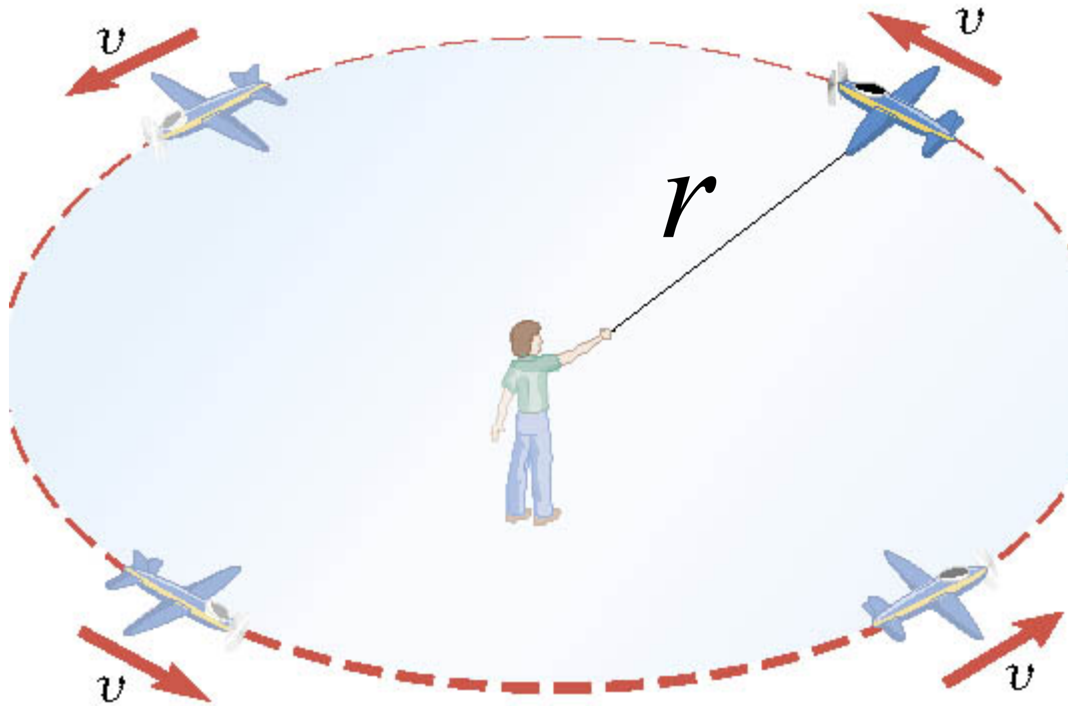
Chapter 3.5

Uniform Circular Motion

DEFINITION OF UNIFORM CIRCULAR MOTION

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.

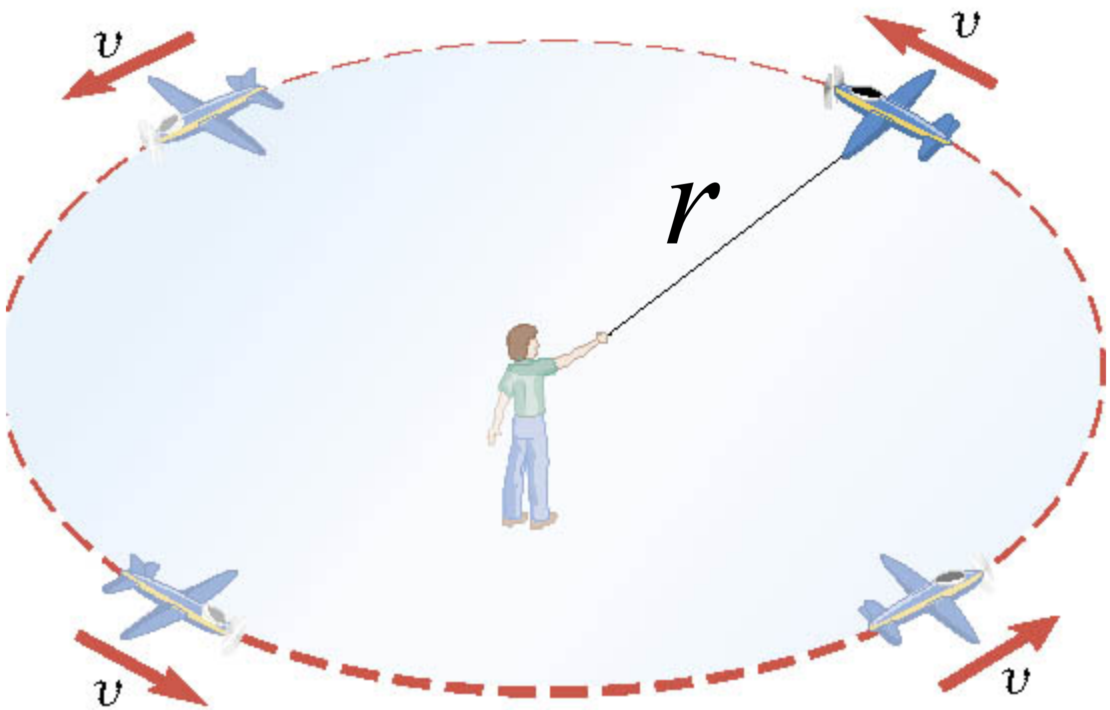


Circumference of the circle is $2\pi r$.

3.5 Uniform Circular Motion

The time it takes the object to travel once around the circle is *T* (a.k.a. the period)

$$\text{Speed around the circle is, } v = \frac{2\pi r}{T}.$$



Example: A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and it being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

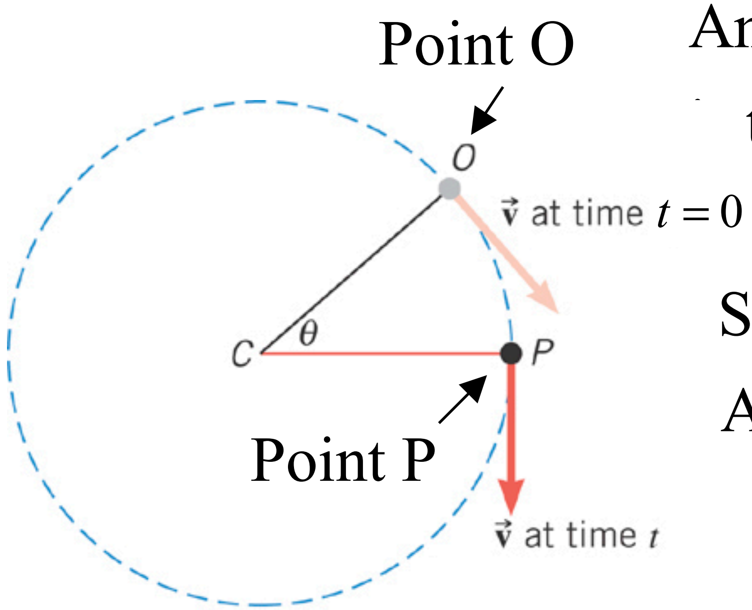
$$\frac{1}{830\,\text{revolutions/min}} = 1.2\times10^{-3}\ \text{min/revolution}$$

$$T = 1.2\times10^{-3}\ \text{min} = 0.072\ \text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29\ \text{m})}{0.072\ \text{s}} = 25\ \text{m/s}$$

3.5 *Centripetal Acceleration*

In uniform circular motion, the speed is *constant*, but the direction of the velocity vector is *not constant.*



(*a*)

Angle between point O and point P

the same as between $\vec{\mathbf{v}}_0$ and $\vec{\mathbf{v}}$.

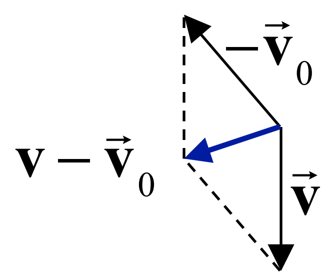
Since velocity vector changes direction

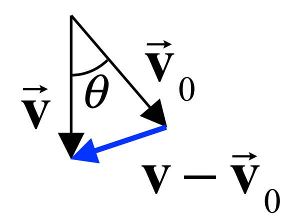
Acceleration vector is NOT ZERO.

$$\mathbf{a} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_0}{t}$$

Need to understand: $\vec{\mathbf{v}} - \vec{\mathbf{v}}_0$

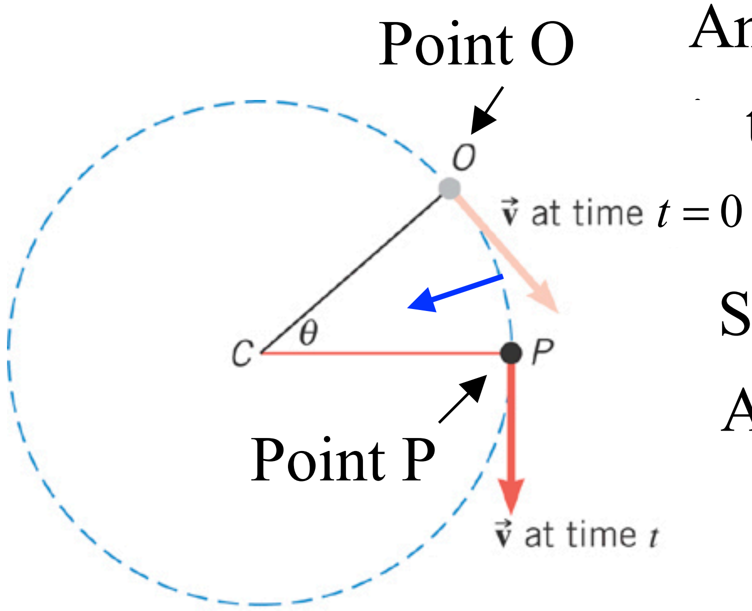
NOTE: $\vec{\mathbf{v}} - \vec{\mathbf{v}}_0$ and $\vec{\mathbf{a}}$ point in toward center of circle!





3.5 Centripetal Acceleration

In uniform circular motion, the speed is *constant*, but the direction of the velocity vector is *not constant.*



(a)

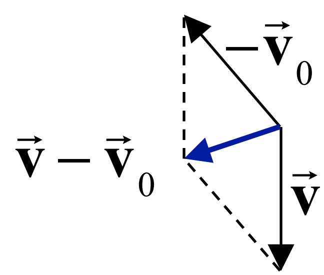
Angle between point O and point P the same as between $\vec{\mathbf{v}}_0$ and $\vec{\mathbf{v}}$.

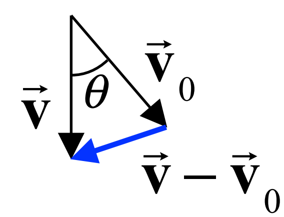
Since velocity vector changes direction Acceleration vector is NOT ZERO.

$$\mathbf{a} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_0}{t}$$

Need to understand: $\vec{\mathbf{v}} - \vec{\mathbf{v}}_0$

NOTE: $\vec{\mathbf{v}} - \vec{\mathbf{v}}_0$ and $\vec{\mathbf{a}}$ point in toward center of circle!

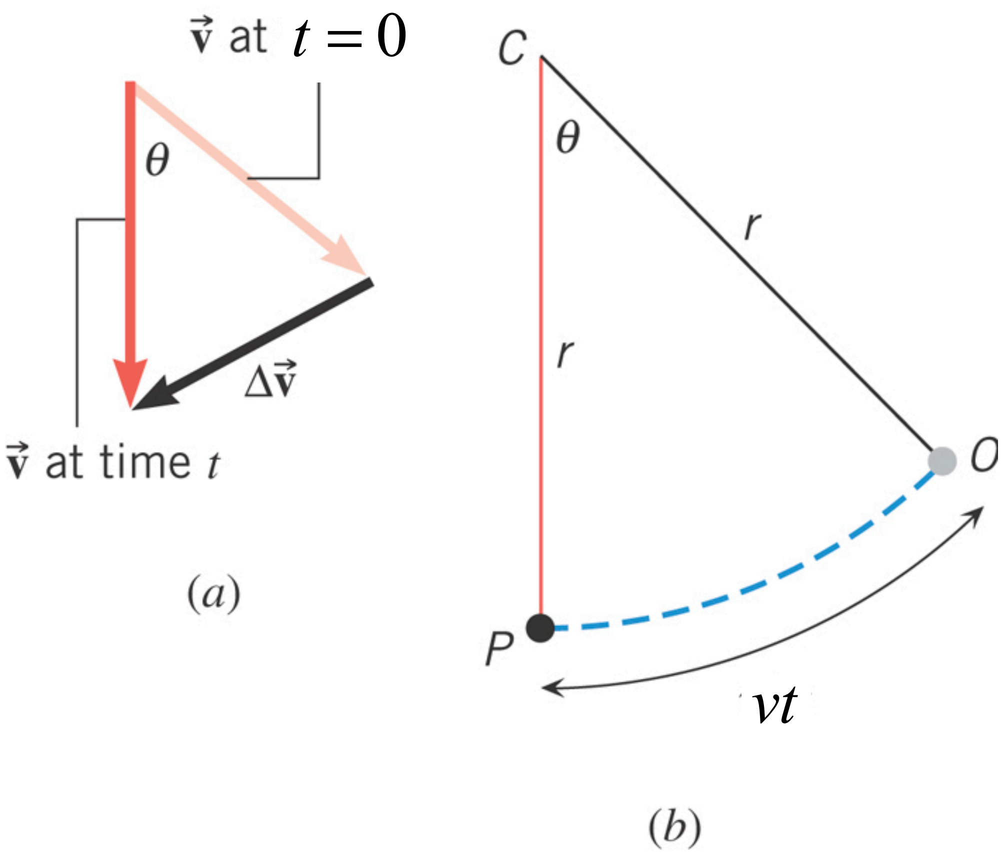




3.5 Centripetal Acceleration

Compare geometry of velocity vectors and the portion of the circle.

$$\theta = \frac{\Delta v}{v} \qquad \theta = \frac{vt}{r}$$



Magnitudes

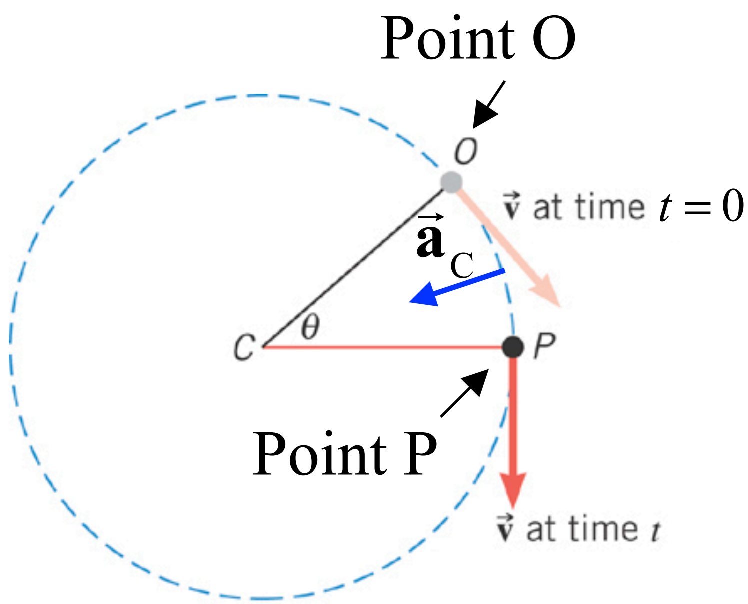
$$\frac{\Delta v}{v} = \frac{vt}{r}$$

$$\frac{\Delta v}{t} = \frac{v^2}{r}$$

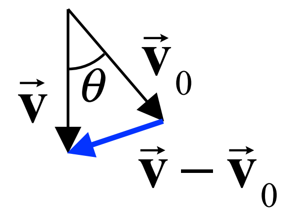
$$a_c = \frac{v^2}{r}$$

3.5 Centripetal Acceleration

The direction of the centripetal acceleration is towards the center of the circle; in the same direction as the change in velocity.



(a)



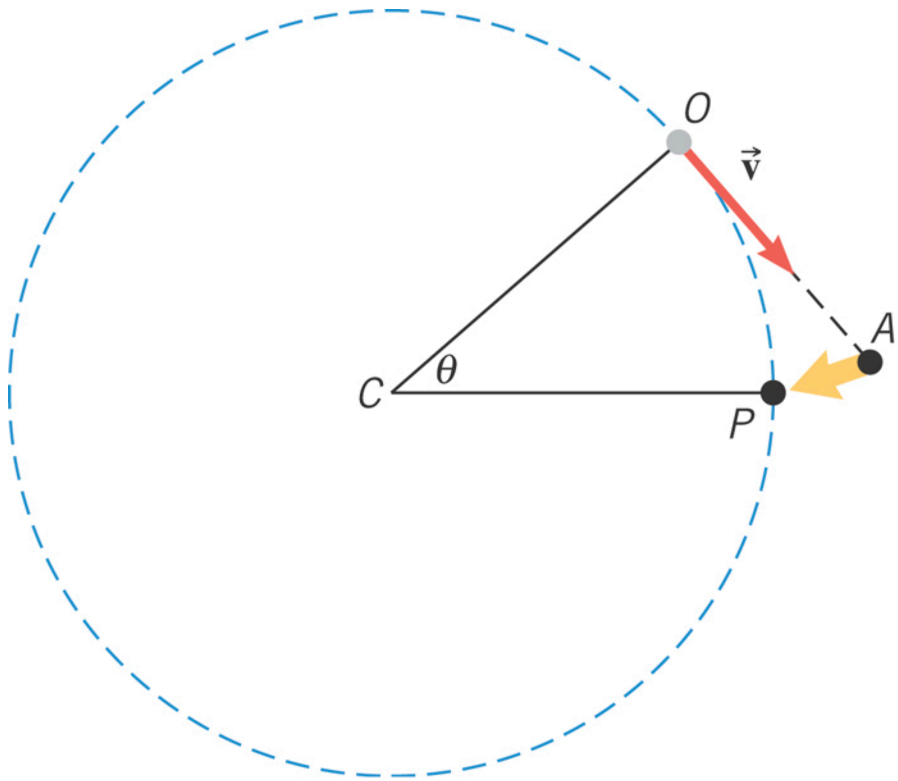
$$a_C = \frac{v^2}{r}$$

Centripetal acceleration
vector points *inward*
at ALL points on the circle

Conceptual Example: Which Way Will the Object Go?

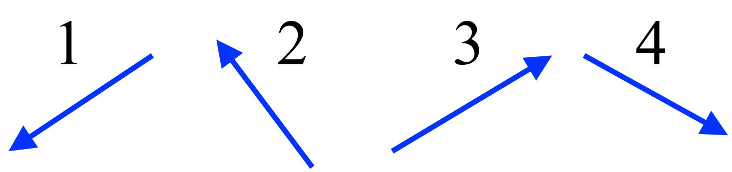
An object (•) is in uniform circular motion. At point *O* it is released from its circular path.

Does the object move along the
(A) Straight path between *O* and *A*
or
(B) Along the circular arc between points *O* and *P* ?



Example: The Effect of Radius on Centripetal Acceleration

The bobsled track contains turns with radii of 33 m and 24 m.
Match the acceleration vector directions below to the points A,B,C,D.

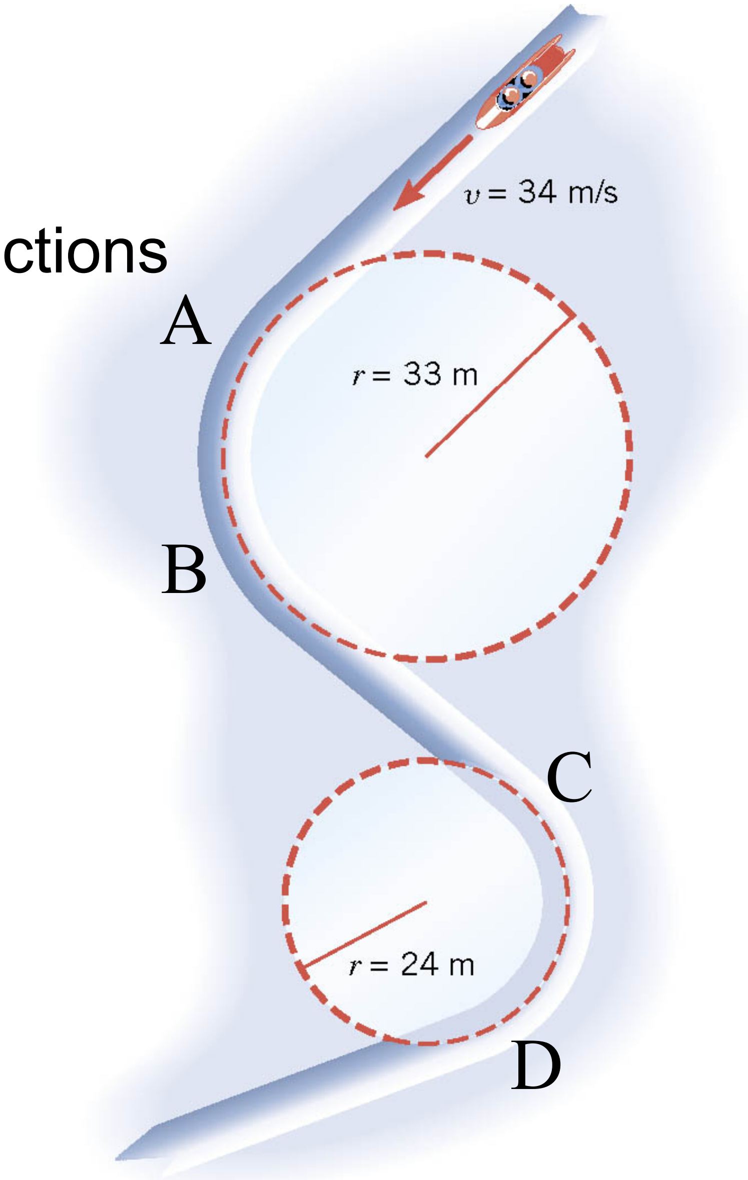


A –

B –

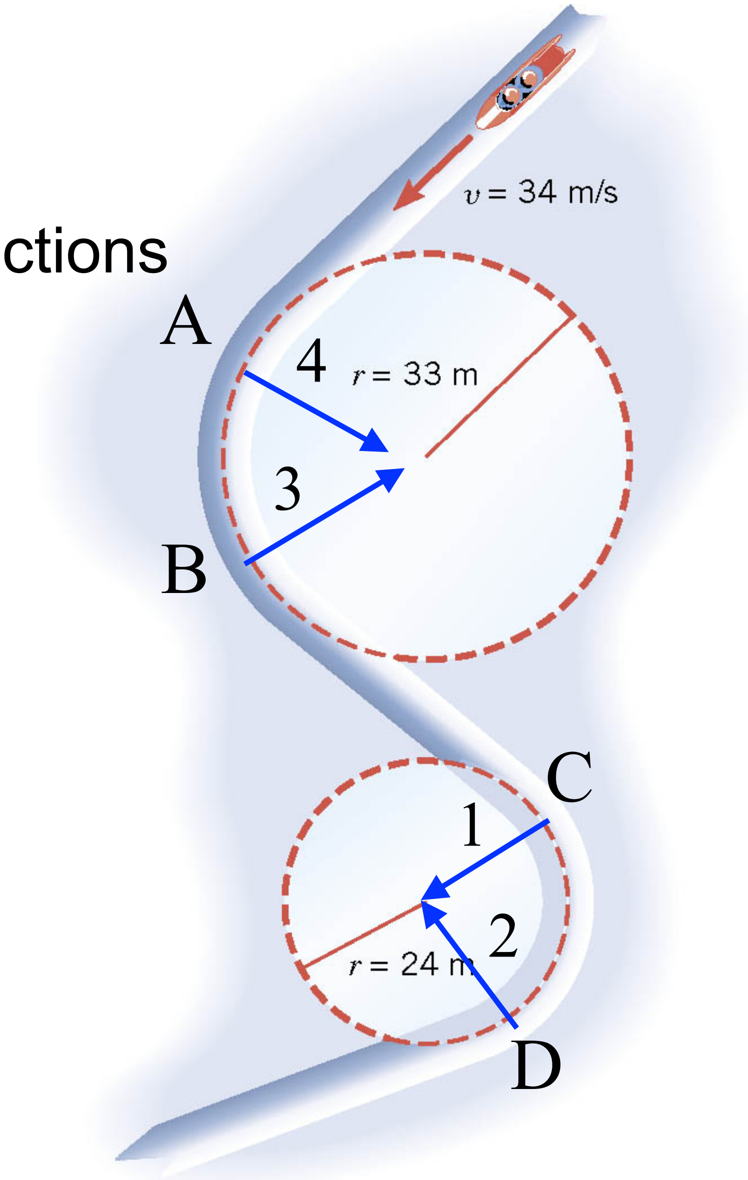
C –

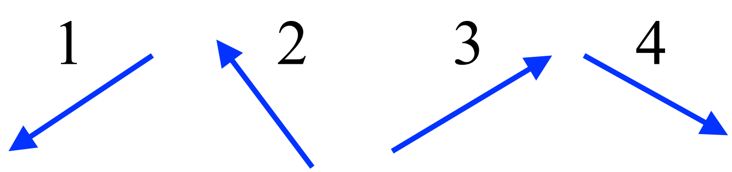
D –



Example: The Effect of Radius on Centripetal Acceleration

The bobsled track contains turns with radii of 33 m and 24 m.
Match the acceleration vector directions below to the points A,B,C,D.





A – 4

B – 3

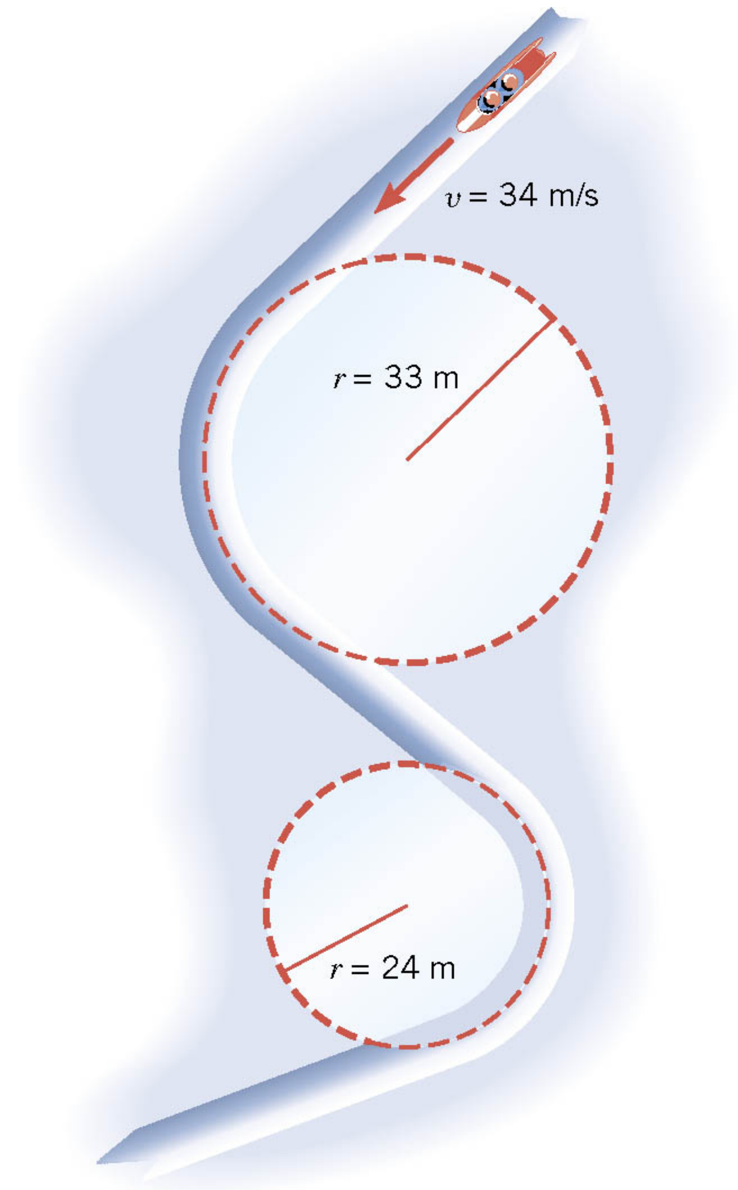
C – 1

D – 2

3.5 Centripetal Acceleration

$$a_C = v^2/r$$

Find the centripetal acceleration at each turn for a speed of 34 m/s. Express answers as multiples of $g = 9.8\,\mathrm{m/s^2}$.



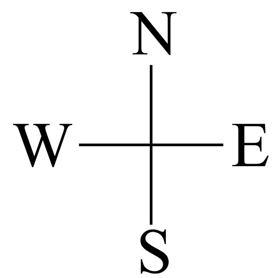
$$a_C = \frac{(34\,\mathrm{m/s})^2}{33\ \mathrm{m}} = 35\,\mathrm{m/s^2} = 3.6g$$

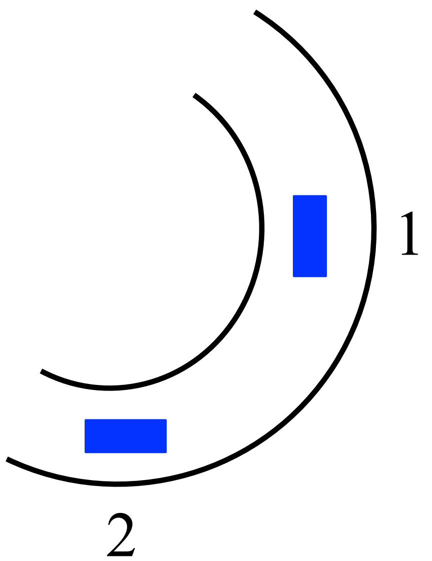
$$a_C = \frac{(34\,\mathrm{m/s})^2}{24\ \mathrm{m}} = 48\,\mathrm{m/s^2} = 4.9g$$

Clicker Question 3.5.1

A car is moving counter-clockwise around a circular section of road at constant speed. What are the directions of its velocity and acceleration at position 1.

	v	**a**
a)	N	S
b)	N	E
c)	N	W
d)	N	N
e)	S	E



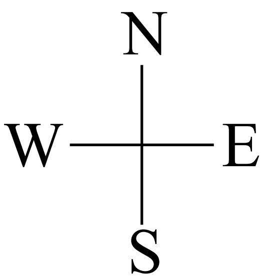


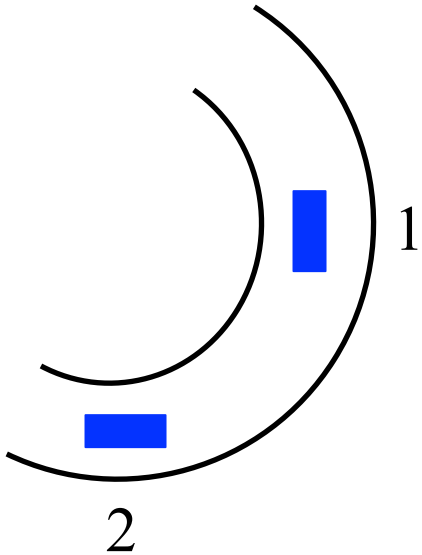
Clicker Question 3.5.2

A car is moving counter-clockwise around a circular section of road at constant speed. What are the directions of its velocity and acceleration

at position 2?

	v	**a**
a)	E	S
b)	E	E
c)	E	N
d)	E	W
e)	W	S





Chapter 8

Accelerated Circular Motion

8.1 Rotational Motion and Angular Displacement

Why are there 360 degrees in a circle?
Why are there 60 minutes in an hour?
Why are there 60 seconds in a minute?

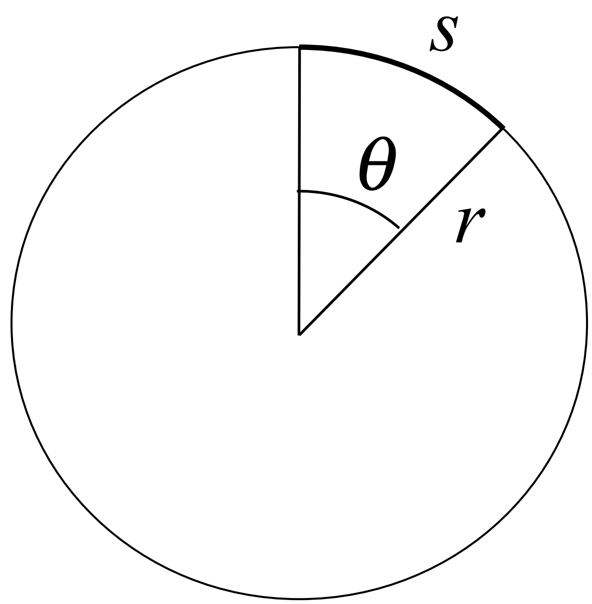
Because the Greeks, who invented these units were enamored with numbers that are divisible by most whole numbers, 12 or below (except 7 and 11).

Strange, because later it was the Greeks who discovered that the ratio of the radius to the circumference of a circle was a number known as 2π.

A new unit, radians, is really useful for angles.

8.1 Rotational Motion and Angular Displacement

A new unit, radians, is really useful for angles.



Radian measure

$$\theta(\text{radians}) = \frac{s\ (\text{arc length})}{r\ (\text{radius})}$$

$$s = r\theta$$

(s in same units as r)

Full circle

$$\theta = \frac{s}{r} = \frac{2\pi \cancel{r}}{\cancel{r}}$$

$$= 2\pi\ (\text{radians})$$

Conversion of degree to radian measure

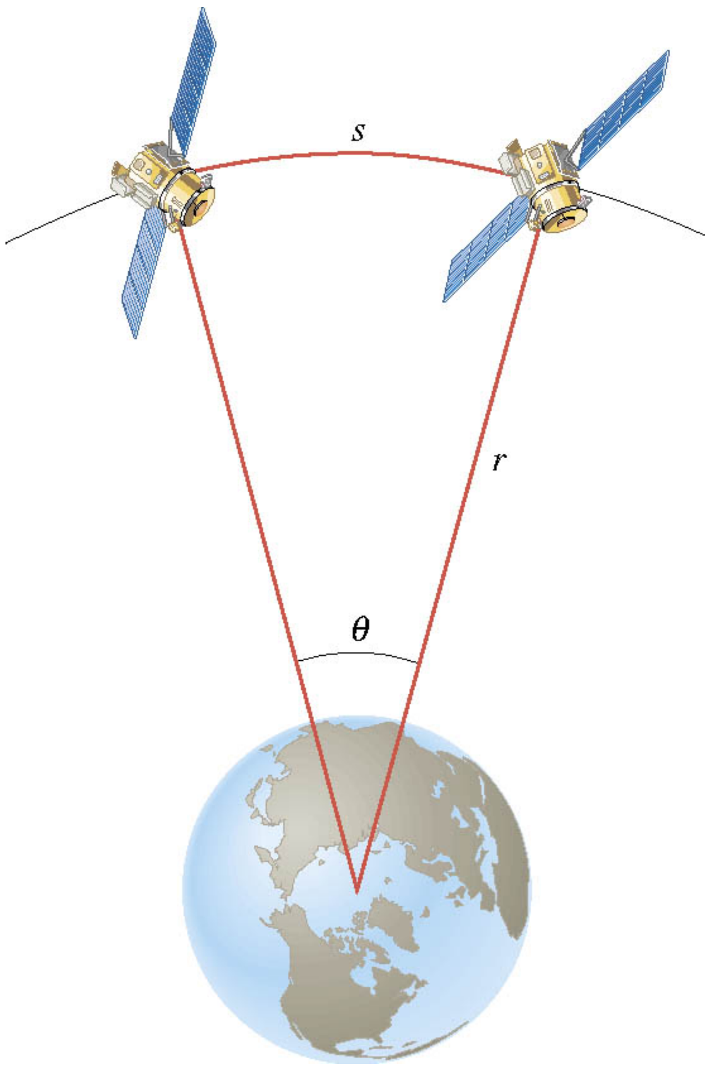
$$\theta(\text{rad}) = \theta(\text{deg.})\left(\frac{2\pi}{360}\frac{\text{rad}}{\text{deg.}}\right)$$

$$\left(\frac{2\pi}{360}\frac{\text{rad}}{\text{deg.}}\right) = 1$$

Example: Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^{7}m.

If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.



Convert degree to radian measure

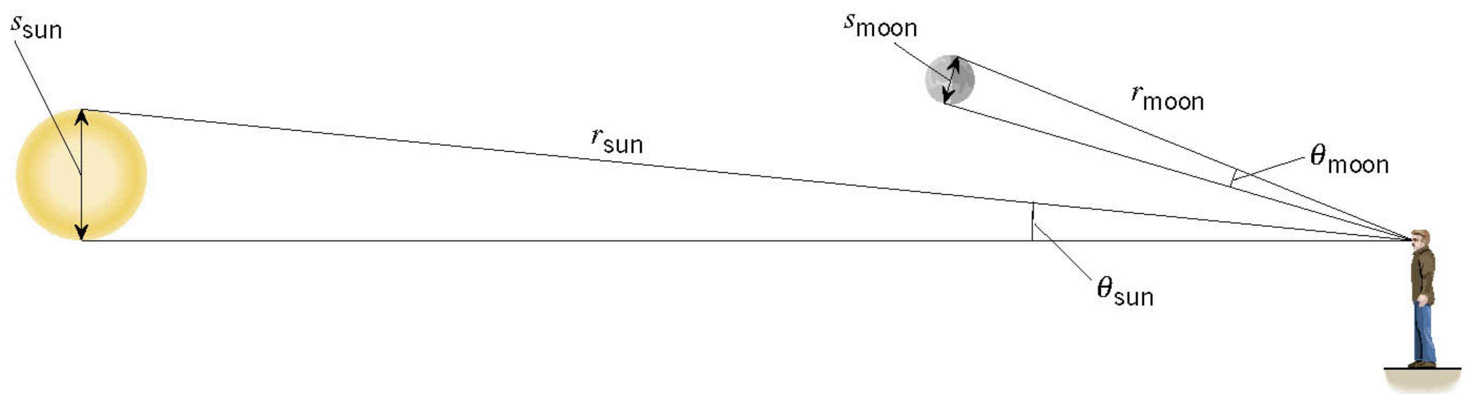
$$2.00\,\text{deg}\left(\frac{2\pi\ \text{rad}}{360\,\text{deg}}\right)=0.0349\ \text{rad}$$

Determine arc length

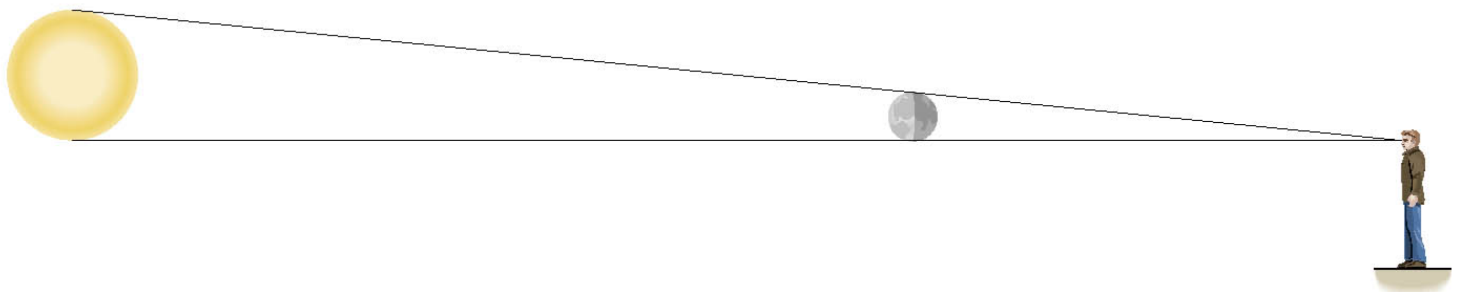
$$s=r\theta=\left(4.23\times10^{7}\,\text{m}\right)\left(0.0349\ \text{rad}\right)$$

$$=1.48\times10^{6}\,\text{m}\ \ (920\ \text{miles})$$

8.1 Rotational Motion and Angular Displacement



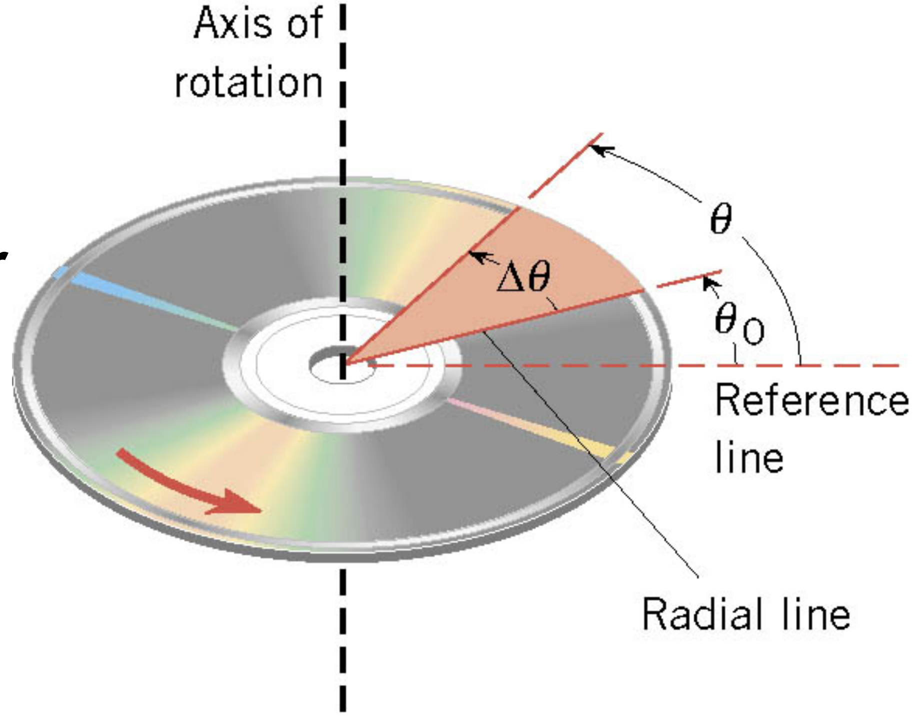
(a)


(b)

For an observer on the earth, an eclipse can occur because angles subtended by the sun and the moon are the same.

$$\theta = \frac{s_{\text{Sun}}}{r_{\text{Sun}}} \approx \frac{s_{\text{Moon}}}{r_{\text{Moon}}} \approx 9.3 \text{ mrad}$$

8.1 Rotational Motion and Angular Displacement

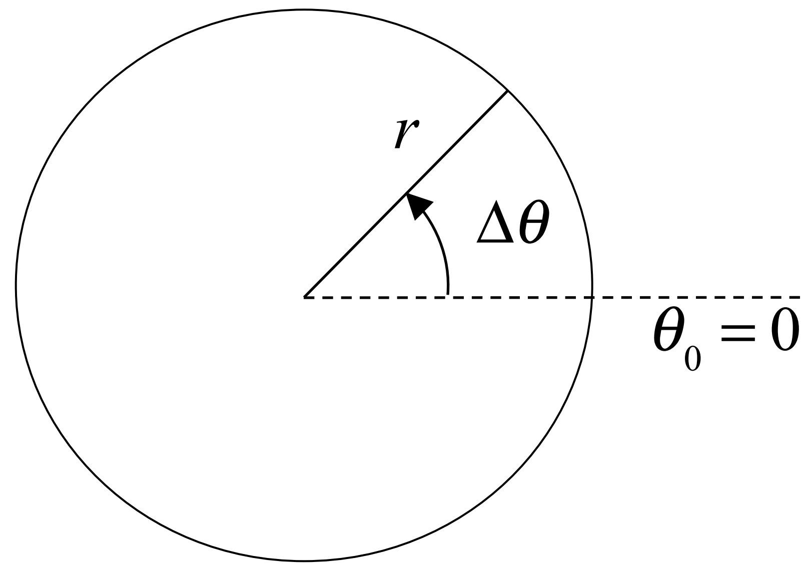


The angle through which the object rotates is called the ***angular displacement vector***

$$\Delta\theta = \theta - \theta_o$$

SI unit of angular displacement, radian (rad)

Simplified using $\theta_o = 0$, and
$\Delta\theta = \theta$, angular displacement vector.



Vector

Counter-clockwise is + displacement
Clockwise is – displacement

Clicker Question 8.1 Radian measure for angles

Over the course of a day (twenty-four hours), what is the angular displacement of the second hand of a wrist watch in radians?

a) 1440 rad

b) 2880 rad

c) 4520 rad

d) 9050 rad

e) 543,000 rad

8.2 *Angular Velocity and Angular Acceleration*

DEFINITION OF AVERAGE ANGULAR VELOCITY

$$\text{Average angular velocity} = \frac{\text{Angular displacement}}{\text{Elapsed time}}$$

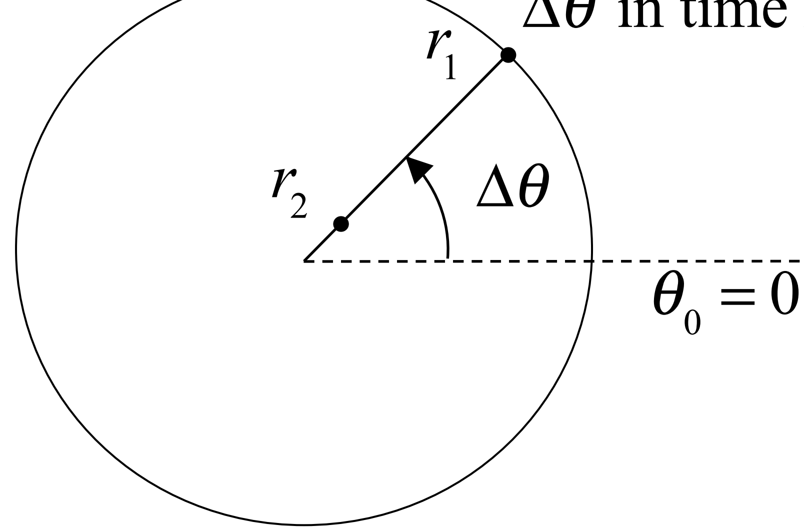
$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \qquad \text{where } \Delta t = t - t_o$$

SI Unit of Angular Velocity: radian per second (rad/s)

$\Delta\theta$ is the same at all radii.

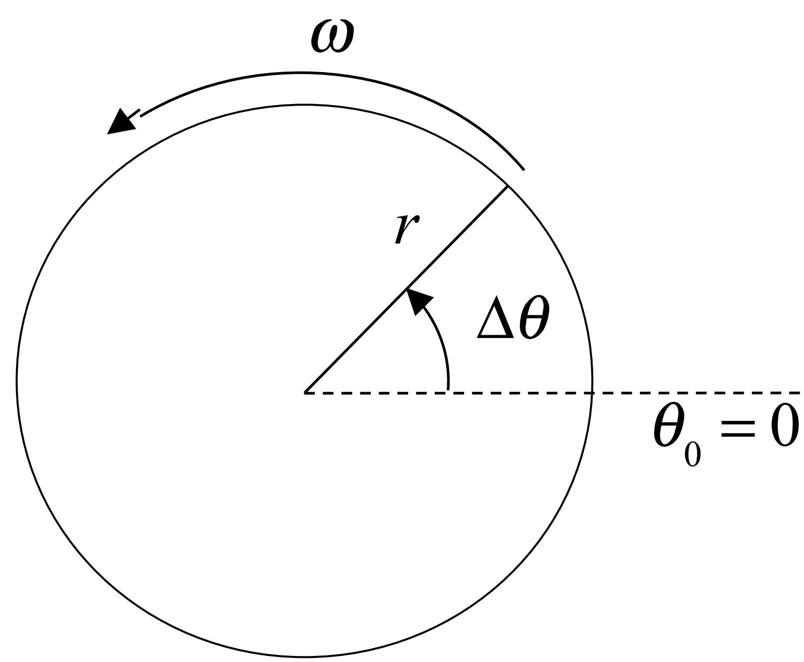
Δt is the same at all radii.

$$\omega = \frac{\Delta\theta}{\Delta t} \text{ is the same at all radii.}$$



8.1 Angular Velocity and Angular Acceleration

Case 1: Constant angular velocity, ω.



$$\omega = \frac{\Delta\theta}{\Delta t} \qquad\qquad \Delta\theta = \omega\,\Delta t$$

Example: A disk rotates with a constant angular velocity of +1 rad/s.

What is the angular displacement of the disk in 13 seconds?

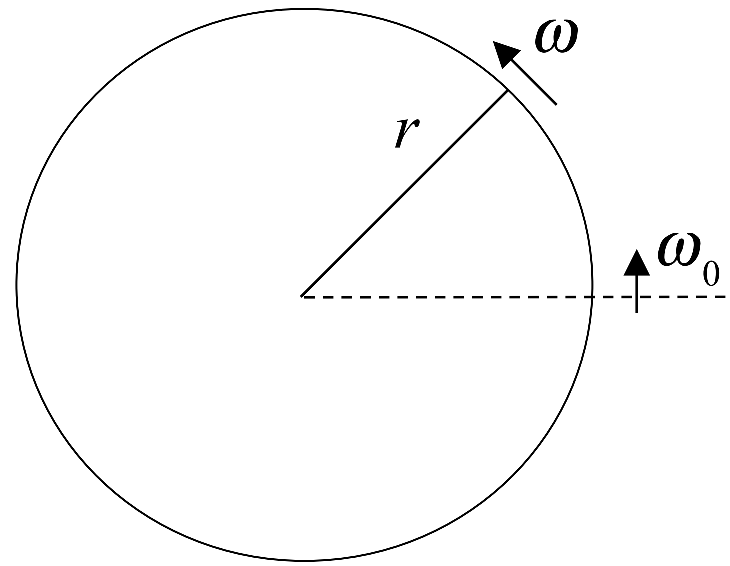
How many rotations has the disk made in that time?

$$\Delta\theta = \omega\,\Delta t = (+1\text{ rad/s})(13\text{ s}) = +13\text{ rad}$$

$$2\pi\text{ radians} = 1\text{ rotation} \Rightarrow 2\pi\text{ rad/rot.}$$

$$n_{rot} = \frac{\Delta\theta}{2\pi\text{ rad/rot.}} = \frac{13\text{ rad}}{6.3\text{ rad/rot}} = 2.1\text{ rot.}$$

Case 2: Angular velocity, ω, changes in time.



Instantaneous angular velocity at time t.

$$\omega = \lim_{\Delta t = 0} \frac{\Delta \theta}{\Delta t}$$

DEFINITION OF AVERAGE ANGULAR ACCELERATION

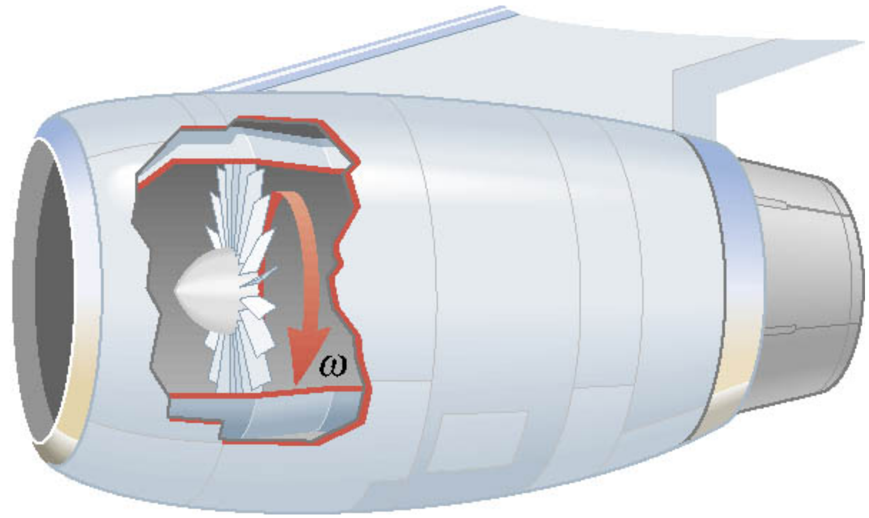
$$\text{Average angular acceleration} = \frac{\text{Change in angular velocity}}{\text{Elapsed time}}$$

$$\overline{\alpha} = \frac{\omega - \omega_o}{t - t_o} = \frac{\Delta \omega}{\Delta t}$$

SI Unit of Angular acceleration: radian per second squared (rad/s^2)

Example: A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular speed of –110 rad/s. As the plane takes off, the angular velocity of the blades reaches –330 rad/s in a time of 14 s.



Rotation is clockwise (negative)

Find the angular acceleration, assuming it to be constant.

$$\bar{\alpha} = \frac{(-330\,\text{rad/s}) - (-110\,\text{rad/s})}{14\ \text{s}} = -16\,\text{rad/s}^2$$

8.2 *The Equations of Rotational Kinematics*

The equations of rotational kinematics for constant angular acceleration:

ANGULAR ACCELERATION

ANGULAR VELOCITY

$$\omega = \omega_0 + \alpha t$$

$$\theta = \tfrac{1}{2}\left(\omega_0 + \omega\right) t$$

TIME

ANGULAR DISPLACEMENT

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

8.2 The Equations of Rotational Kinematics

Table 8.2 Symbols Used in Rotational and Linear Kinematics

Rotational Motion	Quantity	Linear Motion
θ	Displacement	x
ω_0	Initial velocity	v_0
ω	Final velocity	v
α	Acceleration	a
t	Time	t

Table 8.1 The Equations of Kinematics for Rotational and Linear Motion

Rotational Motion (α = constant)		Linear Motion (a = constant)	
$\omega = \omega_0 + \alpha t$	(8.4)	$v = v_0 + at$	(2.4)
$\theta = \frac{1}{2}(\omega_0 + \omega)t$	(8.6)	$x = \frac{1}{2}(v_0 + v)t$	(2.7)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	(8.7)	$x = v_0 t + \frac{1}{2}at^2$	(2.8)
$\omega^2 = \omega_0{}^2 + 2\alpha\theta$	(8.8)	$v^2 = v_0{}^2 + 2ax$	(2.9)

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (–).

3. Write down the values that are given for any of the five kinematic variables.

4. Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final angular velocity of one segment is the initial angular velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

Clicker Question 8.2 Rotational motion kinematics

Given the initial and final angular velocity of a disk, and the total angular displacement of the disk, with which single equation can the angular acceleration of the disk be obtained?

a) $\omega = \omega_0 + \alpha t$

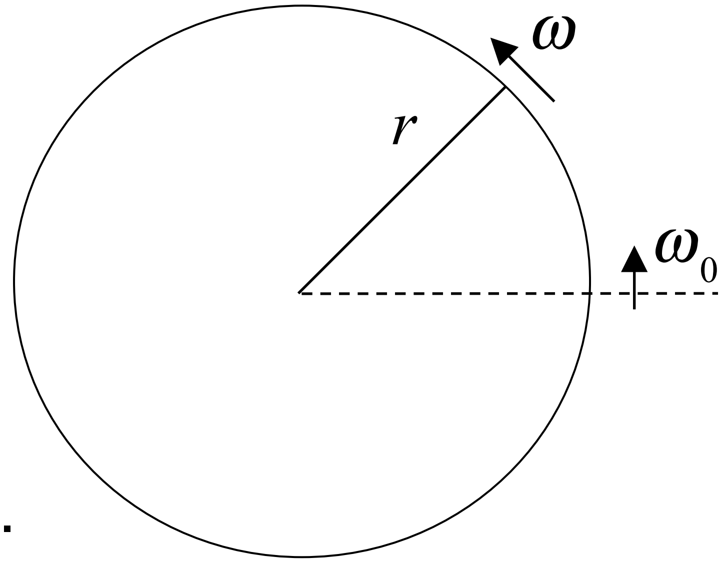
b) $\theta = \frac{1}{2}(\omega + \omega_0)t$

c) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

d) $\omega^2 = \omega_0^2 + 2\alpha\theta$

e) none of the above

Example: A disk has an initial angular velocity of +375 rad/s. The disk accelerates and reaches a greater angular velocity after rotating through an angular displacement of +44.0 rad.



If the angular acceleration has a constant value of +1740 rad/s^2, find the final angular velocity of the disk.

Given: $\omega_0 = +375 \text{ rad/s}, \theta = +44 \text{ rad}, \alpha = 1740 \text{ rad/s}^2$

Want: final angular velocity, ω.

No time!

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$= (375 \text{ rad/s})^2 + 2(1740 \text{ rad/s}^2)(+44 \text{ rad})$$

$$\omega = 542 \text{ rad/s}$$

8.3 Angular Variables and Tangential Variables

$\omega =$ angular velocity is the same at all radii

$\alpha =$ angular acceleration is the same at all radii

$\vec{\mathbf{v}}_T =$ tangential velocity is different at each radius

$\vec{\mathbf{a}}_T =$ tangential acceleration is different at each radius

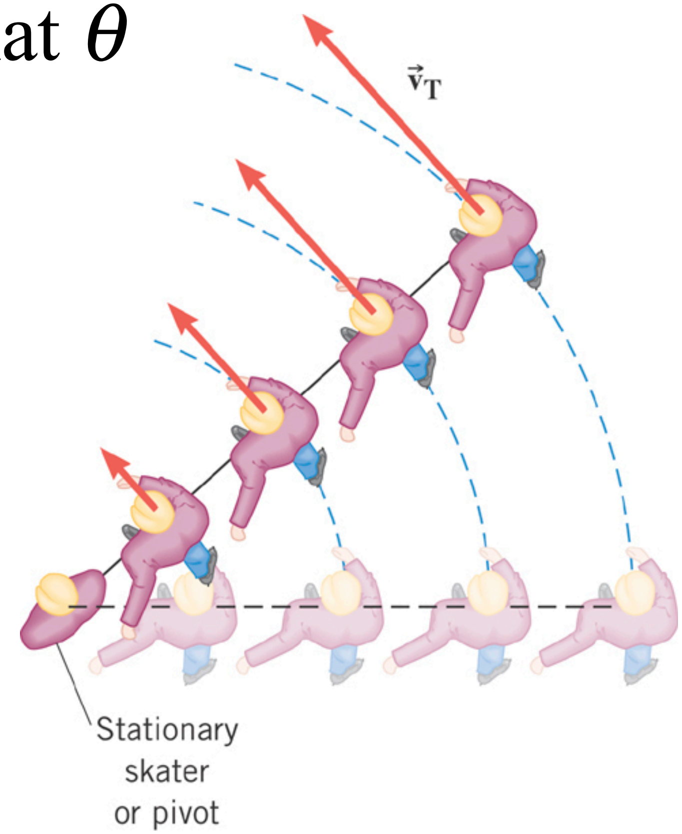
Direction is tangent to circle at that θ

$$\vec{\mathbf{v}}_T = \omega r \qquad \vec{\mathbf{a}}_T = \alpha r$$

$\vec{\mathbf{v}}_T$ (m/s) $\qquad \vec{\mathbf{a}}_T$ (m/s^2)

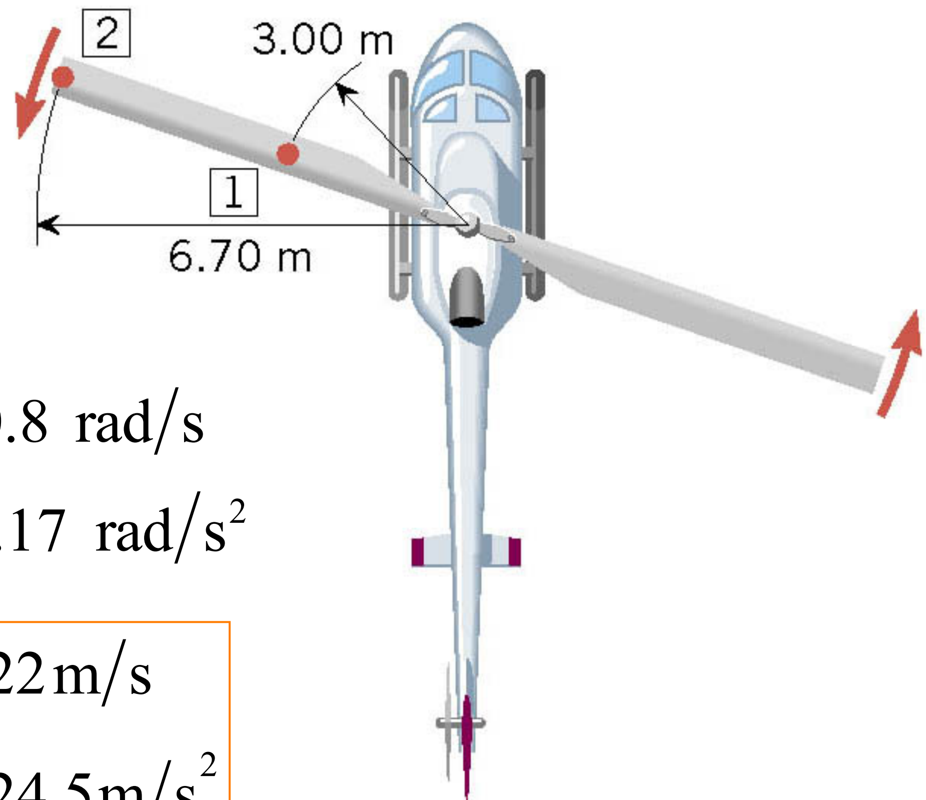
ω (rad/s) $\qquad \alpha$ (rad/s^2)

r (m) $\qquad r$ (m)



Example: A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2.
For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



Convert revolutions to radians

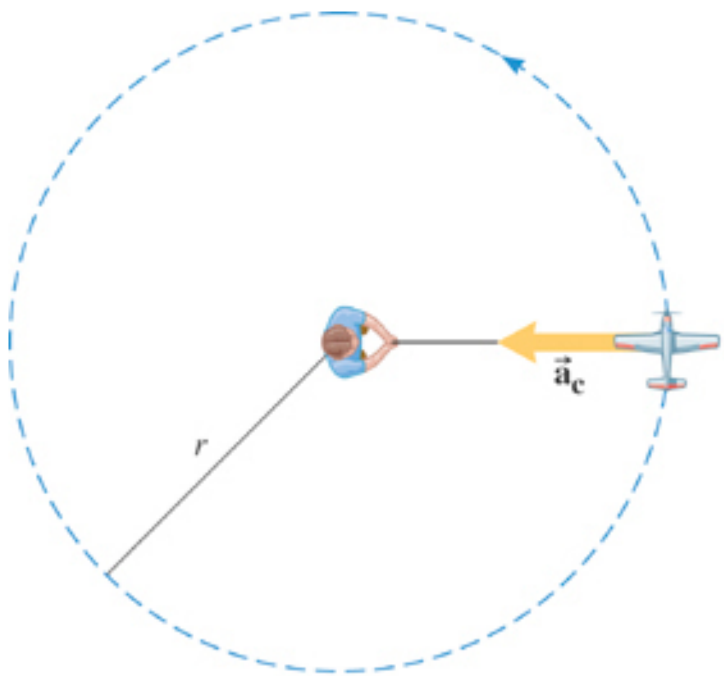
$$\omega = (6.50\ \text{rev/s})(2\pi\ \text{rad/rev}) = 40.8\ \text{rad/s}$$

$$\alpha = (1.30\ \text{rev/s}^2)(2\pi\ \text{rad/rev}) = 8.17\ \text{rad/s}^2$$

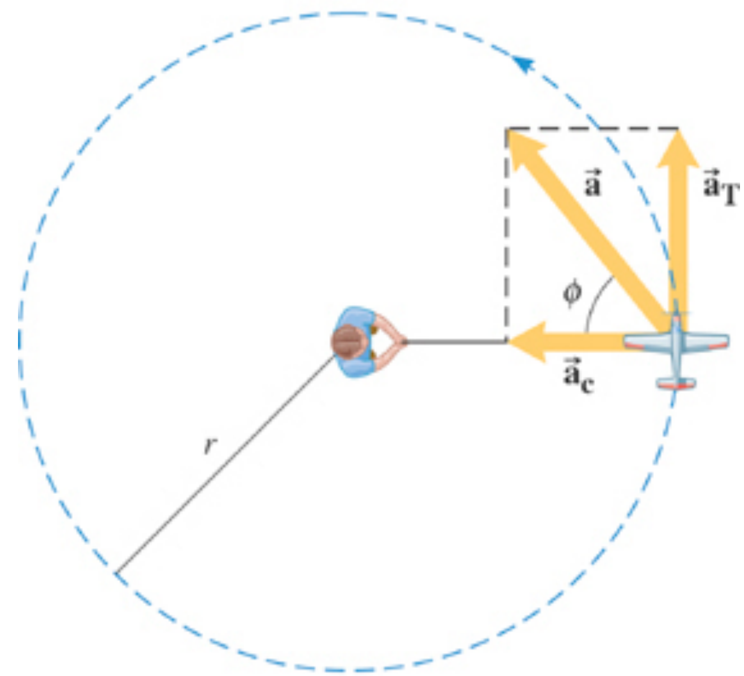
$$v_T = \omega r = (40.8\,\text{rad/s})(3.00\ \text{m}) = 122\,\text{m/s}$$

$$a_T = \alpha r = (8.17\,\text{rad/s}^2)(3.00\ \text{m}) = 24.5\,\text{m/s}^2$$

8.3 Centripetal Acceleration and Tangential Acceleration



Uniform circular motion



Non-uniform circular motion

$\underline{\omega \text{ in rad/s constant}}$

$$a_c = \frac{v_T^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

$$a_T = \alpha r$$

$$a_{total} = \sqrt{a_c^2 + \alpha^2 r^2}$$

8.3 Rolling Motion

The tangential speed of a point on the outer edge of the tire is equal to the speed of the car over the ground.

$$v = v_T = \omega r$$

$$a = a_T = \alpha r$$