the barrel of the rifle?



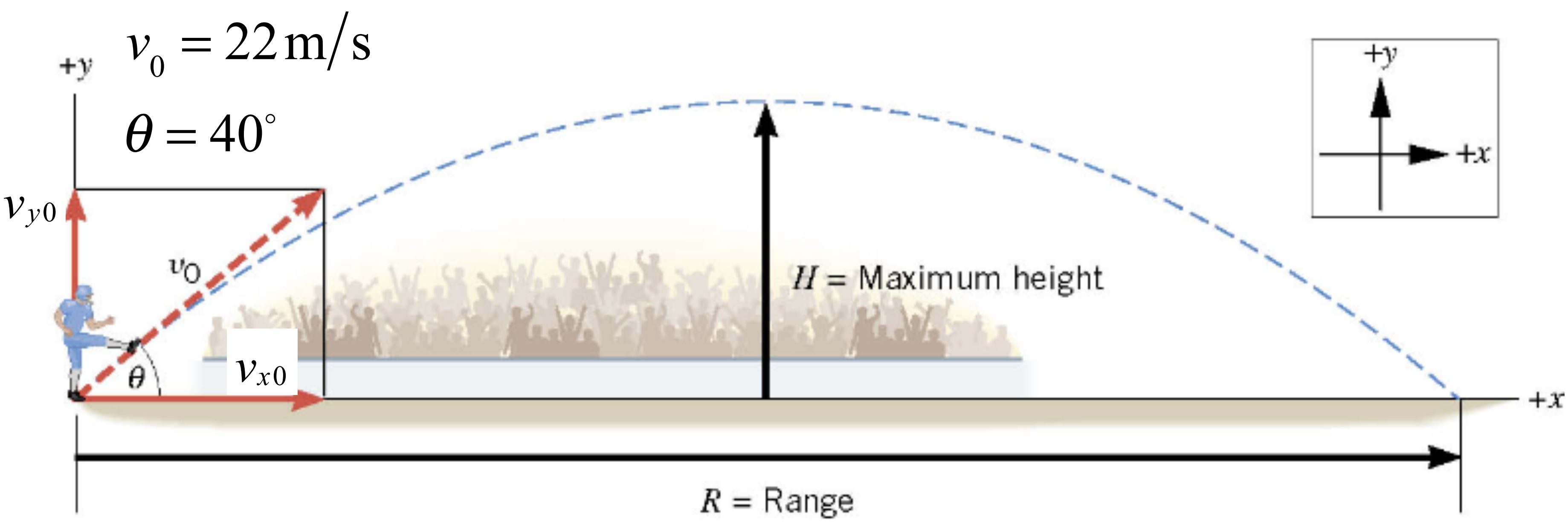
Ballistic Cart Demonstration

3.3 Projectile Motion

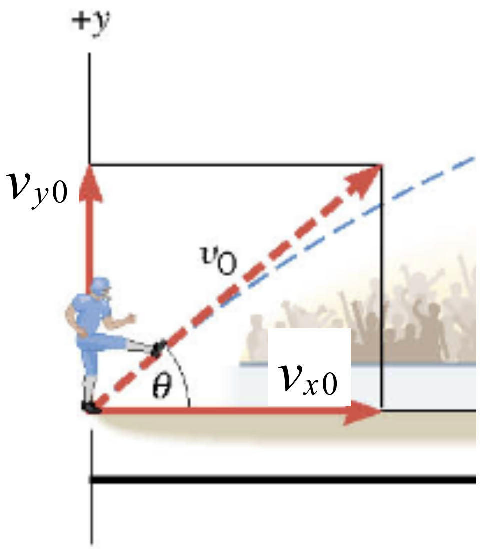
Example: The Height of a Kickoff

A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.

maximum height and "hang time" depend only on the y-component of initial velocity

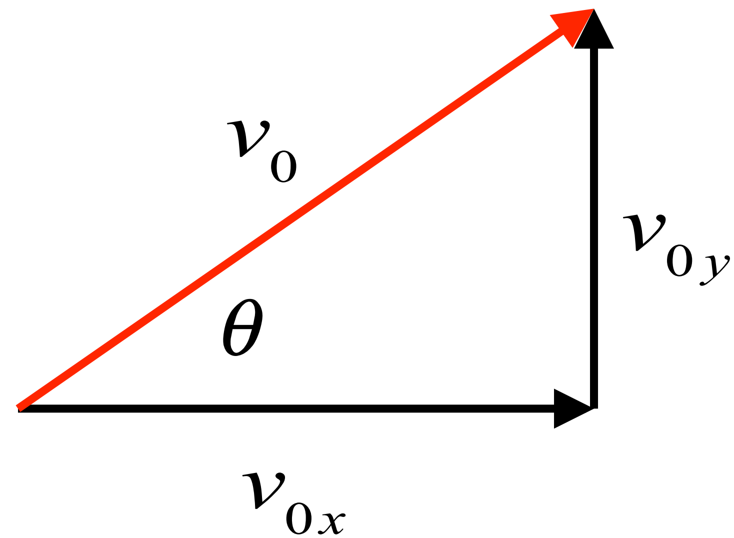


3.3 Projectile Motion



$$v_0 = 22\,\mathrm{m/s}$$

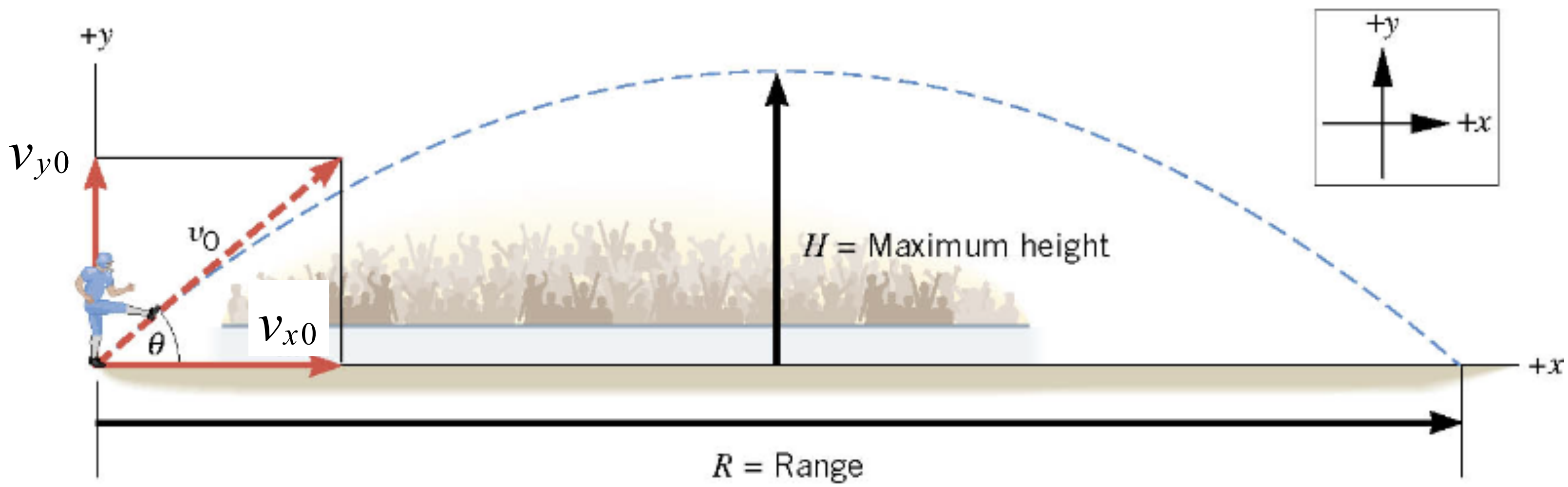
$$\theta = 40^\circ$$



$$v_{y0} = v_0 \sin\theta = \left(22\,\mathrm{m/s}\right)\sin 40^\circ = 14\,\mathrm{m/s}$$

$$v_{x0} = v_0 \cos\theta = \left(22\,\mathrm{m/s}\right)\cos 40^\circ = 17\,\mathrm{m/s}$$

3.3 Projectile Motion



Δy	a_y	v_y	v_{y0}	t
?	–9.80 m/s²	0	14 m/s	

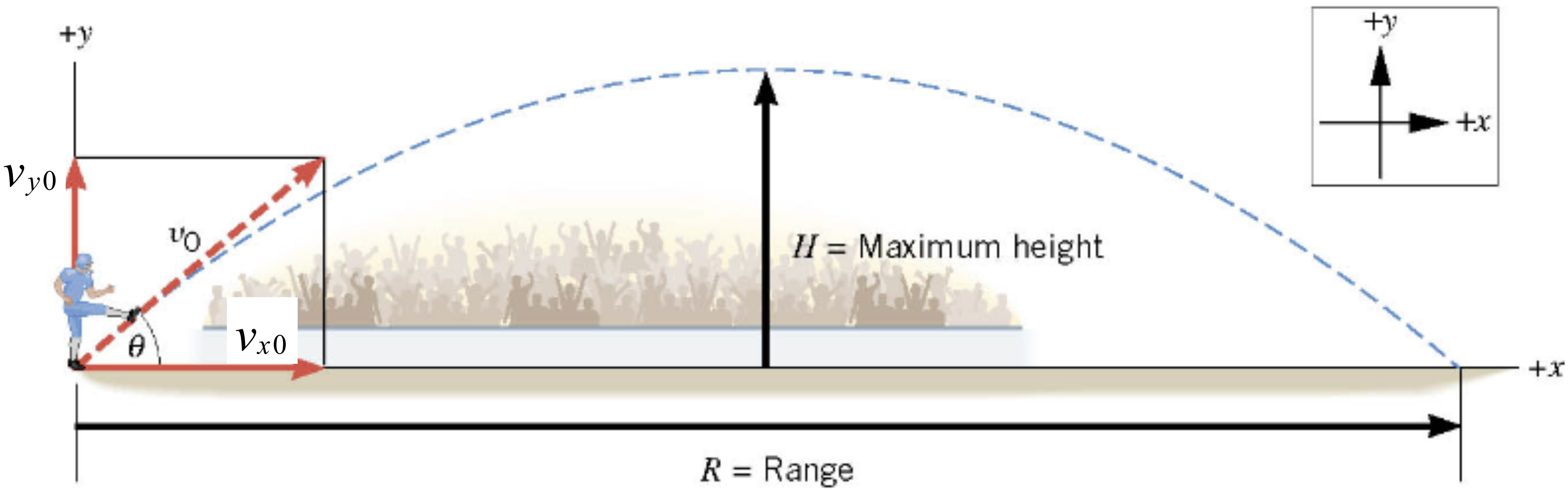
$$v_y^2 = v_{0y}^2 + 2a_y\Delta y \quad \Rightarrow \quad \Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

$$\Delta y = \frac{0-(14\,\text{m/s})^2}{2(-9.8\,\text{m/s}^2)} = +10\ \text{m}$$

3.3 Projectile Motion

Example: The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



Δy	a_y	v_y	v_{y0}	t
0	–9.80 m/s^2		14 m/s	?

3.3 Projectile Motion

Δy	a_y	v_y	v_{y0}	t
0	–9.81 m/s^2		14 m/s	?

$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

$$0 = \left(14\,\mathrm{m/s}\right)t + \tfrac{1}{2}\left(-9.81\,\mathrm{m/s^2}\right)t^2$$

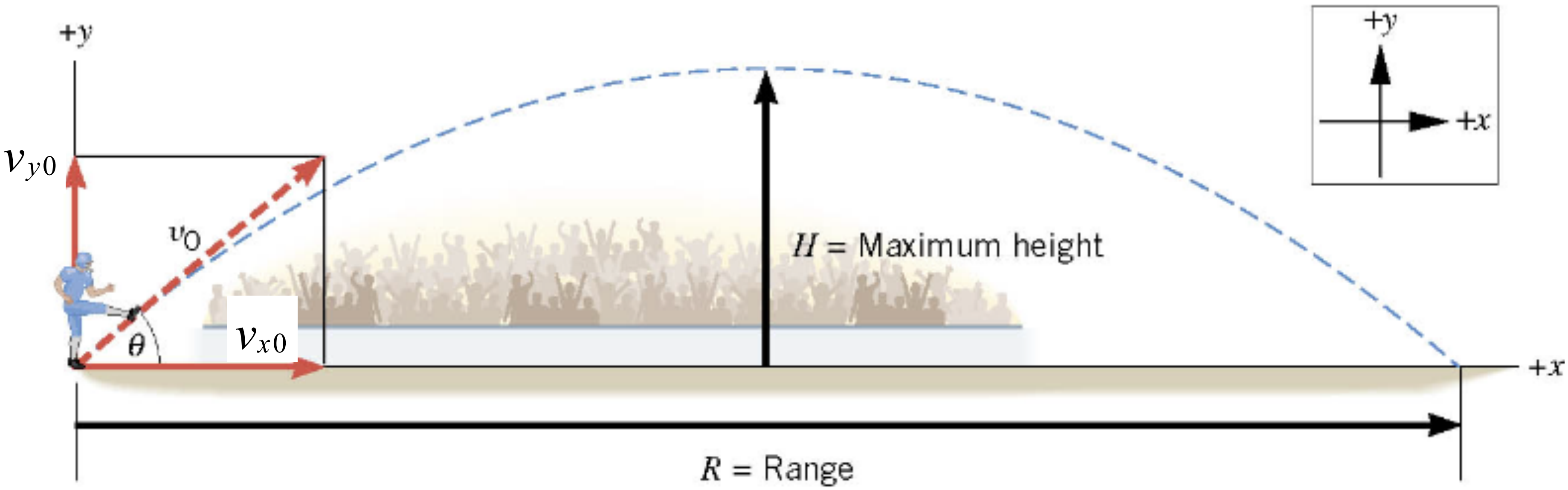
$$0 = 2\left(14\,\mathrm{m/s}\right) + \left(-9.81\,\mathrm{m/s^2}\right)t$$

$$t = 2.9\,\mathrm{s}$$

3.3 Projectile Motion

Example: The Range of a Kickoff
Calculate the range R of the projectile.

Range depends on the hang time and x-component of initial velocity

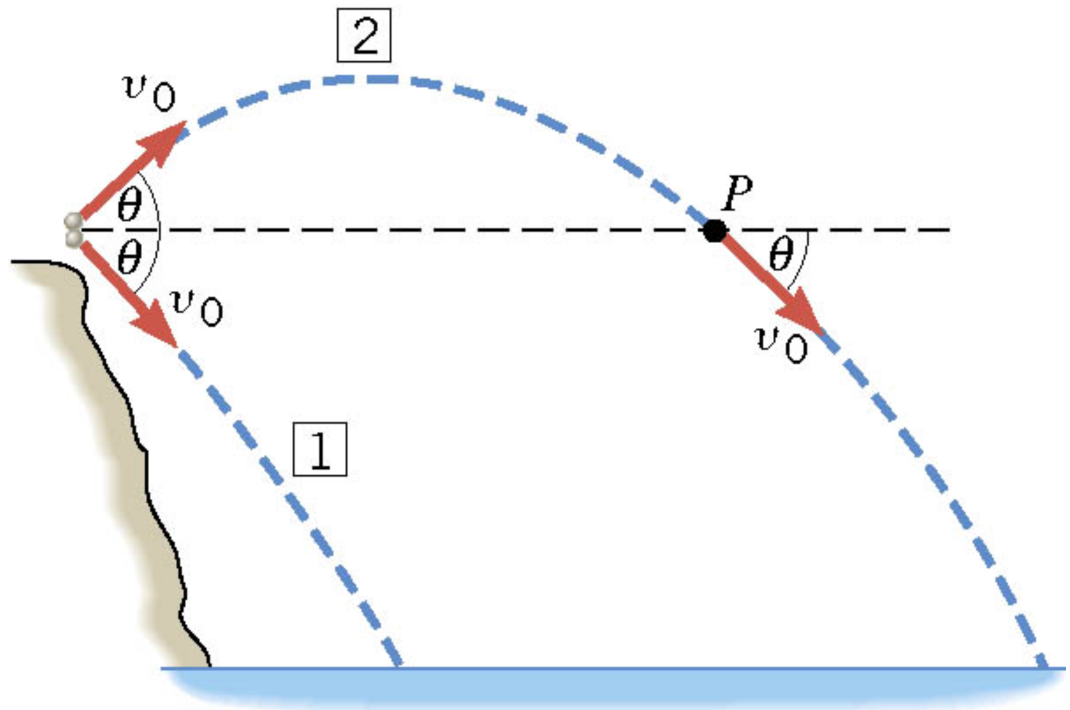


$$\Delta x = v_{x0}t + \tfrac{1}{2}a_x t^2 = v_{x0}t$$

$$= (17\,\text{m/s})(2.9\text{ s}) = +49\text{ m}$$

Conceptual Example: Two Ways to Throw a Stone

From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle above the horizontal and stone 2 is thrown at the same angle below the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?

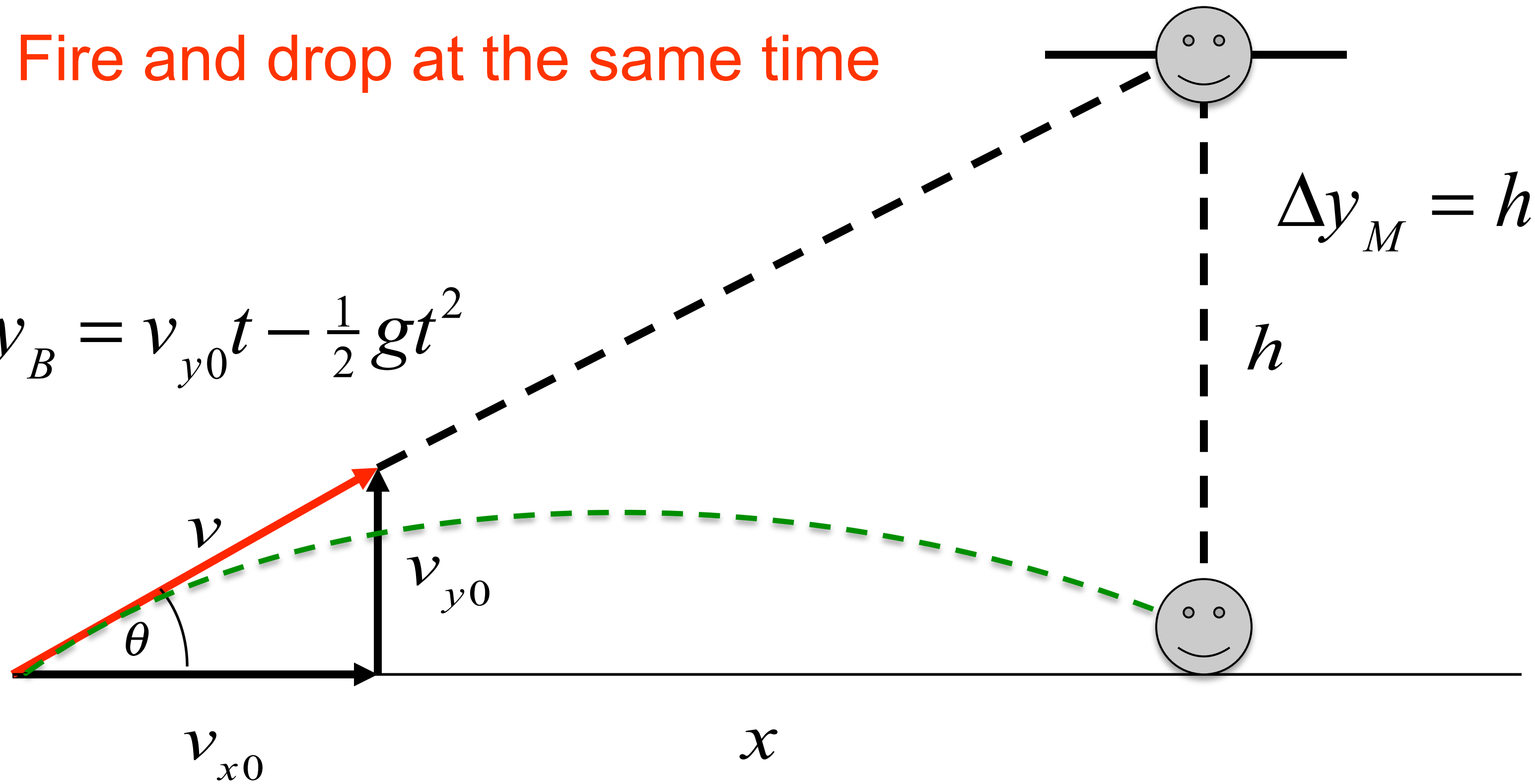


Shoot the Monkey Demonstration

Fire and drop at the same time

$$\Delta y_M = h - \tfrac{1}{2}gt^2$$

$$\Delta y_B = v_{y0}t - \tfrac{1}{2}gt^2$$



Shoot at the Monkey !

Hit height: $\Delta y_B = \Delta y_M \Rightarrow \quad v_{0y}t = h$

Hit time: $t = \dfrac{\Delta x}{v_{x0}}$ $\qquad \dfrac{v_{y0}}{v_{x0}}x = h$

$$\boxed{\frac{v_{y0}}{v_{x0}} = \frac{h}{x} = \tan\theta}$$