Chapter 8

Rotational Dynamics

continued

New Seat Assignments for Thursday - www.pa.msu.edu/courses/phy231

Chapter 8 Rotational Kinematics/Dynamics

Angular motion variables
(with the usual motion equations)

displacement $\theta = s / r$ (rad.)

velocity $\omega = v / r$ (rad./s)

acceleration $\alpha = a / r$ (rad./s^2)

Uniform circular motion

centripetal acceleration $a_C = \dfrac{v^2}{r} = \omega^2 r$

centripetal force $F_C = m a_C = \dfrac{m v^2}{r}$

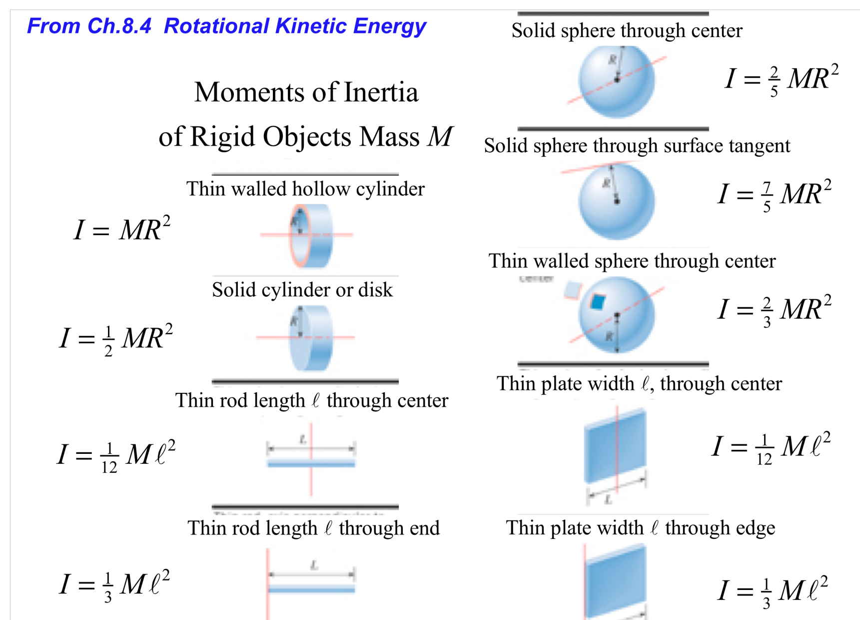
torque $\tau = rF \sin\theta$
($\theta : \sphericalangle$ between $\vec{\mathbf{F}}$ & $\vec{\mathbf{r}}$)

Newton's 2nd Law $\tau_{\text{Net}} = I\,\alpha$

rot. kinetic energy $K_{rot} = \frac{1}{2} I \omega^2$

angular momentum $L = I\omega$

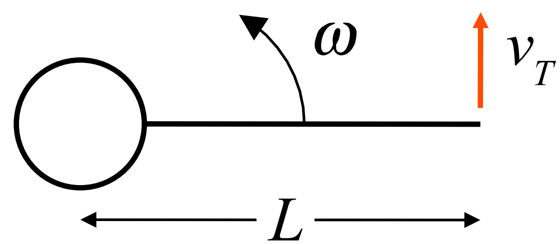
I Moment of Inertia



New Seat Assignments for Thursday - www.pa.msu.edu/courses/phy231

Clicker Question 8.4

A weed wacker is a length of nylon string that rotates rapidly around one end. The rotation angular speed is 47.0 rev/s, and the tip has tangential speed of 54.0 m/s. What is length of the nylon string?



a) 0.030 m

b) 0.120 m

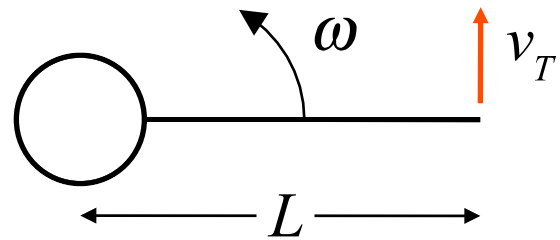
c) 0.180 m

d) 0.250 m

e) 0.350 m

Clicker Question 8.4

A weed wacker is a length of nylon string that rotates rapidly around one end. The rotation angular speed is 47.0 rev/s, and the tip has tangential speed of 54.0 m/s. What is length of the nylon string?



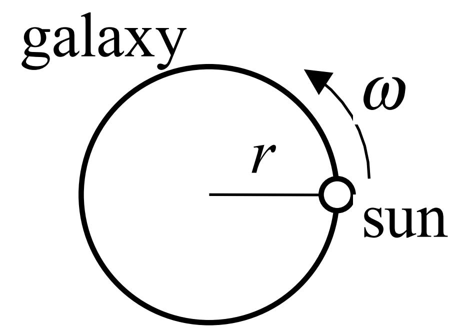
- **a)** 0.033 m
- **b)** 0.123 m
- **c)** 0.183 m
- **d)** 0.253 m
- **e)** 0.353 m

$$v_T = \omega r$$
$$= \omega L$$
$$L = \frac{v_T}{\omega}; \quad \omega = 47.0 \text{ rev/s} = 2\pi(47.0) \text{ rad/s} = 295 \text{ rad/s}$$
$$= \frac{54.0 \text{ m/s}}{295 \text{ rad/s}} = 0.183\text{m}$$

New Seat Assignments for Thursday - www.pa.msu.edu/courses/phy231

Clicker Question 8.5

The sun moves in circular orbit with a radius of 2.30×10^4 light-yrs, (1 light-yr = 9.50×10^{15} m) around the center of the galaxy at an angular speed of 1.10×10^{-15} rad/s. Find the tangential speed of sun.



a) 2.40×10^5 m/s

b) 3.40×10^5 m/s

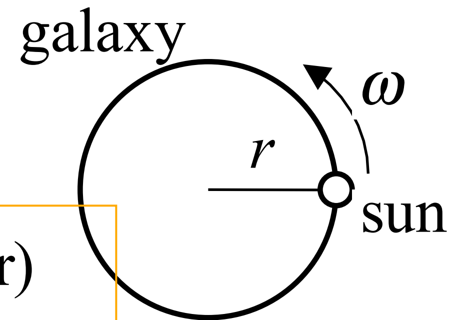
c) 4.40×10^5 m/s

d) 5.40×10^5 m/s

e) 6.40×10^5 m/s

Clicker Question 8.5

The sun moves in circular orbit with a radius of 2.30×10^4 light-yrs, (1 light-yr = 9.50×10^{15} m) around the center of the galaxy at an angular speed of 1.10×10^{-15} rad/s. Find the tangential speed of sun.



a) 2.40×10^5 m/s

b) 3.40×10^5 m/s

c) 4.40×10^5 m/s

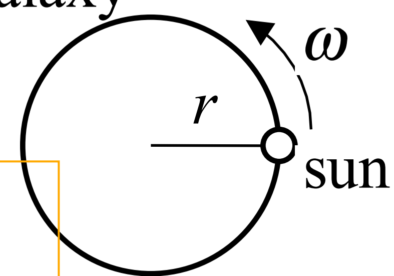
d) 5.40×10^5 m/s

e) 6.40×10^5 m/s

$$r = \left(2.30 \times 10^4 \text{ l-yr}\right)(9.50 \times 10^{15} \text{ m/l-yr})$$
$$= 2.19 \times 10^{20} \text{ m}$$
$$v_T = \omega r = (1.10 \times 10^{-15} \text{ rad/s})(2.19 \times 10^{20} \text{ m})$$
$$= 2.40 \times 10^5 \text{ m/s}$$

Clicker Question 8.5

The sun moves in circular orbit with a radius of 2.30x10^{4} light-yrs, (1 light-yr = 9.50x10^{15} m) around the center of the galaxy at an angular speed of 1.10x10^{-15} rad/s. Find the tangential speed of sun.



a) 2.40×10^{5} m/s

b) 3.40×10^{5} m/s

c) 4.40×10^{5} m/s

d) 5.40×10^{5} m/s

e) 6.40×10^{5} m/s

$$r = \left(2.30\times10^{4}\ \text{l-yr}\right)(9.50\times10^{15}\,\text{m/l-yr})$$
$$= 2.19\times10^{20}\,\text{m}$$
$$v_T = \omega r = (1.10\times10^{-15}\,\text{rad/s})(2.19\times10^{20}\,\text{m})$$
$$= 2.40\times10^{5}\ \text{m/s}$$

Clicker Question 8.6 Find the centripetal force on the sun.

$$m_{\text{Sun}} = \left(1.99\times10^{30}\ \text{kg}\right)$$

a) 2.27×10^{20} N

b) 3.27×10^{20} N

c) 4.27×10^{20} N

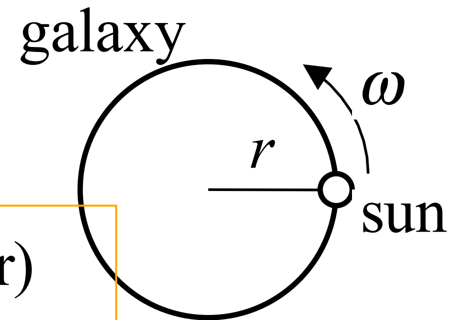
d) 5.27×10^{20} N

e) 6.27×10^{20} N

New Seat Assignments for Thursday - www.pa.msu.edu/courses/phy231

Clicker Question 8.5

The sun moves in circular orbit with a radius of 2.30×10^4 light-yrs, (1 light-yr = 9.50×10^{15} m) around the center of the galaxy at an angular speed of 1.10×10^{-15} rad/s. Find the tangential speed of sun.



a) 2.40×10^5 m/s

b) 3.40×10^5 m/s

c) 4.40×10^5 m/s

d) 5.40×10^5 m/s

e) 6.40×10^5 m/s

$$r = \left(2.30 \times 10^4 \text{ l-yr}\right)(9.50 \times 10^{15} \text{m/l-yr})$$
$$= 2.19 \times 10^{20} \text{m}$$
$$v_T = \omega r = (1.10 \times 10^{-15} \text{rad/s})(2.19 \times 10^{20} \text{m})$$
$$= 2.40 \times 10^5 \text{ m/s}$$

Clicker Question 8.6 Find the centripetal force on the sun

$$m_{\text{Sun}} = \left(1.99 \times 10^{30} \text{ kg}\right)$$

a) 2.27×10^{20} N

b) 3.27×10^{20} N

c) 4.27×10^{20} N

d) 5.27×10^{20} N

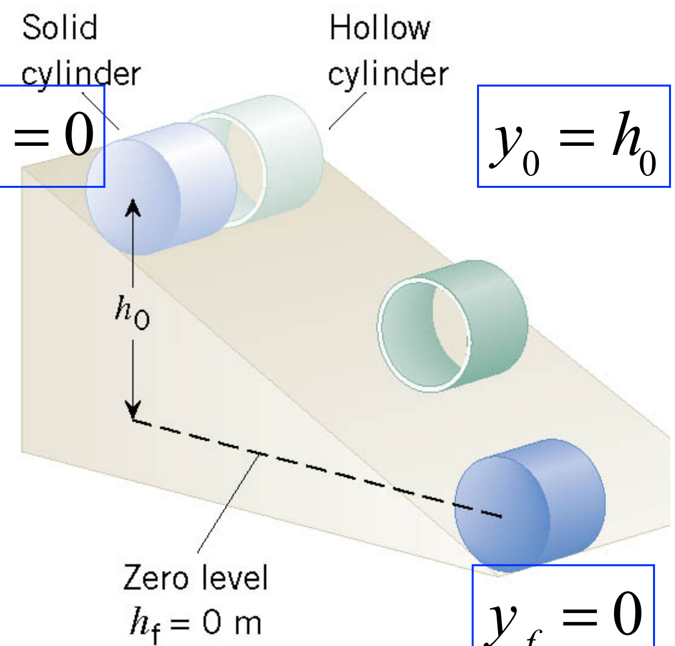
e) 6.27×10^{20} N

$$F_C = ma_C = m\left(\omega^2 r\right)$$
$$= \left(1.99 \times 10^{30} \text{kg}\right)\left(1.10 \times 10^{-15} /\text{s}\right)^2 \left(2.19 \times 10^{20} \text{m}\right)$$
$$= 5.27 \times 10^{20} \text{N}$$

8.4 Rotational Work and Energy

Total Energy = (Translational Kinetic + Rotational Kinetic + Potential) Energy

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy$$



$v_0 = \omega_0 = 0$

$y_0 = h_0$

$y_f = 0$

ENERGY CONSERVATION $E_f = E_0$

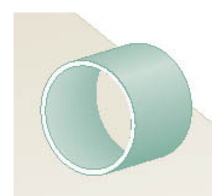
$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgy_f = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgy_0$$

$\omega = v/R$

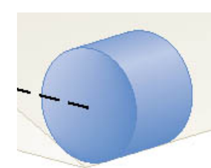
$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\,v_f^2/R^2 = mgh_0$$

$$v_f^2(m + I/R^2) = 2mgh_0$$

$$v_f = \sqrt{\frac{2mgh_0}{m + I/R^2}} = \sqrt{\frac{2gh_0}{1 + I/mR^2}}$$



$I = mR^2$



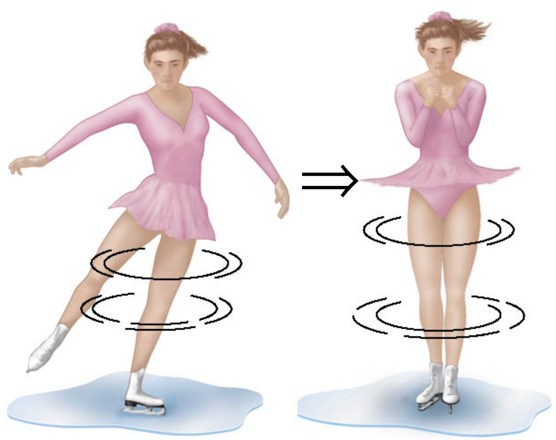
$I = \tfrac{1}{2}mR^2$

The cylinder with the smaller moment of inertia will have a greater final translational speed.

New Seat Assignments for Thursday - www.pa.msu.edu/courses/phy231

PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

The angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.



Angular momentum, L

$$L_i = I_i\omega_i;\quad L_f = I_f\omega_f$$

No external torque

$\Rightarrow$ Angular momentum conserved

$$L_f = L_i$$

$$I_f\omega_f = I_i\omega_i$$

$$\omega_f = \frac{I_i}{I_f}\omega_i;\quad \frac{I_i}{I_f} > 1$$

$$\omega_f > \omega_i \text{ (angular speed increases)}$$

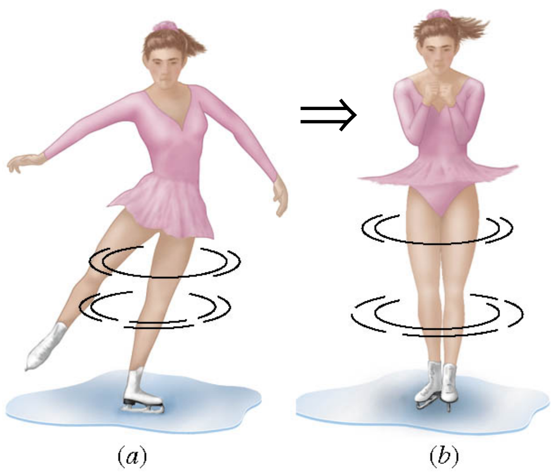
Moment of Inertia decreases

$$I = \sum mr^2,\ r_f < r_i$$

$$I_f < I_i$$

$$\frac{I_i}{I_f} > 1$$

8.9 Angular Momentum



From Angular Momentum Conservation

$$\omega_f = \left(I_i / I_f\right)\omega_i$$

because $I_i / I_f > 1$

Angular velocity increases

Is Energy conserved?

$$K_f = \tfrac{1}{2} I_f \omega_f^2$$

$$= \tfrac{1}{2} I_f \left(I_i / I_f\right)^2 \omega_i^2$$

$$= \left(I_i / I_f\right)\left(\tfrac{1}{2} I_i \omega_i^2\right) \qquad K_i = \tfrac{1}{2} I_i \omega_i^2;$$

$$= \left(I_i / I_f\right) K_i \quad \Rightarrow \quad \text{Kinetic Energy increases}$$

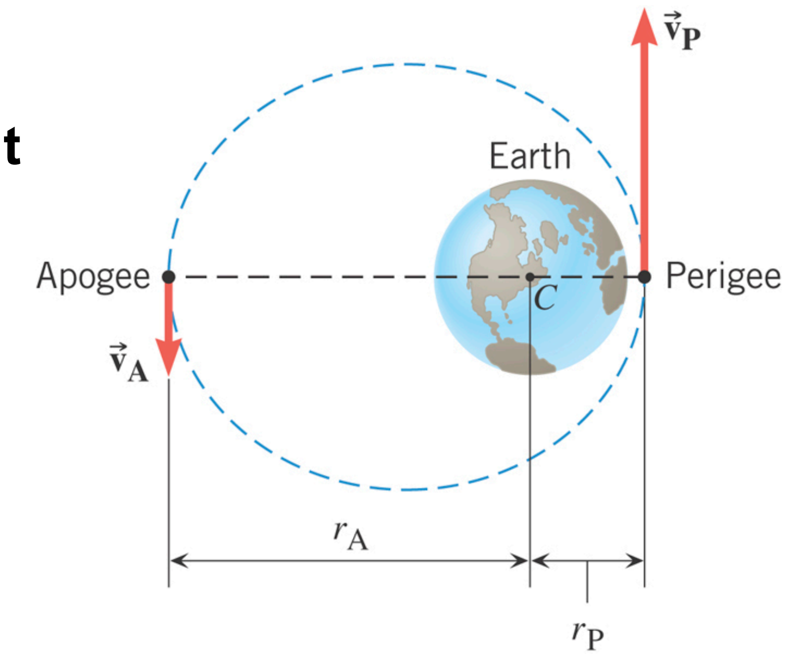
Energy is NOT conserved because pulling in the arms does (NC) work on the mass of each arm and increases the kinetic energy of rotation.

New Seat Assignments for Thursday - www.pa.msu.edu/courses/phy231

8.9 *Angular Momentum*

Example: A Satellite in an Elliptical Orbit

An artificial satellite is placed in an elliptical orbit about the earth. Its point of closest approach is 8.37x10^6 m from the center of the earth, and its point of greatest distance is 25.1x10^6 m from the center of the earth. The speed of the satellite at the perigee is 8450 m/s. Find the speed at the apogee.



$$I_A = mr_A^2; \quad I_P = mr_P^2$$
$$\omega_A = v_A / r_A; \quad \omega_P = v_P / r_P$$

Gravitational force along r (no torque) $\Rightarrow$ Angular momentum conserved

$$I_A \omega_A = I_P \omega_P$$

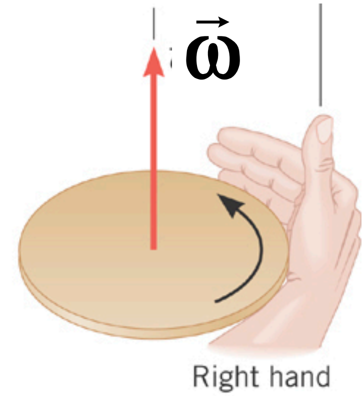
$$mr_A^2 \left(v_A / r_A \right) = mr_P^2 \left(v_P / r_P \right) \quad \Rightarrow \quad r_A v_A = r_P v_P$$

$$v_A = \left(r_P / r_A \right) v_P = \left[(8.37 \times 10^6) / (25.1 \times 10^6) \right] \left(8450 \text{ m/s} \right) = 2820 \text{ m/s}$$

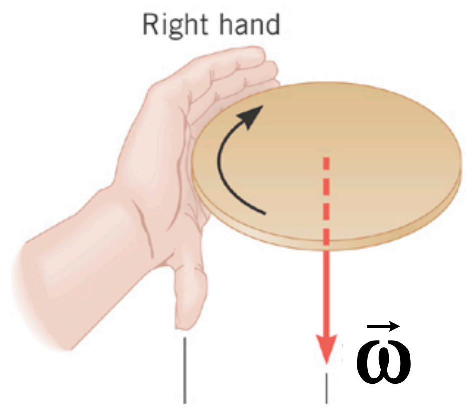
New Seat Assignments for Thursday - www.pa.msu.edu/courses/phy231

8.9 *The Vector Nature of Angular Variables*

Right-Hand Rule: Grasp the axis of rotation with your right hand, so that your fingers circle the axis in the same sense as the rotation.



Your extended thumb points along the axis in the direction of the angular velocity.



Vector Quantities in Rotational Motion

Angular Acceleration $\vec{\alpha} = \dfrac{\Delta\vec{\omega}}{\Delta t}$

Angular Momentum $\vec{\mathbf{L}} = I\vec{\omega}$

Torque $\vec{\tau} = I\,\vec{\alpha} = \dfrac{\Delta\vec{\mathbf{L}}}{\Delta t}$ Changes Angular Momentum

If torque is perpendicular to the angular momentum, only the direction of the angular momentum changes (precession) – no changes to the magnitude.

Rotational/Linear Dynamics Summary

linear		rotational
x	displacement	θ
v	velocity	ω
a	acceleration	α
m	point m inertia	$I = mr^2$
F	force/torque	$\tau = Fr\sin\theta$

linear	rotational
$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$	$\vec{\tau} = I\vec{\alpha}$
$W = F(\cos\theta)x$	$W_{rot} = \tau\theta$
$K = \frac{1}{2}mv^2$	$K_{rot} = \frac{1}{2}I\omega^2$
$W \Rightarrow \Delta K$	$W_{rot} \Rightarrow \Delta K_{rot}$
$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	$\vec{L} = I\vec{\omega}$
$\vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}}$	$\vec{\tau}\Delta t = \Delta\vec{L}$

Potential Energies

$U_G = mgy$

or $U_G = -GM_E m/R_E$

$U_S = \frac{1}{2}kx^2$

Conservation laws

Conserved:

If $W_{NC} = 0$, $E = K + U$

If $\mathbf{F}_{ext} = 0$, $\mathbf{P}_{system} = \sum \mathbf{p}$

If $\tau_{ext} = 0$, $\vec{L} = I\vec{\omega}$

EQUILIBRIUM OF A RIGID BODY

A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

Note: constant linear speed or constant rotational speed are allowed for an object in equilibrium.

Reasoning Strategy

1. Select the object to which the equations for equilibrium are to be applied.

2. Draw a free-body diagram that shows all of the external forces acting on the object.

3. Choose a convenient set of *x*, *y* axes and resolve all forces into components that lie along these axes.

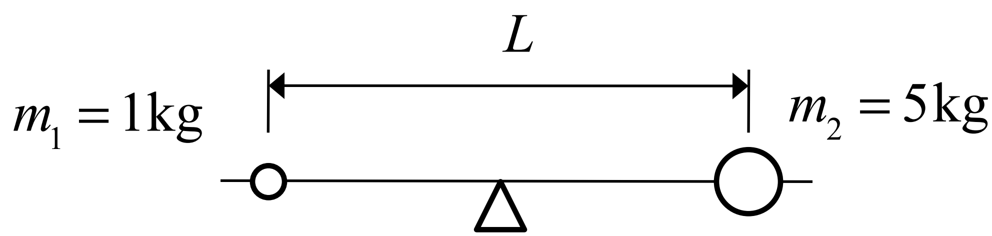
4. Apply the equations that specify the balance of forces at equilibrium. (Set the net force in the *x* and *y* directions equal to zero.)

5. Select a convenient axis of rotation. Set the sum of the torques about this axis equal to zero.

6. Solve the equations for the desired unknown quantities.

Clicker Question 8.7

A 5-kg ball and a 1-kg ball are positioned a distance L apart on a bar of negligible mass. How far from the 5-kg mass should the fulcum be placed to balance the bar?



a) $\frac{1}{2}L$

b) $\frac{1}{3}L$

c) $\frac{1}{4}L$

d) $\frac{1}{5}L$

e) $\frac{1}{6}L$

Clicker Question 8.7

A 5-kg ball and a 1-kg ball are positioned a distance L apart on a bar of negligible mass. How far from the 5-kg mass should the fulcum be placed to balance the bar?

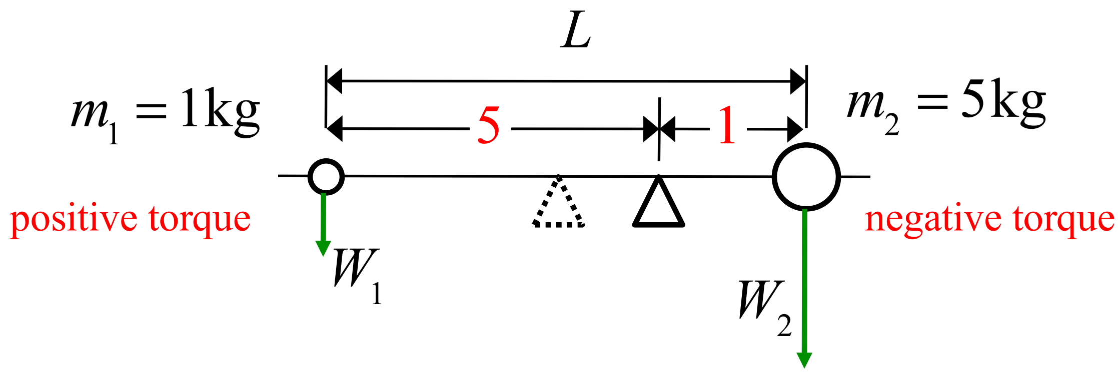
a) $\frac{1}{2}L$

b) $\frac{1}{3}L$

c) $\frac{1}{4}L$

d) $\frac{1}{5}L$

e) $\frac{1}{6}L$



For equilibrium the sum of the torques must be zero

Need to separate length into 5 parts on 1-kg mass side and 1 part on the 5-kg mass side. Total is 6 parts.

Fulcum must be 1/6 of the total length from the 5-kg mass.

Clicker Question 8.7

A 5-kg ball and a 1-kg ball are positioned a distance L apart on a bar of negligible mass. How far from the 5-kg mass should the fulcum be placed to balance the bar?

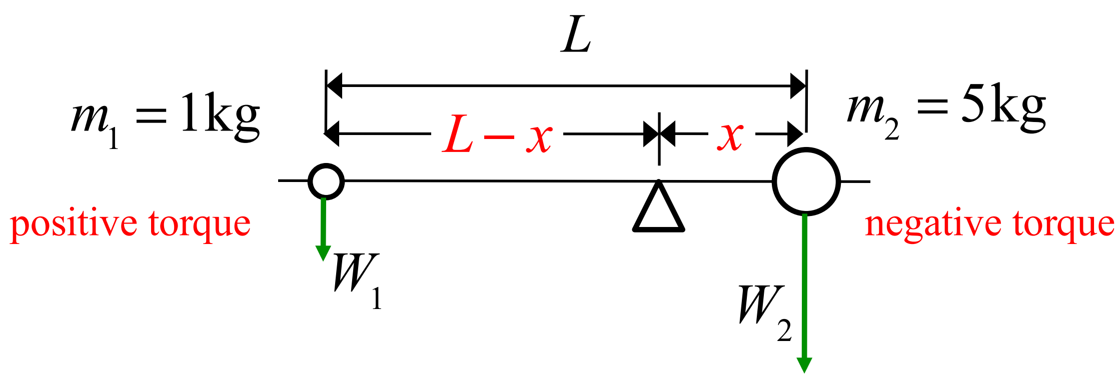
a) $\frac{1}{2}L$

b) $\frac{1}{3}L$

c) $\frac{1}{4}L$

d) $\frac{1}{5}L$

e) $\frac{1}{6}L$



For equilibrium the sum of the torques must be zero

Let x be the distance of fulcum from 5-kg mass.

$$\sum \tau = 0 = m_1 g(L-x) + (-m_2 g x)$$

$$\left(m_1 + m_2\right)x = m_1 L$$

$$x = \frac{m_1}{\left(m_1 + m_2\right)}L = \frac{1}{(1+5)}L = \frac{1}{6}L$$

Clicker Question 8.8

A 0.280 m long wrench at an angle of 50.0° to the floor is used to turn a nut. What horizontal force *F* produces a torque of 45.0 Nm on the nut?

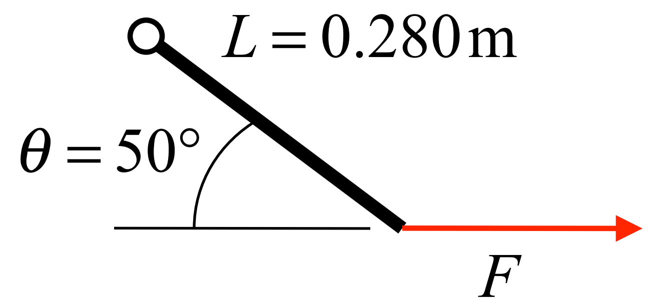
a) 10 N

b) 110 N

c) 210 N

d) 310 N

e) 410 N



Clicker Question 8.8

A 0.280 m long wrench at an angle of 50.0° to the floor is used to turn a nut. What horizontal force *F* produces a torque of 45.0 Nm on the nut?

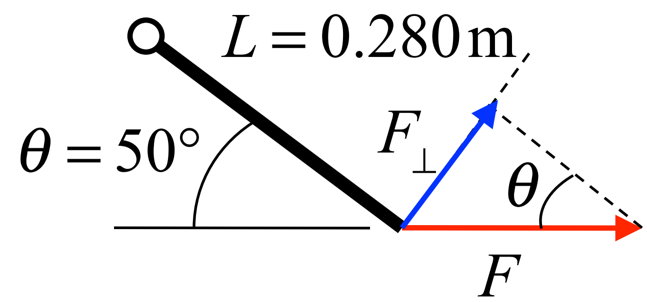
a) 10 N

b) 110 N

c) 210 N

d) 310 N

e) 410 N



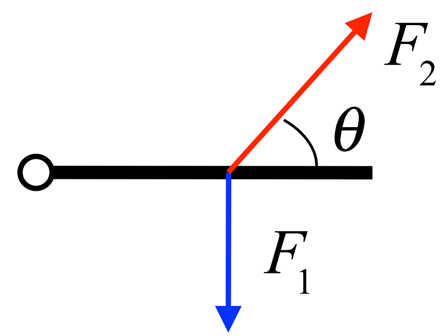
$$\tau = F_\perp L = (F\sin\theta)L$$

$$F = \tau / L\sin\theta = 45\text{Nm}/(0.28\text{ m})\sin 50°$$

$$= 210\text{ N}$$

Clicker Question 8.9

A force F_1 = 38.0 N acts downward on a horizontal arm. A second force, F_2 = 55.0 N acts at the same point but upward at the angle theta. What is the angle of F_2 for a net torque = zero?



a) 23.7°

b) 28.7°

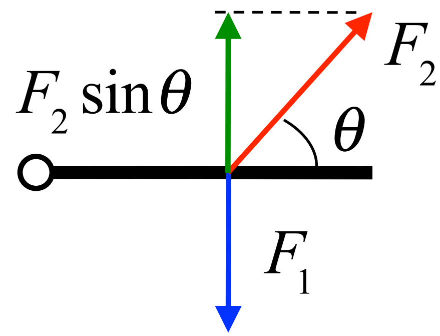
c) 33.7°

d) 38.7°

e) 43.7°

Clicker Question 8.9

A force F_1 = 38.0 N acts downward on a horizontal arm. A second force, F_2 = 55.0 N acts at the same point but upward at the angle theta. What is the angle of F_2 for a net torque = zero?



a) 23.7°

b) 28.7°

c) 33.7°

d) 38.7°

e) 43.7°

$$\vec{\tau}_{Net} = \vec{\tau}_1 + \vec{\tau}_2 = 0 \quad (\vec{\tau}_1 \text{ is clockwise } \Rightarrow -)$$

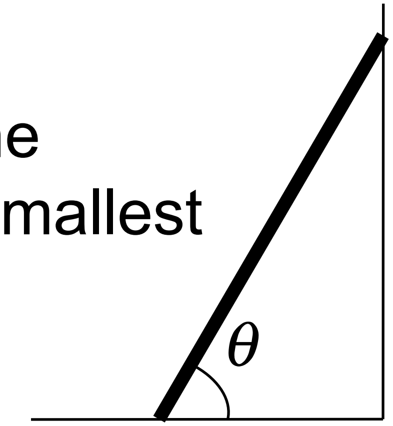
$$\tau_{Net} = -F_1 L + \left(F_2 \sin\theta\right) L = 0$$

$$\sin\theta = F_1 / F_2 = 38.0 / 55.0 = 0.691$$

$$\theta = 43.7°$$

8.7 Rigid Objects in Equilibrium

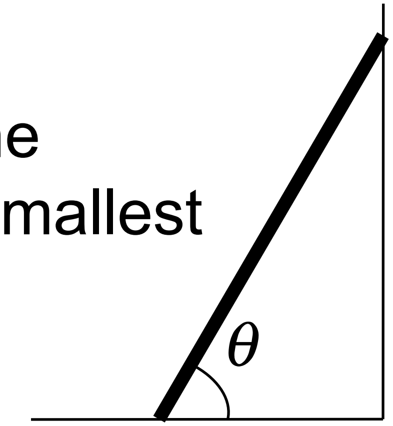
Example: A board length L lies against a wall. The coefficient of friction with ground 0.650. What is smallest angle the board can be placed without slipping?



1. Determine the forces acting on the board.

8.7 *Rigid Objects in Equilibrium*

Example: A board length L lies against a wall. The coefficient of friction with ground 0.650. What is smallest angle the board can be placed without slipping?



1. Determine the forces acting on the board.

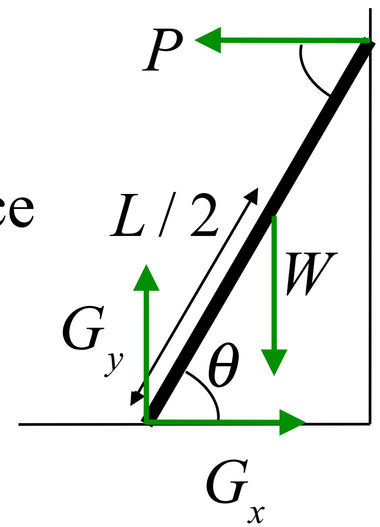
<u>Forces</u>

G_y ground normal force

G_x ground static frictional force

P wall normal force

W gravitational force



Forces:

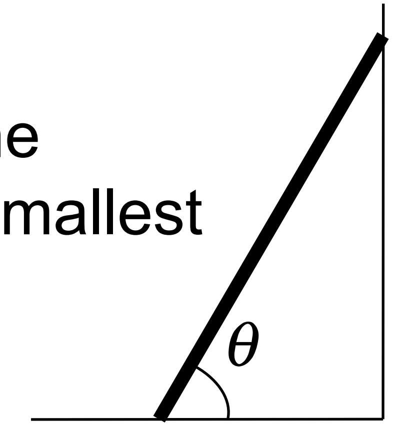
$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$

2. Choose pivot point at ground.

8.7 Rigid Objects in Equilibrium

Example: A board length L lies against a wall. The coefficient of friction with ground 0.650. What is smallest angle the board can be placed without slipping?



1. Determine the forces acting on the board.

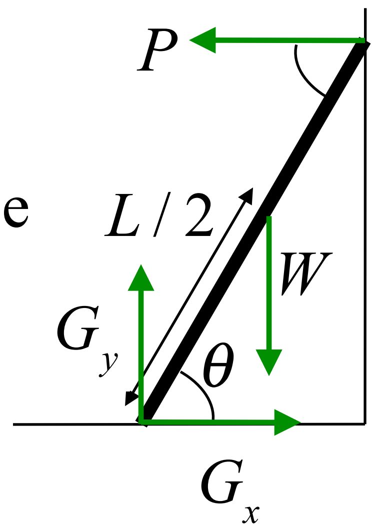
<u>Forces</u>

G_y ground normal force

G_x ground static frictional force

P wall normal force

W gravitational force



Forces:

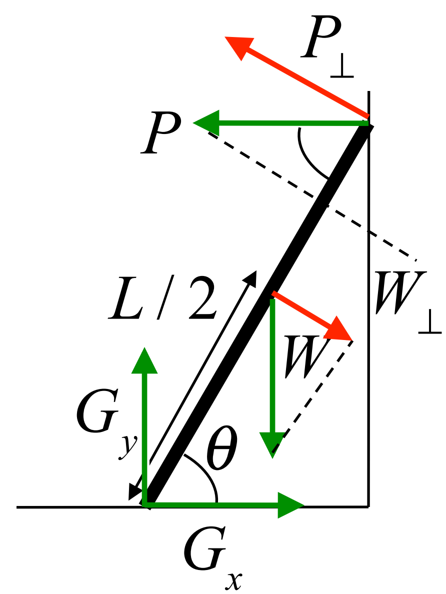
$$G_y = W$$

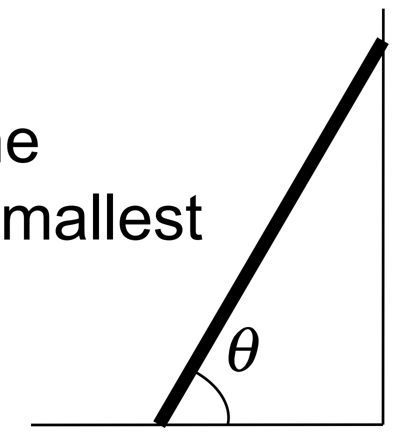
$$P = G_x = \mu G_y = \mu W$$

2. Choose pivot point at ground.

3. Find components normal to board

$P_\perp$ and $W_\perp$ are forces producing torque





Example: A board length L lies against a wall. The coefficient of friction with ground 0.650. What is smallest angle the board can be placed without slipping?

4. Net torque must be zero for equilibrium

Torque:

$$\tau_P = +P_\perp L = (P\sin\theta)L$$

$$\tau_W = -W_\perp (L/2) = -(W\cos\theta)(L/2)$$

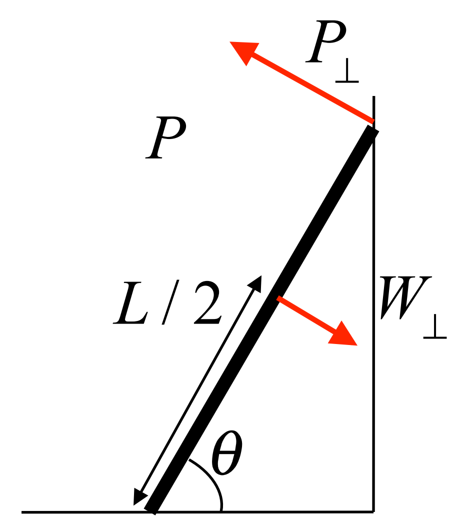
$$\tau_W + \tau_P = 0 \Rightarrow (P\sin\theta) = (W\cos\theta)/2$$

$$W = 2P\sin\theta/\cos\theta$$

Forces:

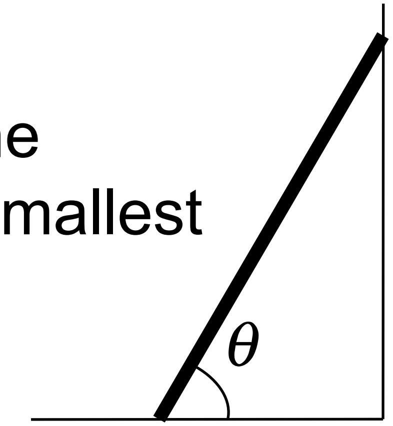
$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$



8.7 Rigid Objects in Equilibrium

Example: A board length L lies against a wall. The coefficient of friction with ground 0.650. What is smallest angle the board can be placed without slipping?



4. Net torque must be zero for equilibrium

Torque:

$$\tau_P = +P_\perp L = \left(P\sin\theta\right)L$$

$$\tau_W = -W_\perp\left(L/2\right) = -\left(W\cos\theta\right)\left(L/2\right)$$

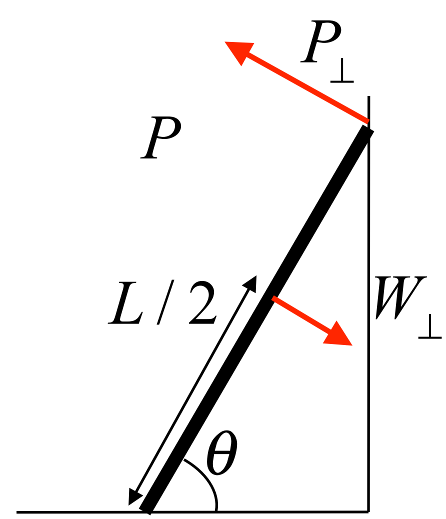
$$\tau_W + \tau_P = 0 \Rightarrow \left(P\sin\theta\right) = \left(W\cos\theta\right)/2$$

$$W = 2P\sin\theta/\cos\theta$$

Forces:

$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$



5. Combine torque and force equations

$$W = 2P\sin\theta/\cos\theta = 2\mu W\tan\theta$$

$$\tan\theta = 1/(2\mu) = 1/(1.3) = 0.77$$

$$\theta = 37.6^\circ$$

Example: A flywheel has a mass of 13.0 kg and a radius of 0.300m. What angular velocity gives it an energy of 1.20×10^9 J ?

$$K = \tfrac{1}{2} I\omega^2; \quad I_{disk} = \tfrac{1}{2} MR^2$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2} MR^2\right)\omega^2$$

$$\omega^2 = \frac{4K}{MR^2} = \frac{4.80 \times 10^9 \,\mathrm{J}}{(13.0\mathrm{kg})(0.300\mathrm{m})^2}$$

$$\omega = \sqrt{4.10 \times 10^9} = 6.40 \times 10^4 \,\mathrm{rad/s}$$

$$= 6.40 \times 10^4 \,\mathrm{rad/s}\left(\mathrm{rev}/(2\pi)\mathrm{rad}\right)$$

$$= 1.02 \times 10^4 \ \mathrm{rev/s}\left(60\,\mathrm{s/min}\right) = 6.12 \times 10^5 \ \mathrm{rpm}$$