Chapter 8

Rotational Dynamics

continued

Chapter 8 Rotational Kinematics/Dynamics

Angular motion variables
(with the usual motion equations)

displacement $\theta = s / r$ (rad.)

velocity $\omega = v / r$ (rad./s)

acceleration $\alpha = a / r$ (rad./s^2)

torque
($\theta : \measuredangle$ between $\vec{\mathbf{F}}$ & $\vec{\mathbf{r}}$) $\tau = rF\sin\theta$

Newton's 2nd Law $\tau_{\text{Net}} = I\,\alpha$

rot. kinetic energy $K_{rot} = \frac{1}{2} I\omega^2$

angular momentum $L = I\omega$

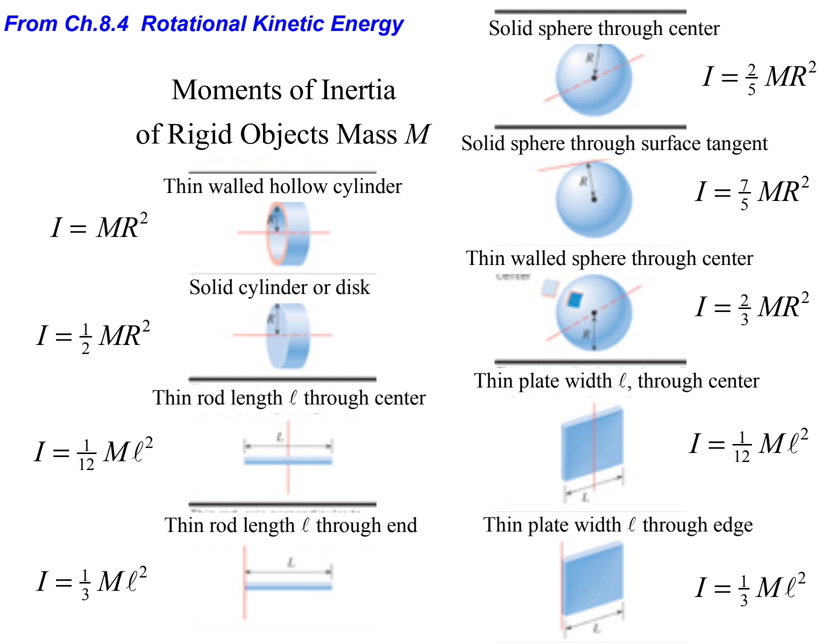
Uniform circular motion

centripetal acceleration $a_C = \dfrac{v^2}{r} = \omega^2 r$

centripetal force $F_C = m a_C = \dfrac{mv^2}{r}$

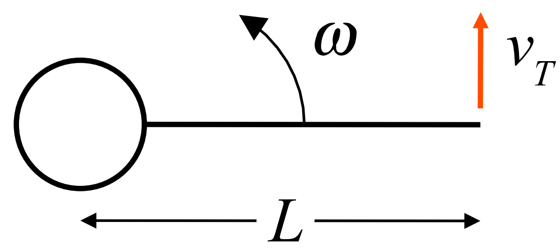
I Moment of Inertia

From Ch.8.4 Rotational Kinetic Energy



Clicker Question 8.4

A weed wacker is a length of nylon string that rotates rapidly around one end. The rotation angular speed is 47.0 rev/s, and the tip has tangential speed of 54.0 m/s. What is length of the nylon string?



a) 0.030 m

b) 0.120 m

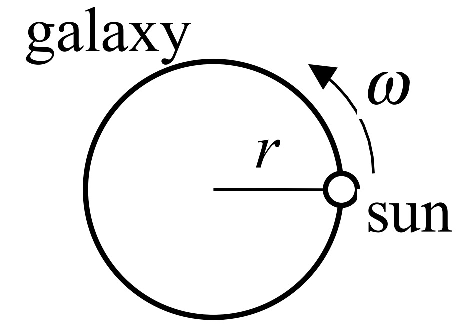
c) 0.180 m

d) 0.250 m

e) 0.350 m

Clicker Question 8.5

The sun moves in circular orbit with a radius of 2.30×10^4 light-yrs, (1 light-yr = 9.50×10^{15} m) around the center of the galaxy at an angular speed of 1.10×10^{-15} rad/s. Find the tangential speed of sun.



a) 2.40×10^5 m/s

b) 3.40×10^5 m/s

c) 4.40×10^5 m/s

d) 5.40×10^5 m/s

e) 6.40×10^5 m/s

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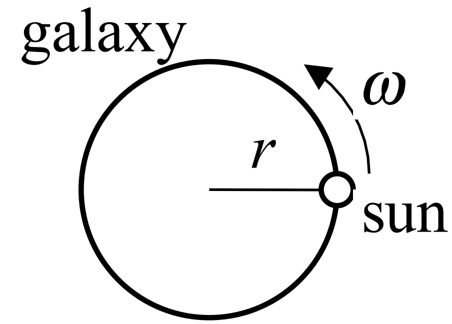


a) 2.40×10^5 m/s

b) 3.40×10^5 m/s

c) 4.40×10^5 m/s

d) 5.40×10^5 m/s

e) 6.40×10^5 m/s

Clicker Question 8.6 Find the centripetal force on the sun.

$$m_{\text{Sun}} = \left(1.99 \times 10^{30}\ \text{kg}\right)$$

a) 2.27×10^{20} N

b) 3.27×10^{20} N

c) 4.27×10^{20} N

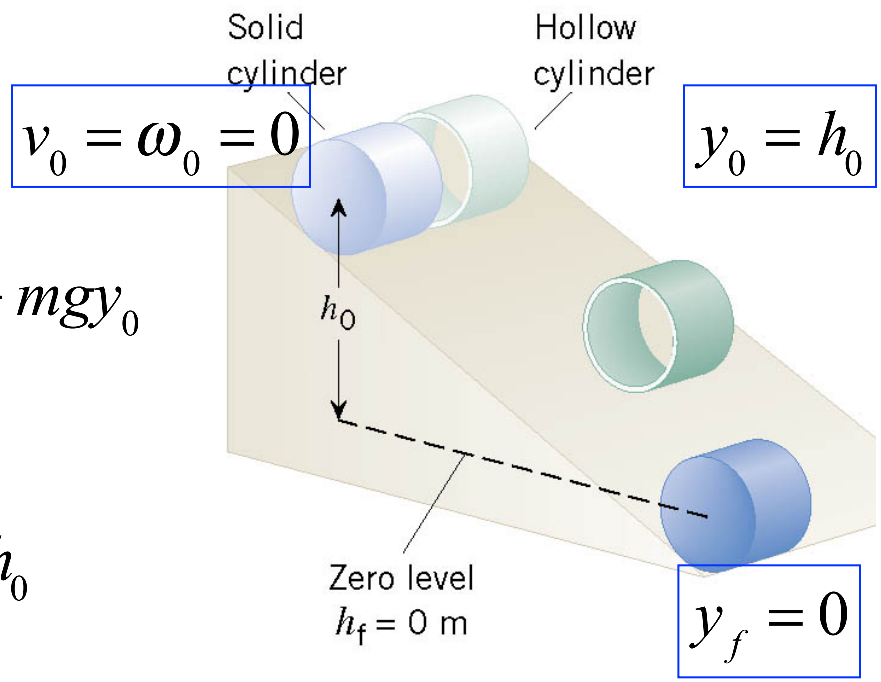
d) 5.27×10^{20} N

e) 6.27×10^{20} N

8.4 *Rotational Work and Energy*

Total Energy = (Translational Kinetic + Rotational Kinetic + Potential) Energy

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy$$



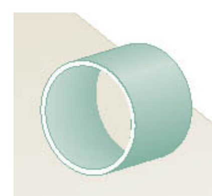
ENERGY CONSERVATION $\quad E_f = E_0$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgy_f = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgy_0$$

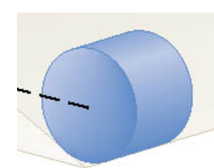
$$\boxed{\omega = v/R} \qquad \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\,v_f^2/R^2 = mgh_0$$

$$v_f^2(m + I/R^2) = 2mgh_0$$

$$v_f = \sqrt{\frac{2mgh_0}{m + I/R^2}} = \sqrt{\frac{2gh_0}{1 + I/mR^2}}$$



$I = mR^2$



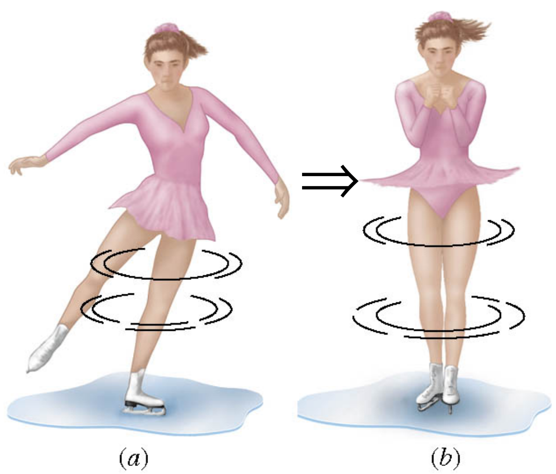
$I = \tfrac{1}{2}mR^2$

The cylinder with the smaller moment of inertia will have a greater final translational speed.

8.9 Angular Momentum

PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

The angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.



Angular momentum, L

$$L_i = I_i\omega_i; \quad L_f = I_f\omega_f$$

No external torque

$\Rightarrow$ Angular momentum conserved

$$L_f = L_i$$

$$I_f\omega_f = I_i\omega_i$$

$$\omega_f = \frac{I_i}{I_f}\omega_i; \quad \frac{I_i}{I_f} > 1$$

$\omega_f > \omega_i$ (angular speed increases)

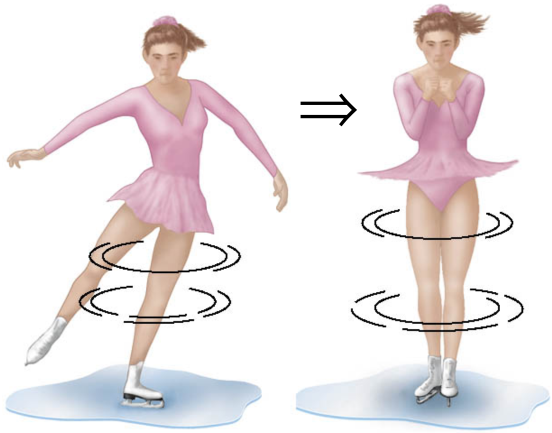
Moment of Inertia decreases

$$I = \sum mr^2, \; r_f < r_i$$

$$I_f < I_i$$

$$\frac{I_i}{I_f} > 1$$

8.9 Angular Momentum



From Angular Momentum Conservation

$$\omega_f = \left(I_i / I_f\right)\omega_i$$

because $I_i / I_f > 1$

Angular velocity increases

Is Energy conserved?

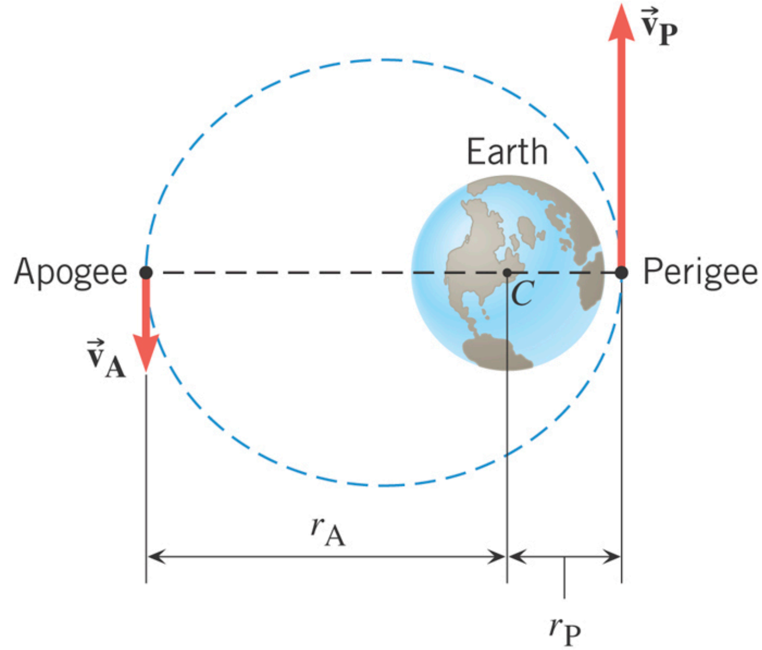
$$K_f = \tfrac{1}{2} I_f \omega_f^2$$
$$= \tfrac{1}{2} I_f \left(I_i / I_f\right)^2 \omega_i^2$$
$$= \left(I_i / I_f\right)\left(\tfrac{1}{2} I_i \omega_i^2\right) \qquad K_i = \tfrac{1}{2} I_i \omega_i^2;$$
$$= \left(I_i / I_f\right) K_i \quad \Rightarrow \quad \text{Kinetic Energy increases}$$

Energy is NOT conserved because pulling in the arms does (NC) work on the mass of each arm and increases the kinetic energy of rotation.

8.9 Angular Momentum

Example: A Satellite in an Elliptical Orbit

An artificial satellite is placed in an elliptical orbit about the earth. Its point of closest approach is 8.37×10^6 m from the center of the earth, and its point of greatest distance is 25.1×10^6 m from the center of the earth. The speed of the satellite at the perigee is 8450 m/s. Find the speed at the apogee.



$$I_A = mr_A^2; \quad I_P = mr_P^2$$
$$\omega_A = v_A / r_A; \quad \omega_P = v_P / r_P$$

Gravitational force along r (no torque) $\Rightarrow$ Angular momentum conserved

$$I_A \omega_A = I_P \omega_P$$

$$mr_A^2 \left(v_A / r_A\right) = mr_P^2 \left(v_P / r_P\right) \quad \Rightarrow \quad r_A v_A = r_P v_P$$

$$v_A = \left(r_P / r_A\right) v_P = \left[(8.37 \times 10^6) / (25.1 \times 10^6)\right]\left(8450 \text{ m/s}\right) = 2820 \text{ m/s}$$

Rotational/Linear Dynamics Summary

linear		rotational	linear	rotational
x	displacement	θ	$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$	$\vec{\tau} = I\vec{\alpha}$
v	velocity	ω	$W = F(\cos\theta)x$	$W_{rot} = \tau\theta$
a	acceleration	α	$K = \frac{1}{2}mv^2$	$K_{rot} = \frac{1}{2}I\omega^2$
m	inertia	$I = mr^2$	$W \Rightarrow \Delta \mathrm{K}$	$W_{rot} \Rightarrow \Delta \mathrm{K}_{rot}$
F	force/torque	$\tau = Fr\sin\theta$	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	$\vec{L} = I\vec{\omega}$
			$\vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}}$	$\vec{\tau}\Delta t = \Delta\vec{L}$

Potential Energies

$U_G = mgy$

or $U_G = -GM_E m/R_E$

$U_S = \frac{1}{2}kx^2$

Conservation laws

Conserved quantity:	If $W_{\mathrm{NC}} = 0$, $E = K + U$	If $\mathbf{F}_{ext} = 0$, $\mathbf{P}_{\mathrm{system}} = \sum \mathbf{p}$	If $\tau_{ext} = 0$, $\vec{L} = I\vec{\omega}$

EQUILIBRIUM OF A RIGID BODY

A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

Note: constant linear speed or constant rotational speed are allowed for an object in equilibrium.

Reasoning Strategy

1. Select the object to which the equations for equilibrium are to be applied.

2. Draw a free-body diagram that shows all of the external forces acting on the object.

3. Choose a convenient set of *x*, *y* axes and resolve all forces into components that lie along these axes.

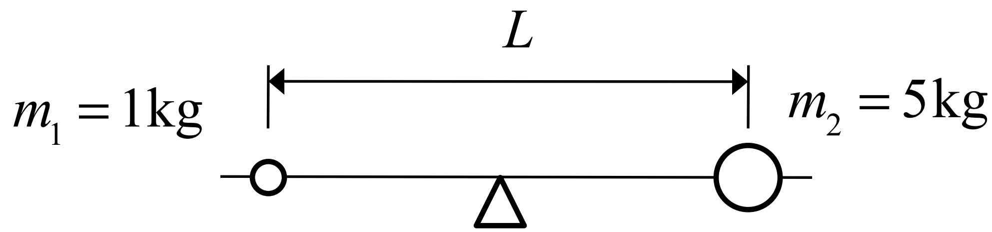
4. Apply the equations that specify the balance of forces at equilibrium. (Set the net force in the *x* and *y* directions equal to zero.)

5. Select a convenient axis of rotation. Set the sum of the torques about this axis equal to zero.

6. Solve the equations for the desired unknown quantities.

Clicker Question 8.7

A 5-kg ball and a 1-kg ball are positioned a distance L apart on a bar of negligible mass. How far from the 5-kg mass should the fulcum be placed to balance the bar?



a) $\frac{1}{2}L$

b) $\frac{1}{3}L$

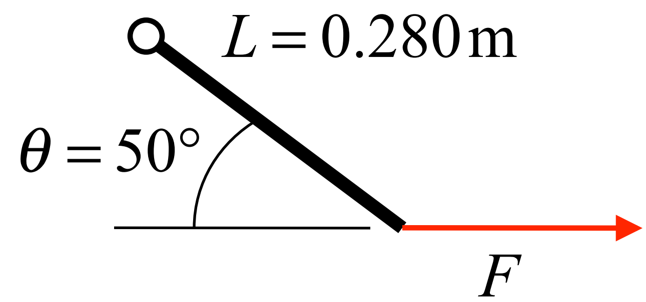
c) $\frac{1}{4}L$

d) $\frac{1}{5}L$

e) $\frac{1}{6}L$

Clicker Question 8.8

A 0.280 m long wrench at an angle of 50.0° to the floor is used to turn a nut. What horizontal force *F* produces a torque of 45.0 Nm on the nut?



a) 10 N

b) 110 N

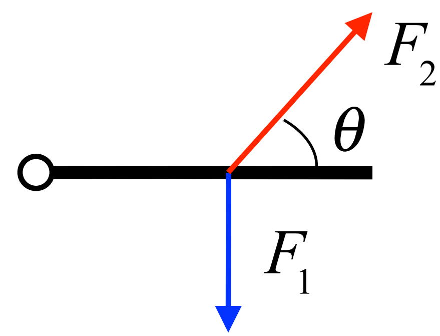
c) 210 N

d) 310 N

e) 410 N

Clicker Question 8.9

A force F_1 = 38.0 N acts downward on a horizontal arm. A second force, F_2 = 55.0 N acts at the same point but upward at the angle theta. What is the angle of F_2 for a net torque = zero?



a) 23.7°

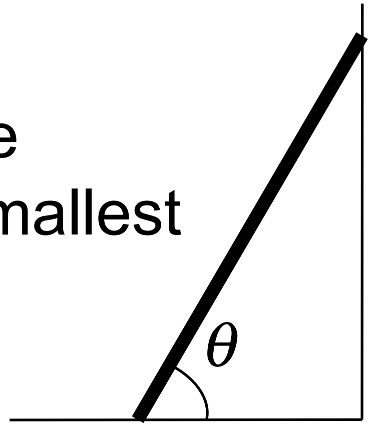
b) 28.7°

c) 33.7°

d) 38.7°

e) 43.7°

Example: A board length L lies against a wall. The coefficient of friction with ground 0.650. What is smallest angle the board can be placed without slipping?



1. Determine the forces acting on the board.

8.7 Rigid Objects in Equilibrium

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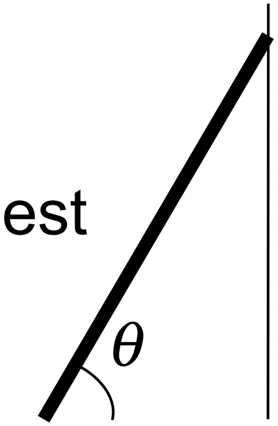


1. Determine the forces acting on the board.

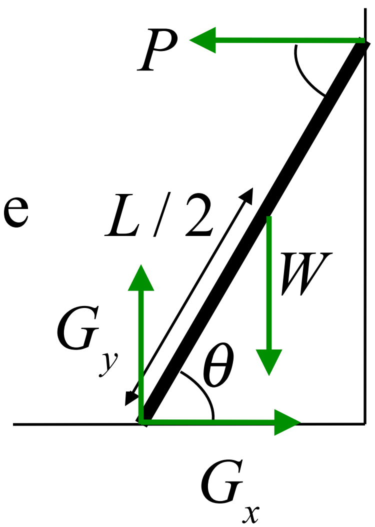
<u>Forces</u>

G_y ground normal force

G_x ground static frictional force

P wall normal force

W gravitational force



Forces:

$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$

2. Choose pivot point at ground.

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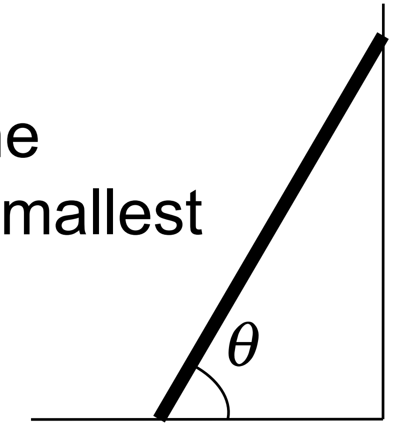

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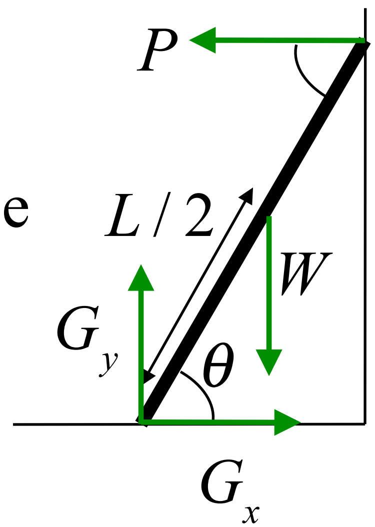

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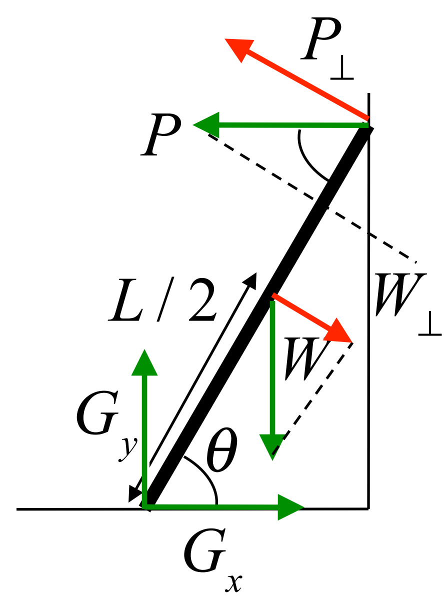
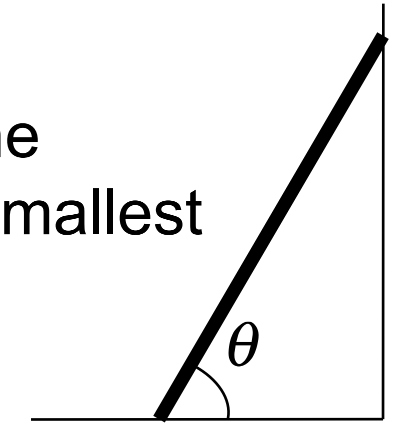


2. Choose pivot point at ground.

3. Find components normal to board

$P_\perp$ and $W_\perp$ are forces producing torque

8.7 Rigid Objects in Equilibrium



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4. Net torque must be zero for equilibrium

Torque:

$$\tau_P = +P_\perp L = \left(P\sin\theta\right)L$$

$$\tau_W = -W_\perp\left(L/2\right) = -\left(W\cos\theta\right)\left(L/2\right)$$

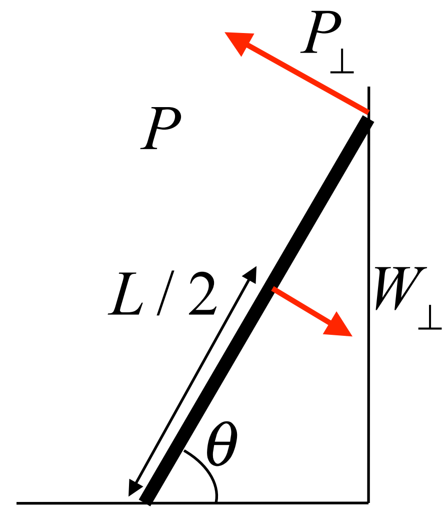
$$\tau_W + \tau_P = 0 \Rightarrow \left(P\sin\theta\right) = \left(W\cos\theta\right)/2$$

$$W = 2P\sin\theta/\cos\theta$$

Forces:

$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$



8.7 *Rigid Objects in Equilibrium*

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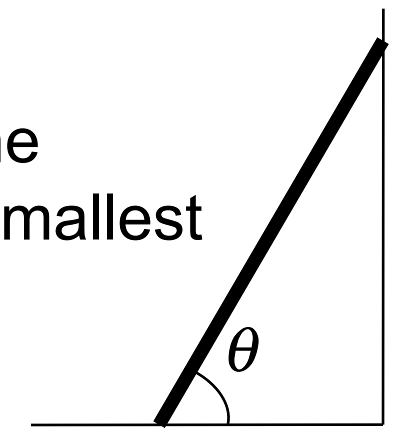

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Forces:

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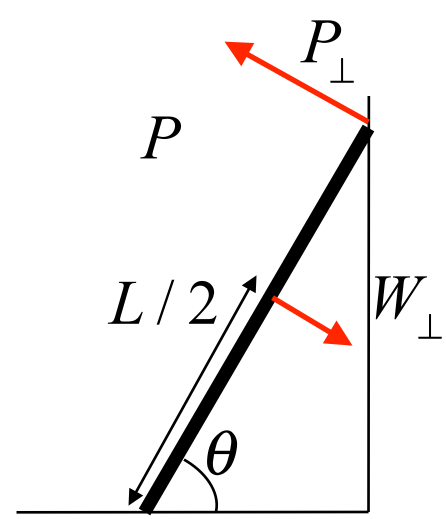

5. Combine torque and force equations

$$W = 2P\sin\theta/\cos\theta = 2\mu W\tan\theta$$

$$\tan\theta = 1/(2\mu) = 1/(1.3) = 0.77$$

$$\theta = 37.6^\circ$$

Example: A flywheel has a mass of 13.0 kg and a radius of 0.300m. What angular velocity gives it an energy of 1.20×10^9 J ?

$$K = \tfrac{1}{2} I\omega^2; \quad I_{disk} = \tfrac{1}{2} MR^2$$

$$= \tfrac{1}{2}\left(\tfrac{1}{2} MR^2\right)\omega^2$$

$$\omega^2 = \frac{4K}{MR^2} = \frac{4.80\times10^9\,\text{J}}{(13.0\text{kg})(0.300\text{m})^2}$$

$$\omega = \sqrt{4.10\times10^9} = 6.40\times10^4\,\text{rad/s}$$

$$= 6.40\times10^4\,\text{rad/s}\left(\text{rev/}(2\pi)\text{rad}\right)$$

$$= 1.02\times10^4\ \text{rev/s}\left(60\,\text{s/min}\right) = 6.12\times10^5\ \text{rpm}$$