Chapter 6

Impulse and Momentum

Continued

6.4 Collisions in Two Dimensions

Momentum conservation can be used to solve collision problems if there are no external forces affecting the motion of the masses.

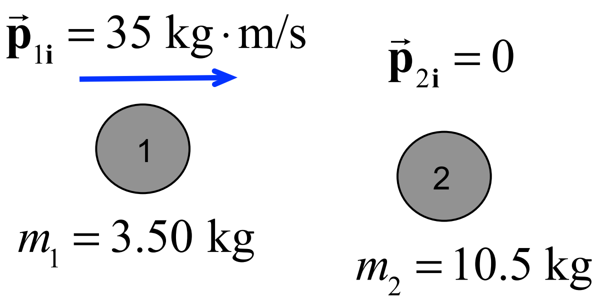
Energy conservation can be used to solve a collision problem if it is stated explicity that the collision is ELASTIC.

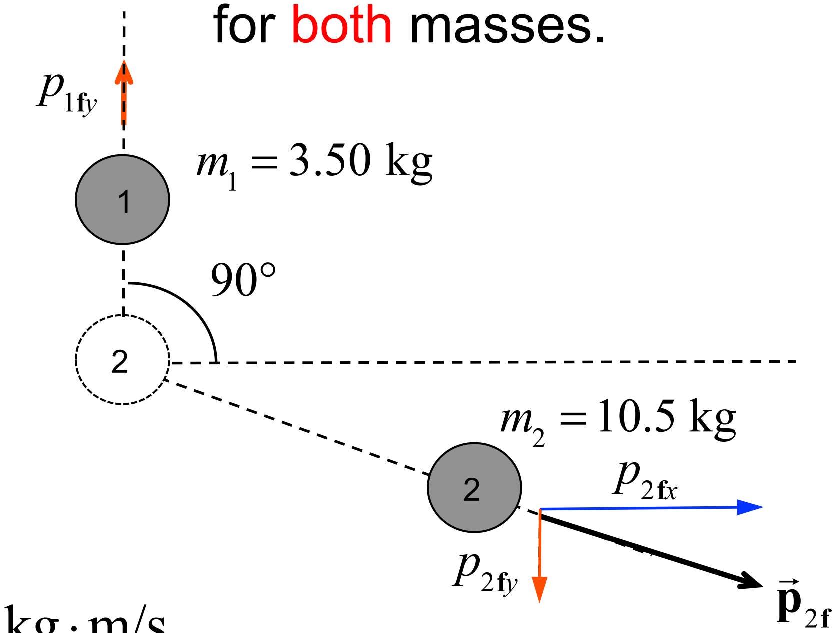
If two masses in a collision stick together, the collision must be INELASTIC. Energy conservation cannot be used unless the energy lost or gained is provided.

6.4 Collisions in Two Dimensions

In the elastic collision, m_1 is deflected upward at 90°.

Determine the final momentum vector for both masses.





Momentum conservation

x-components: $p_{1i} = p_{2fx} = 35\ \text{kg}\cdot\text{m/s}$

y-components: $p_{1fy} = -p_{2fy}$ (need this)

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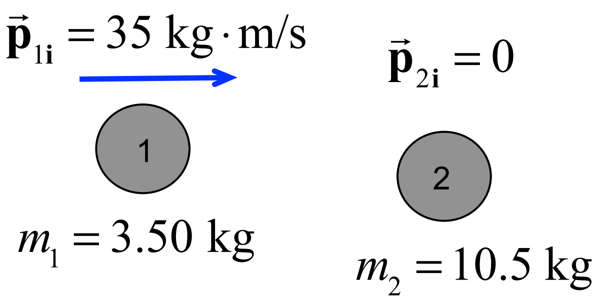

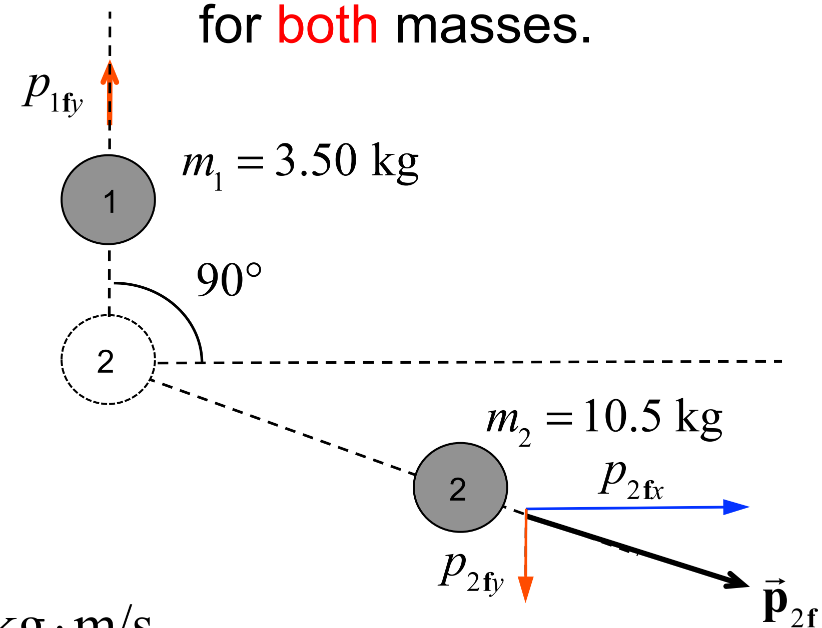

Momentum conservation

x-components: $p_{1\mathrm{i}} = \underline{p_{2\mathrm{f}x} = 35\ \mathrm{kg \cdot m/s}}$

y-components: $p_{1\mathrm{f}y} = \underline{-p_{2\mathrm{f}y}}$ (need this)

Kinetic Energies

$$K_\mathrm{i} = \frac{p_{1\mathrm{i}}^2}{2m_1}$$

$$K_\mathrm{f} = \frac{p_{1\mathrm{f}y}^2}{2m_1} + \frac{p_{2\mathrm{f}x}^2 + p_{2\mathrm{f}y}^2}{2m_2}$$

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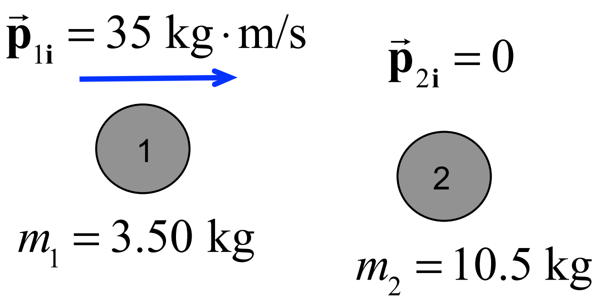

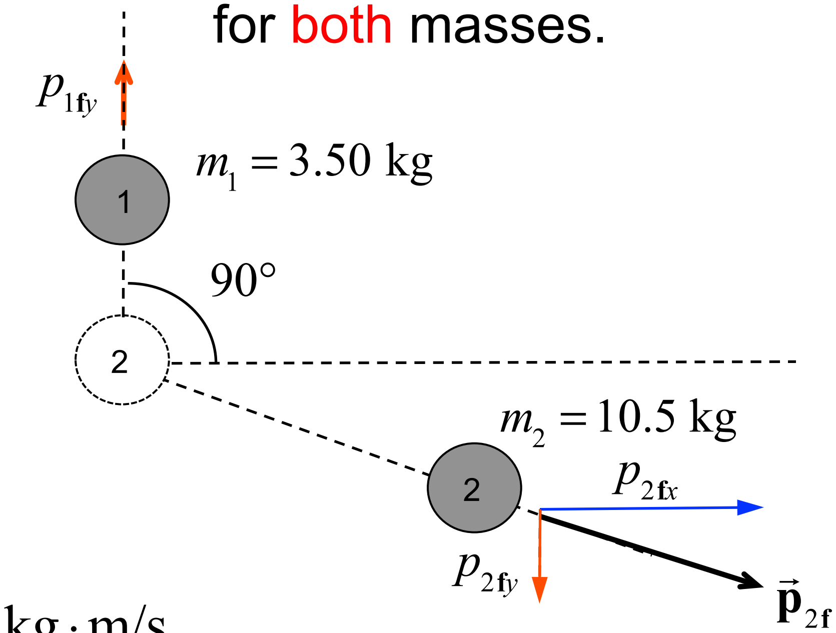

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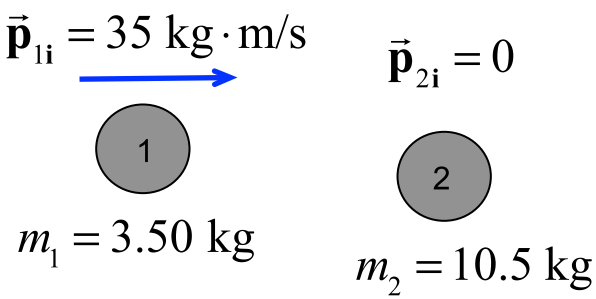

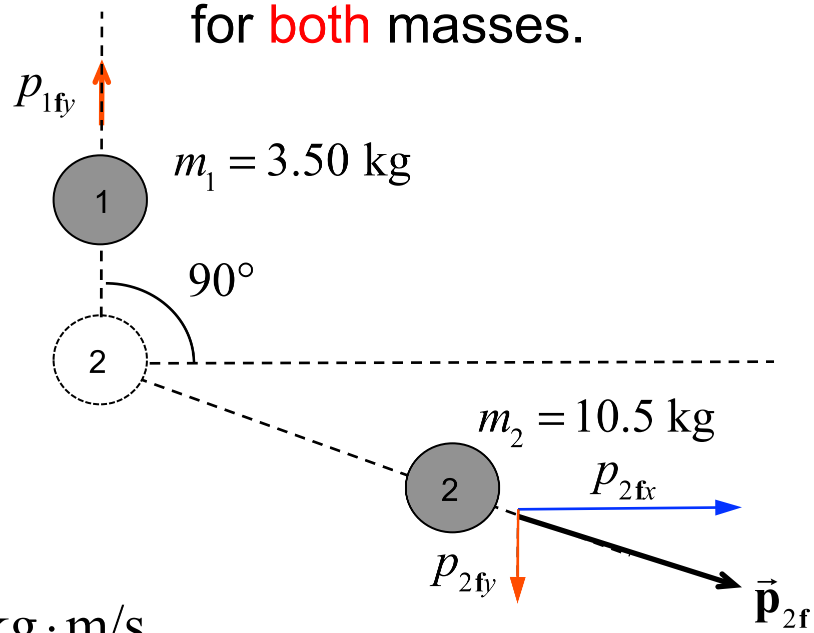

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x-components: $p_{1i} = p_{2fx} = 35\ \text{kg} \cdot \text{m/s}$

y-components: $p_{1fy} = -p_{2fy}$ (need this)

Kinetic Energies

$$K_i = \frac{p_{1i}^2}{2m_1}$$

$$K_f = \frac{p_{1fy}^2}{2m_1} + \frac{p_{2fx}^2 + p_{2fy}^2}{2m_2}$$

$$\frac{p_{2fy}^2}{2m_1} + \frac{p_{1i}^2 + p_{2fy}^2}{2m_2}$$

Energy conservation

$$K_f = K_i$$

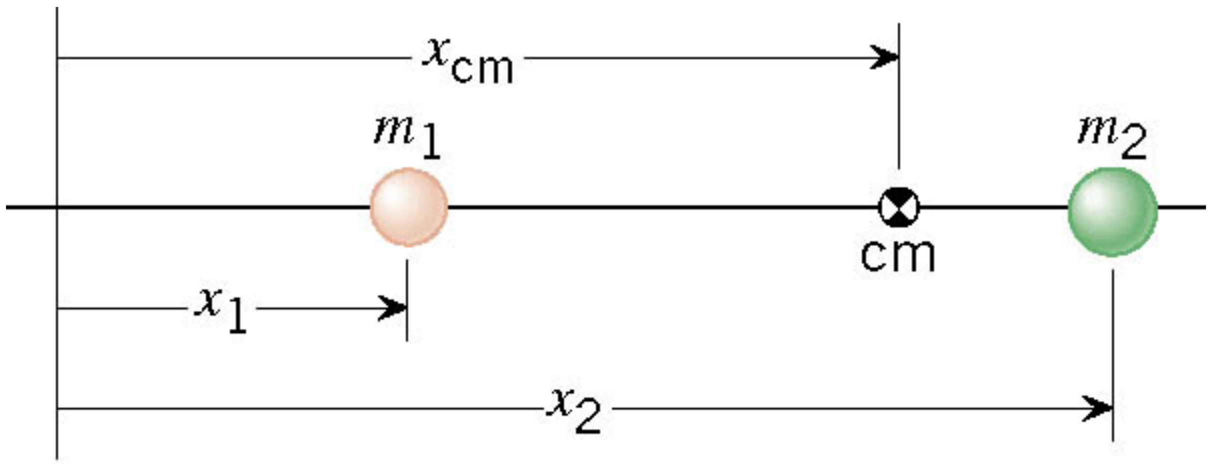
$$p_{2fy}^2\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = p_{1i}^2\left(\frac{1}{m_1} - \frac{1}{m_2}\right)$$

$$p_{2fy} = \pm\frac{1}{\sqrt{2}}p_{1i} \quad \Rightarrow \quad p_{2fy} = -24.7\ \text{kg} \cdot \text{m/s}$$

therefore, $p_{1fy} = +24.7\ \text{kg} \cdot \text{m/s}$

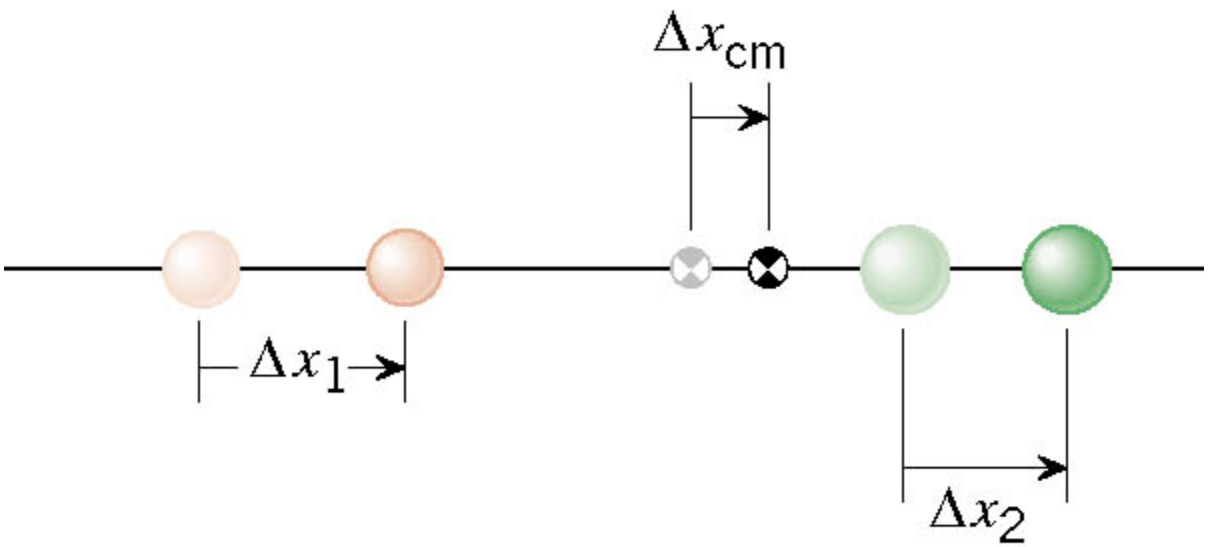
6.5 Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

6.5 Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \quad \Rightarrow \quad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

6.5 Center of Mass

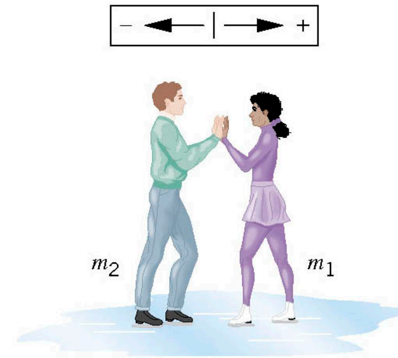
$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

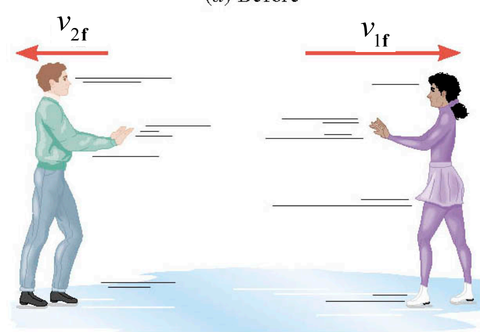
6.5 Center of Mass

BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$



(a) Before



(b) After

AFTER

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{(88\text{ kg})(-1.5\text{m/s}) + (54\text{ kg})(+2.5\text{m/s})}{88\text{ kg} + 54\text{ kg}}$$

$$= 0.00$$

Summaries and Examples
Energy and energy conservation
Momentum and momentum conservation

Energy and energy conservation

Work: $W = F(\cos\theta)\Delta x$

Kinetic Energy: $K = \frac{1}{2}mv^2$

Work changes kinetic energy: $W = K - K_0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

Conservative forces $\Rightarrow$ Potential Energy

Gravitational Potential Energy: $U_G = mgy$

Ideal Spring Potential Energy: $U_S = \frac{1}{2}kx^2$

Total Energy: $E = K + U$

Work by non-conservative forces (friction, humans, explosions)

changes total energy: $W_{\text{NC}} = \left(K - K_0\right) + \left(U - U_0\right)$

If $W_{\text{NC}} = 0$, there is total energy conservation:

$$E = E_0 \quad \Rightarrow \quad K + U = K_0 + U_0$$

Average power = Work/time = (Energy change)/time = $F\,\overline{v}$

Momentum and momentum conservation

Impulse: $\vec{\mathbf{J}} = \vec{\mathbf{F}}t$

Momentum: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

Net average impulse changes momentum:

$$\sum \bar{\vec{\mathbf{F}}}\Delta t = \vec{\mathbf{p}} - \vec{\mathbf{p}}_0 = m\vec{\mathbf{v}}_{\mathrm{f}} - m\vec{\mathbf{v}}_0$$

Momentum of 2 masses in collision: $\vec{\mathbf{p}}_{\mathrm{total}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$

No net external force active, momentum is conserved

$$\vec{\mathbf{p}}_{Total,\mathbf{f}} = \vec{\mathbf{p}}_{Total.\mathbf{i}} \quad \Rightarrow \quad m_1\vec{\mathbf{v}}_{1\mathbf{f}} + m_2\vec{\mathbf{v}}_{2\mathbf{f}} = m_1\vec{\mathbf{v}}_{1\mathbf{i}} + m_2\vec{\mathbf{v}}_{2\mathbf{i}}$$

Center of mass position: $x_{\mathrm{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$, and velocity $\vec{\mathbf{v}}_{\mathrm{cm}} = \dfrac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$

Conservation of momentum $\Rightarrow$ $\vec{\mathbf{v}}_{\mathrm{cm}}$ remains constant

Clicker Question 6.7

A space probe travels with a momentum of $+6.0 \times 10^{7}\ \mathrm{kg \cdot m/s}$.
A retrorocket fires for 10 s, with a force magnitude of $2.0 \times 10^{6}\ \mathrm{N}$, that slows the speed of the probe. What is the momentum of the probe after the rocket ceases to fire?

a) $2.0 \times 10^{7}\ \mathrm{kg \cdot m/s}$

b) $4.0 \times 10^{7}\ \mathrm{kg \cdot m/s}$

c) $6.0 \times 10^{7}\ \mathrm{kg \cdot m/s}$

d) $8.0 \times 10^{7}\ \mathrm{kg \cdot m/s}$

e) $10.0 \times 10^{7}\ \mathrm{kg \cdot m/s}$

Clicker Question 6.8

Two identical 2.0-kg objects are involved in a collision. The objects are on a frictionless track. The initial velocity of object A is +12 m/s and the initial velocity of object B is – 6.0 m/s. What is the final velocity of the two objects?

a) Object A moves at -6.0 m/s and B moves at +12 m/s.

b) Object A moves at $+9.0$ m/s and B moves at +9.0 m/s.

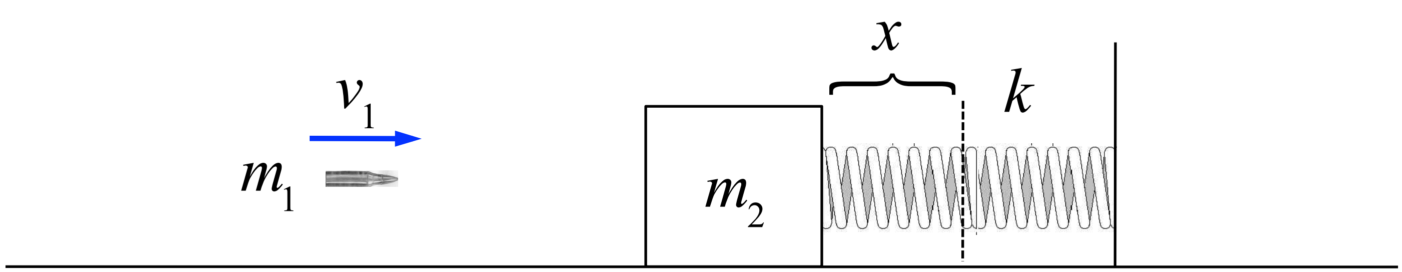
c) Object A moves at -6.0 m/s and B moves at +6.0 m/s.

d) Object A moves at -12 m/s and B moves at +6.0 m/s.

e) This cannot be determined without more information.

Example:

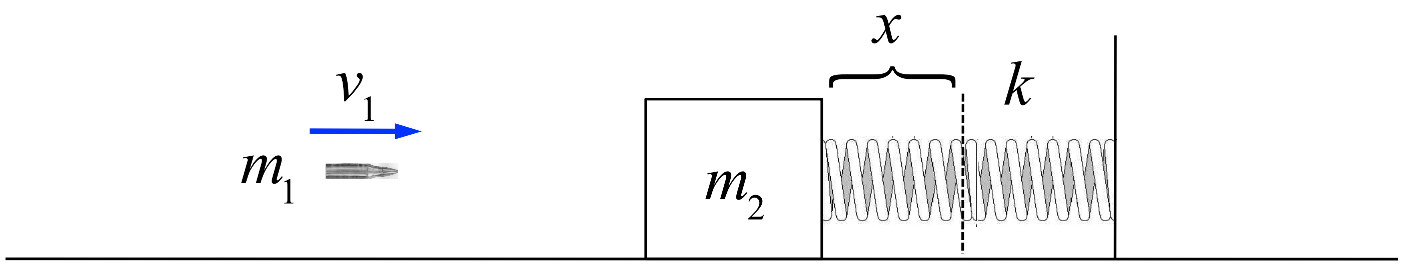
An 11-g bullet is fired horizontally into a 110-g wooden block that is initially at rest on a frictionless horizontal surface and connected to a spring having spring constant 183 N/m. The bullet becomes embedded in the block. The bullet-block system compresses the spring by a maximum amount 85 cm. What was the initial velocity of the bullet?



Strategy:

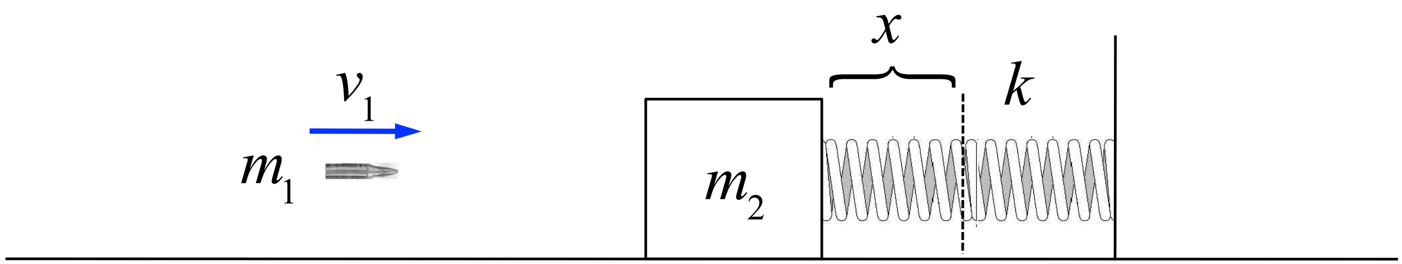
1) Momentum conservation bullet + block

2) Energy conservation in spring compression



1) Momentum conservation bullet + block

$$m_1 v_1 = (m_1 + m_2) v \quad \Rightarrow \quad v = \left[m_1 / (m_1 + m_2) \right] v_1$$



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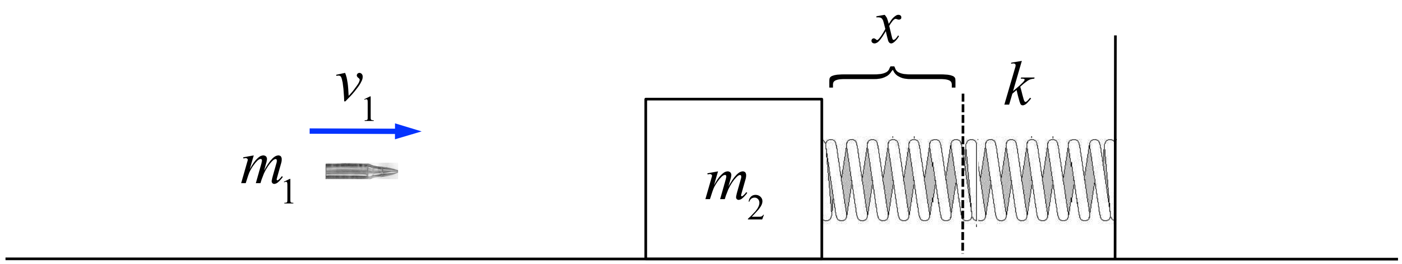
2) Energy conservation in spring compression

Kinetic Energy of block and bullet : $K_i = \frac{1}{2}(m_1 + m_2)v^2$

Potential Energy of compressed spring : $U = \frac{1}{2}kx^2$



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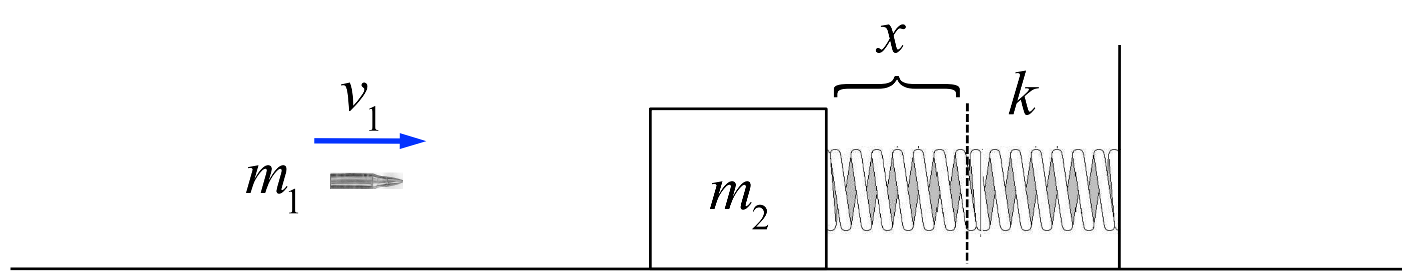
Energy Conservation : $K + U = K_\mathrm{i} + U_\mathrm{i} \quad ; \quad U_\mathrm{i} = 0, K = 0$



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$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}(m_1 + m_2)v^2 = \tfrac{1}{2}(m_1 + m_2)\left[m_1 / (m_1 + m_2) \right]^2 v_1^2$$

$$v_1 = \sqrt{k(m_1 + m_2)/m_1^2}\, x$$

$$= \sqrt{(183\,\text{N/m})(0.121\,\text{kg})/(0.011\,\text{kg})^2}\,(0.85\ \text{m}) = 364\ \text{m/s}$$