Chapter 10

Solids & Liquids

DEFINITION OF MASS DENSITY

The mass density of a substance is the mass of a substance divided by its volume:

$$\rho = \frac{m}{V}$$

SI Unit of Mass Density: kg/m^3

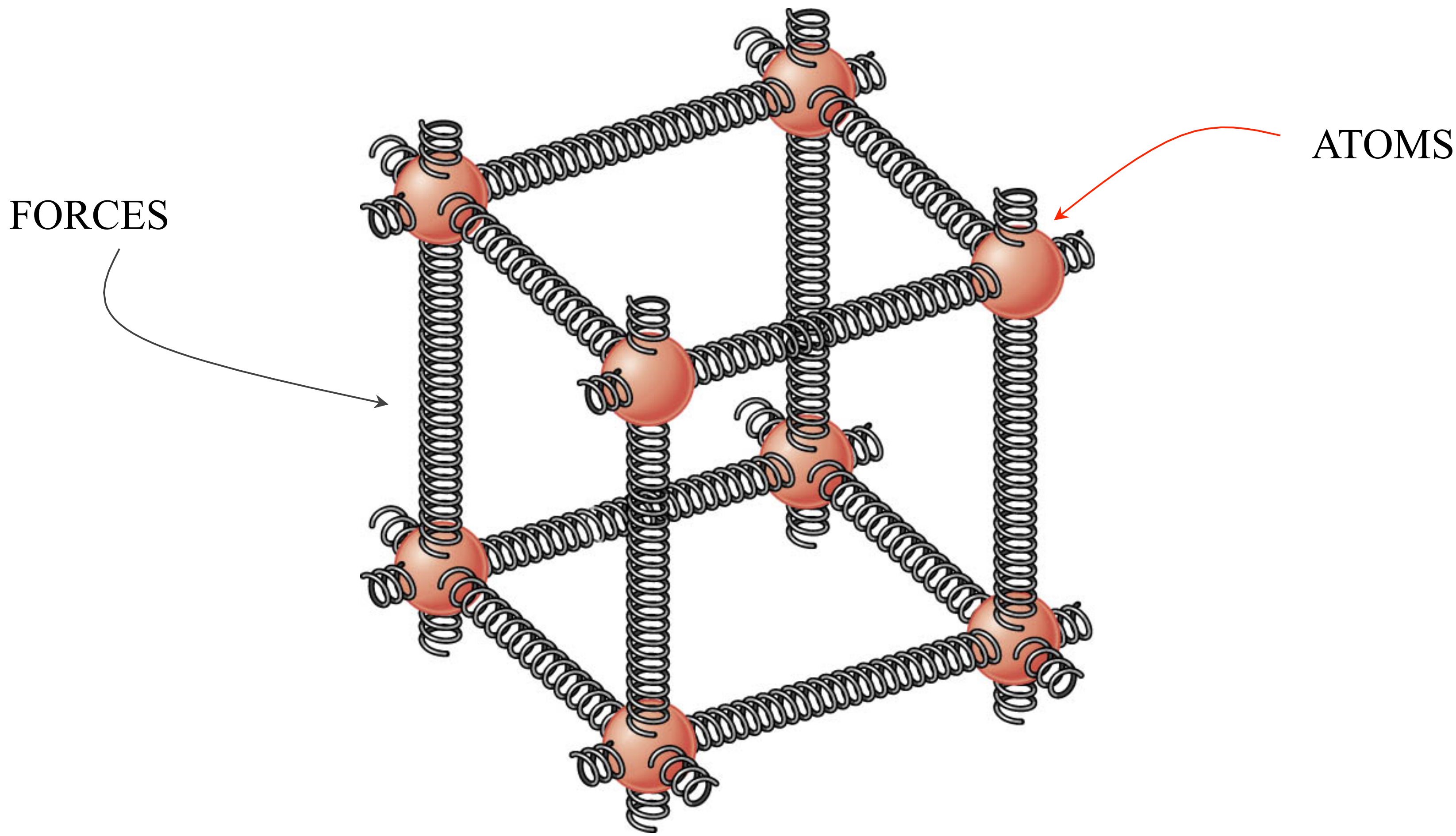
Mass Densities[a] of Common Substances

Substai	Mass Density ρ (kg/m^3)
Solids	
Aluminum	2700
Brass	8470
Concrete	2200
Copper	8890
Diamond	3520
Gold	19 300
Ice	917
Iron (steel)	7860
Lead	11 300
Quartz	2660
Silver	10 500
Wood (yellow pine)	550
Liquids	
Blood (whole, 37 °C)	1060
Ethyl alcohol	806
Mercury	13 600
Oil (hydraulic)	800
Water (4 °C)	1.000×10^3
Gases	
Air	1.29
Carbon dioxide	1.98
Helium	0.179
Hydrogen	0.0899
Nitrogen	1.25
Oxygen	1.43

[a] Unless otherwise noted, densities are given at 0 °C and 1 atm pressure.

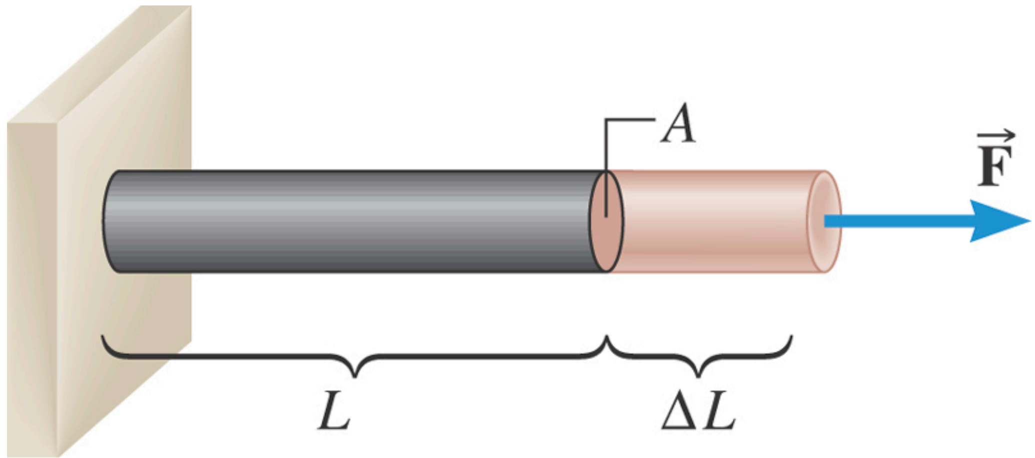
10.2 Solids and Elastic Deformation

Because of these atomic-level “springs”, a material tends to return to its initial shape once forces have been removed.



10.2 Solids and Elastic Deformation

STRETCHING, COMPRESSION, AND YOUNG'S MODULUS



$$F = Y\left(\frac{\Delta L}{L}\right)A$$

Young's modulus has the units of pressure: N/m^2

Values for the Young's Modulus of Solid Materials

Material	Young's Modulus Y (N/m^2)
Aluminum	6.9×10^{10}
Bone	
Compression	9.4×10^{9}
Tension	1.6×10^{10}
Brass	9.0×10^{10}
Brick	1.4×10^{10}
Copper	1.1×10^{11}
Mohair	2.9×10^{9}
Nylon	3.7×10^{9}
Pyrex glass	6.2×10^{10}
Steel	2.0×10^{11}
Teflon	3.7×10^{8}
Titanium	1.2×10^{11}
Tungsten	3.6×10^{11}

Young's modulus is a characteristic of the material (see table 10.2)

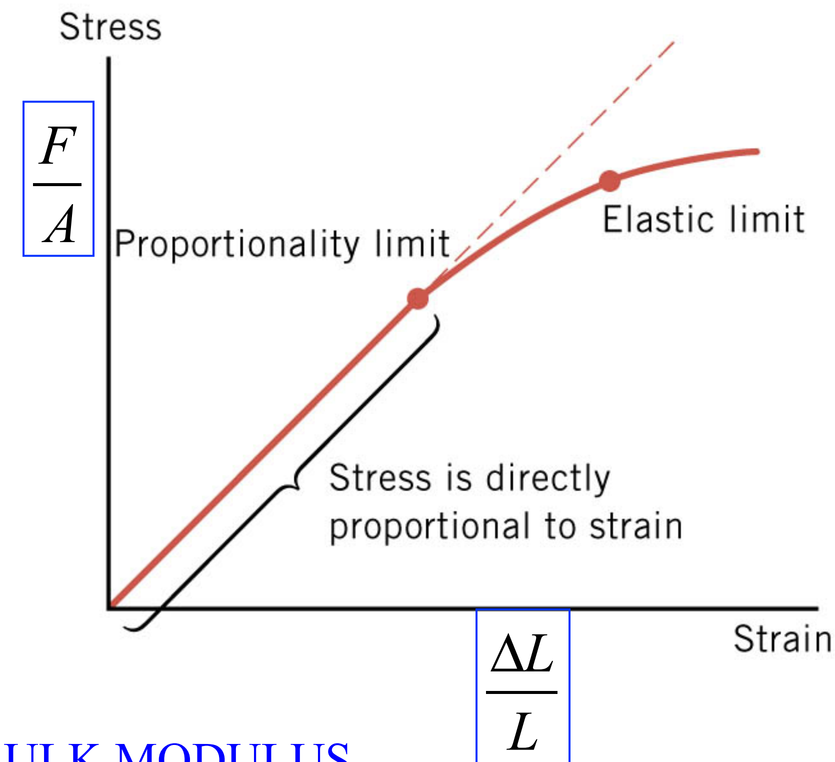
Note: 1 Pascal (Pa) = 1 N/m^2

$$1\ \text{GPa} = 1\times10^{9}\ \text{N/m}^2$$

HOOKE'S LAW
FOR STRESS AND STRAIN

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

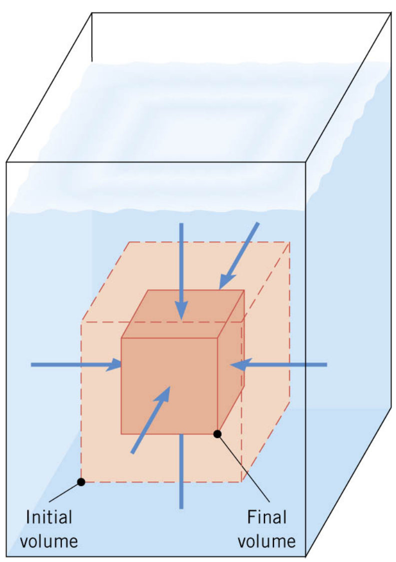
$$\text{stress} = Y \times \text{strain}$$



Slope is Young's modulus *Y*.

Strain is a unitless quantity, and
SI Unit of Stress: N/m^2

VOLUME DEFORMATION AND THE BULK MODULUS



Pressure Change

$$\Delta P = -B\left(\frac{\Delta V}{V}\right)$$

B: Bulk modulus
Table 10.2

Example: **Bone Compression**

In a circus act, a performer supports the combined weight (1080 N) of a number of colleagues. Each thighbone of this performer has a length of 0.55 m and an effective cross sectional area of 7.7×10^{-4} m^2. Determine the amount that each thighbone compresses under the extra weight.

$$F = Y\left(\frac{\Delta L}{L}\right)A$$

$$\text{each leg} = \frac{1080\ \text{N}}{2}$$

$$\Delta L = \frac{FL}{YA}$$

$$= \frac{(540\ \text{N})(0.55\ \text{m})}{(9.4\times10^{9}\ \text{N/m}^2)(7.7\times10^{-4}\ \text{m}^2)}$$

$$= 4.1\times10^{-5}\,\text{m} = 0.041\text{mm}$$

Clicker Question 10.1

A cylindrical, 0.500-m rod has a diameter of 0.02 m. The rod is stretched to a length of 0.501 m by a force of 3000 N. What is the Young's modulus of the material?

$$F = Y\left(\frac{\Delta L}{L}\right)A$$

a) $1.5\times10^{8}\ \mathrm{N/m^2}$

b) $1.2\times10^{9}\ \mathrm{N/m^2}$

c) $7.5\times10^{7}\ \mathrm{N/m^2}$

d) $4.8\times10^{9}\ \mathrm{N/m^2}$

e) $1.5\times10^{7}\ \mathrm{N/m^2}$

Clicker Question 10.2

A cube made of brass (bulk modulus $B = 6.70 \times 10^{10}$ N/m^2) is taken by submarine from the surface where the pressure is 1.01×10^5 N/m^2 to the deepest part of the ocean at a depth of 1.10×10^4 m where it is exposed to a pressure is 1.25×10^8 N/m^2. What is the percent change in volume as a result of this movement?

$$\Delta P = -B\left(\frac{\Delta V}{V}\right)$$

a) 0.413%

b) 0.297%

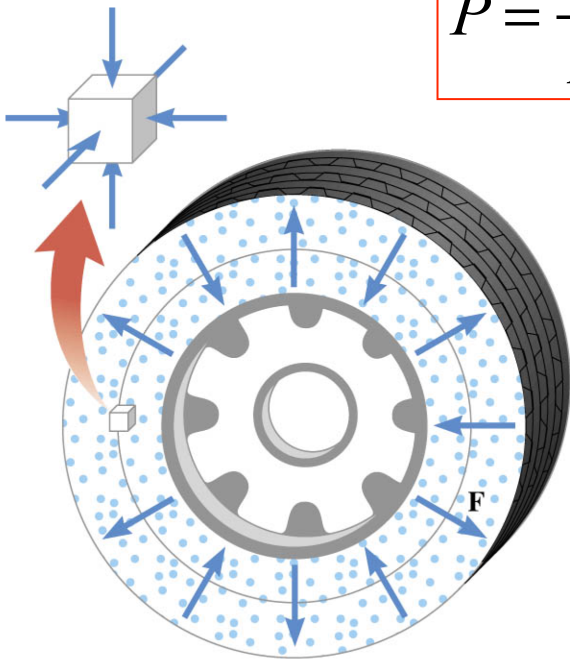
c) 0.187%

d) 0.114%

e) Need to know the initial size of the cube

10.3 ***Pressure***

$$P = \frac{F}{A}$$

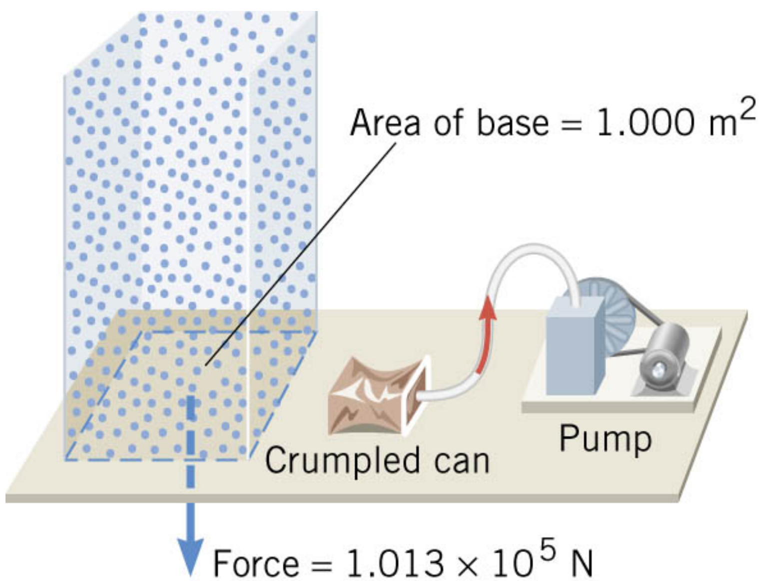


Pressure = Force per unit Area

The same pressure acts inward in every direction on a small volume.

SI Unit of Pressure: 1 N/m^2 = 1Pa

Pascal

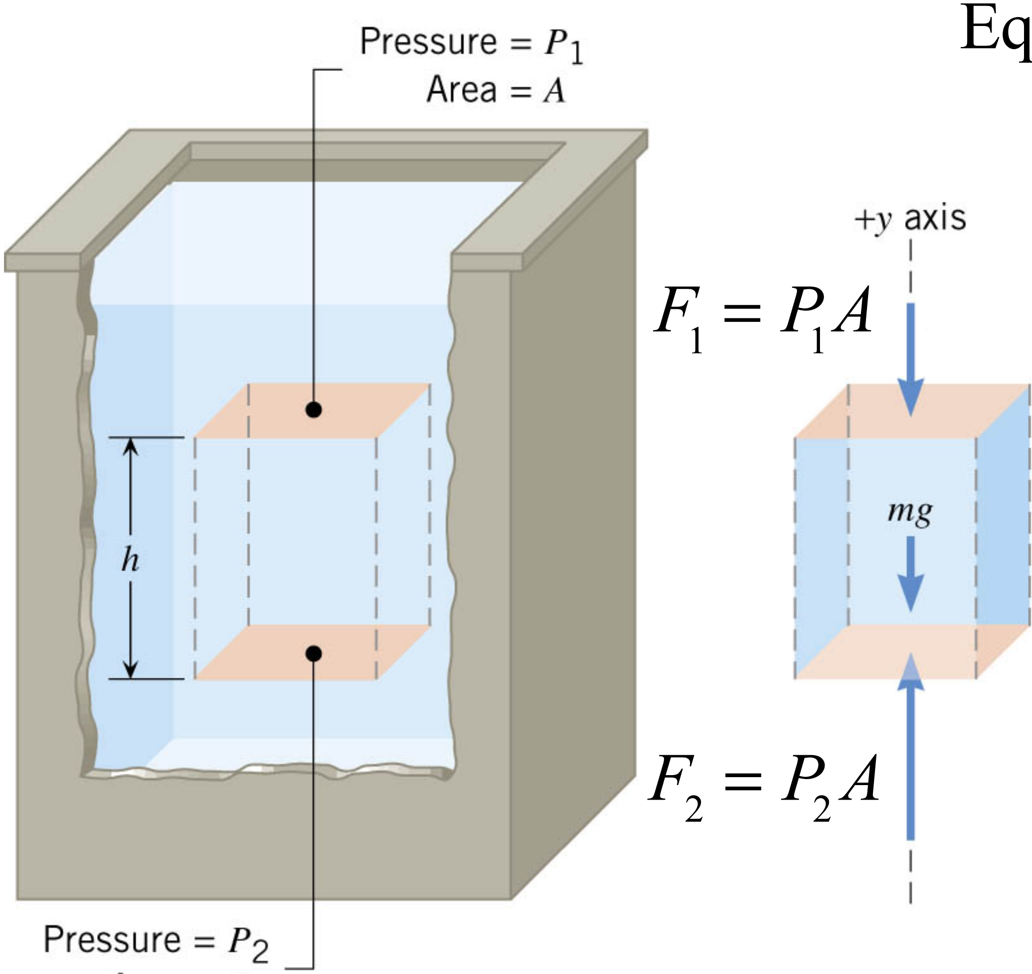


Atmospheric Pressure at Sea Level:

1.013×10^5 Pa = 1 atmosphere

Fluid density is ρ

Equilibrium of a volume of fluid



(*a*)

(*b*) Free-body diagram of the column

$$F_2 = F_1 + mg$$

with $F = PA$, $m = \rho V$

$$P_2 A = P_1 A + \rho V g$$

with $V = Ah$

$$P_2 \cancel{A} = P_1 \cancel{A} + \rho \cancel{A} h g$$

$$\boxed{P_2 = P_1 + \rho g h}$$

Pressure grows linearly with depth (h)

Clicker Question 10.3

The density of mercury is 13.6×10^3 kg/m^3. The pressure 100 cm below the surface of a pool of mercury is how much higher than at the surface?

$$P_2 = P_1 + \rho g h$$

a) $13\ \text{N/m}^2$

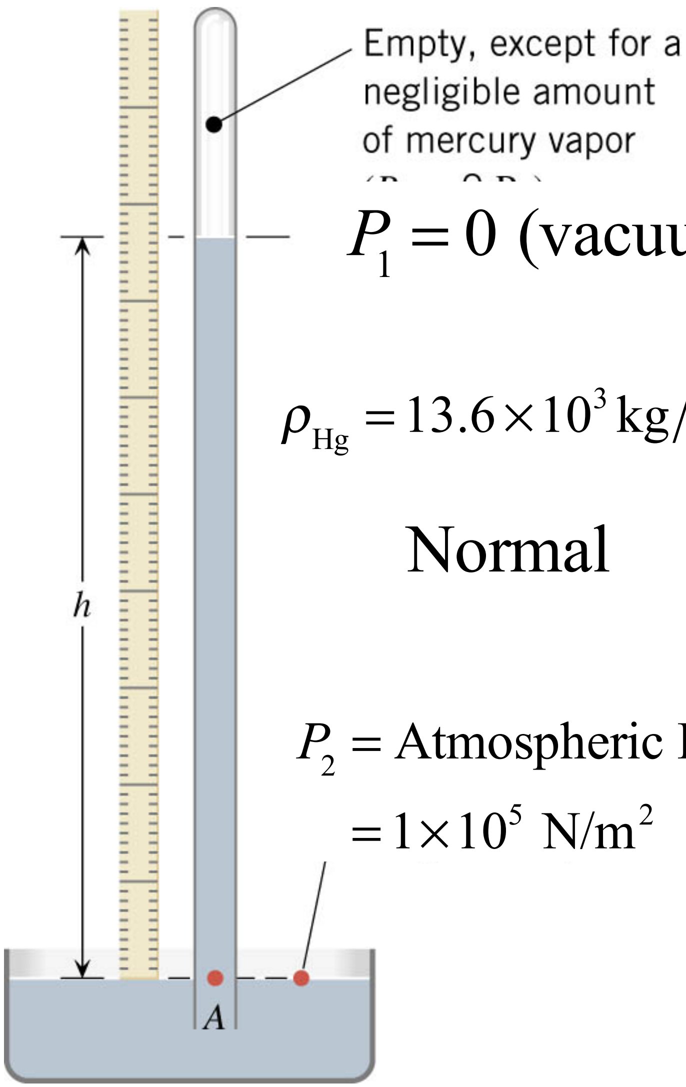
b) $130\ \text{N/m}^2$

c) $1.3\times10^3\ \text{N/m}^2$

d) $1.3\times10^4\ \text{N/m}^2$

e) $1.3\times10^5\ \text{N/m}^2$

Mercury Barometer



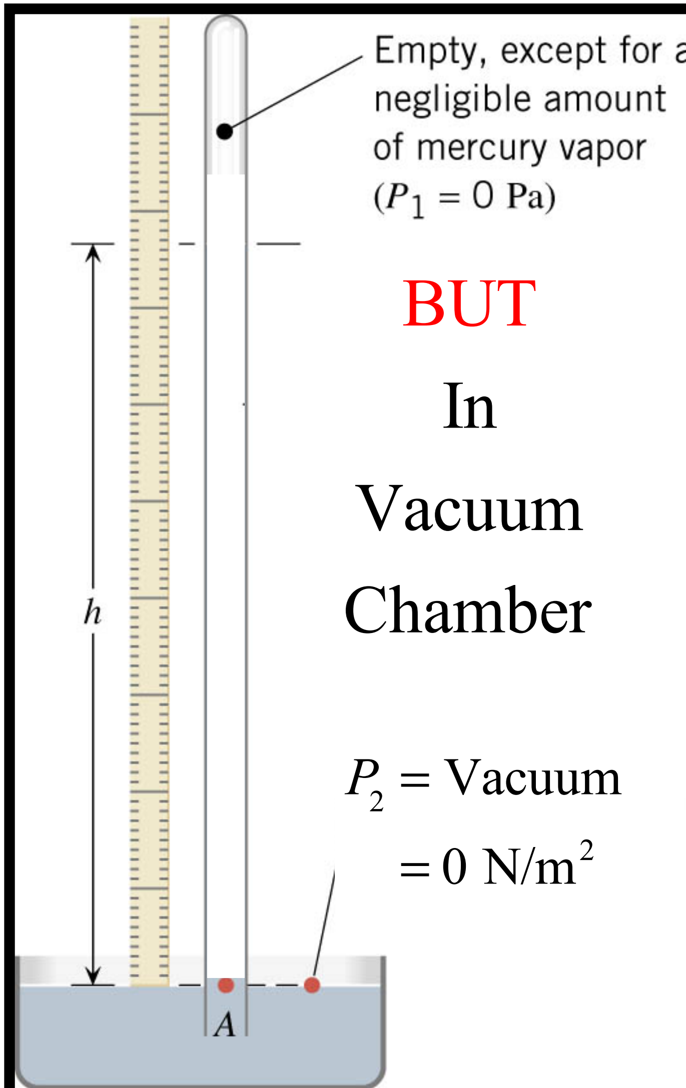
$P_1 = 0$ (vacuum)

$\rho_{Hg} = 13.6\times10^3\,\mathrm{kg/m^3}$

Normal

P_2 = Atmospheric Pressure

$= 1\times10^5\ \mathrm{N/m^2}$



BUT

In

Vacuum

Chamber

P_2 = Vacuum

$= 0\ \mathrm{N/m^2}$

$$P_{atm} = \rho g h \Rightarrow h = \frac{P_{atm}}{\rho g} = \frac{\left(1.01\times10^5\ \mathrm{Pa}\right)}{\left(13.6\times10^3\,\mathrm{kg/m^3}\right)\left(9.81\,\mathrm{m/s^2}\right)} = 0.760\,\mathrm{m}\,(760\ \text{mm of Hg})$$

Clicker Question 10.4

What is the force that causes a liquid to move upward in a drinking straw a a person takes a drink?

a) the force due to a low pressure generated by sucking

b) the force due to the pressure within the liquid

c) the force due to the atmospheric pressure

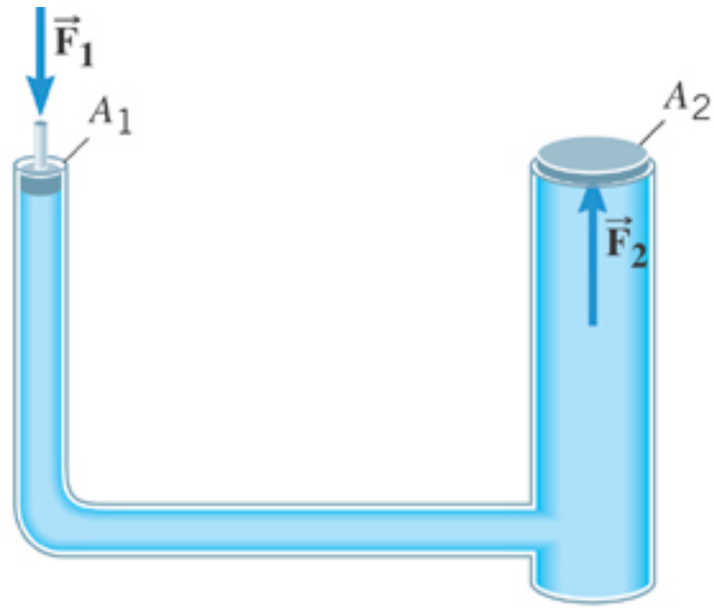
d) the force due to the low pressure in the lungs

e) the force due to friction on the surface of the straw

10.3 Pascal's Principle

PASCAL'S PRINCIPLE

Any change in the pressure applied to a completely enclosed fluid is transmitted <u>undiminished</u> to all parts of the fluid and enclosing walls.



(a)

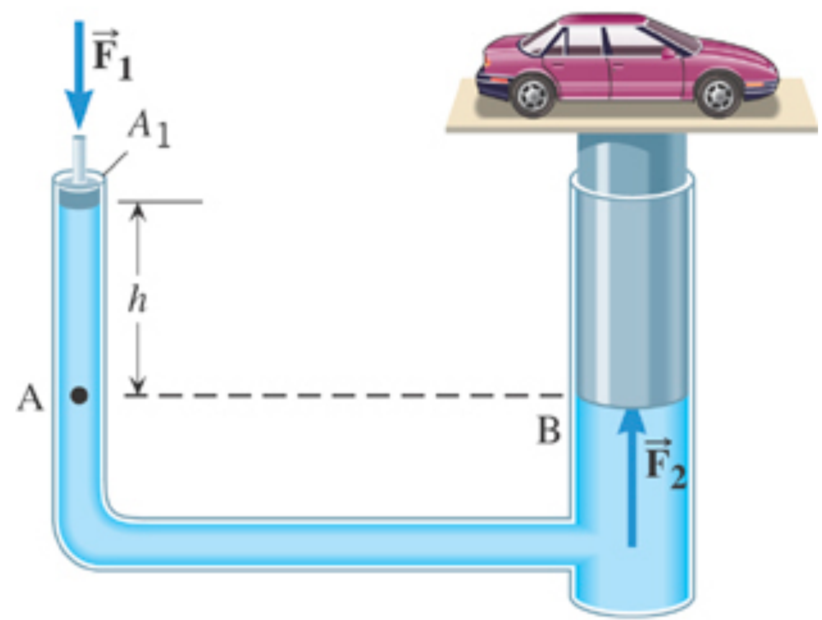
$$P_2 = \frac{F_2}{A_2}; \quad P_1 = \frac{F_1}{A_1}$$

Assume weight of fluid in the tube is negligible

$$\rho g h << P$$

$$P_2 = P_1 + \rho g h$$

$$P_2 = P_1$$



Small ratio

$$\frac{F_2}{A_2} = \frac{F_1}{A_1} \Rightarrow$$

$$F_1 = F_2\left(\frac{A_1}{A_2}\right)$$

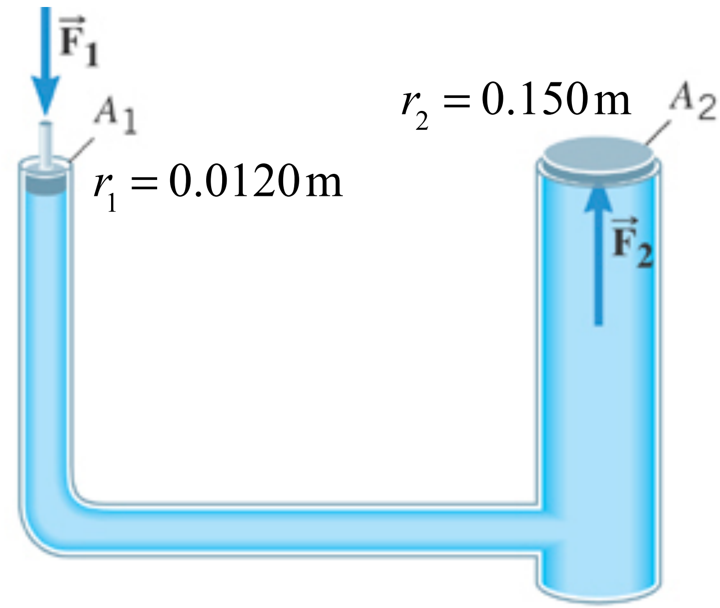
Example: A Car Lift

The input piston has a radius of 0.0120 m and the output plunger has a radius of 0.150 m.

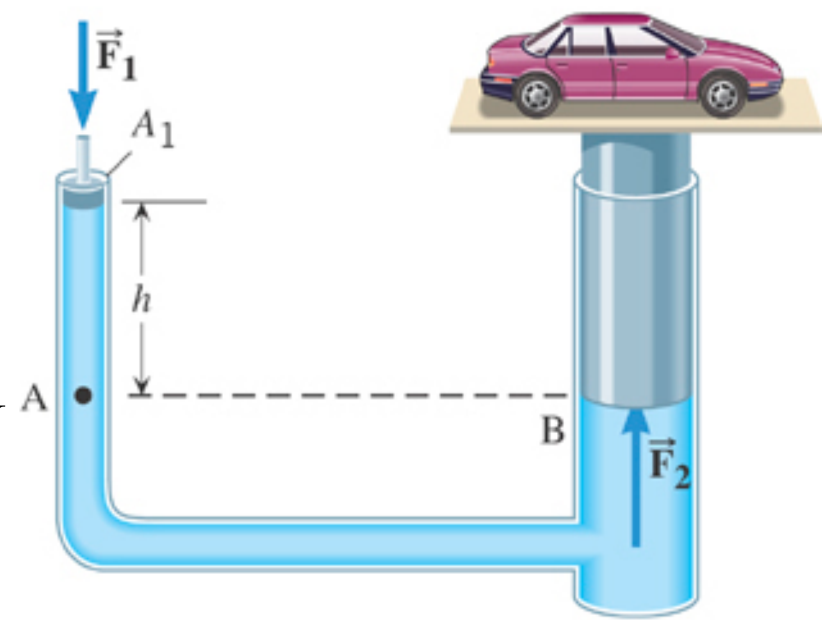
The combined weight of the car and the plunger is 20500 N. Suppose that the input piston has a negligible weight and the bottom surfaces of the piston and plunger are at the same level. What is the required input force?

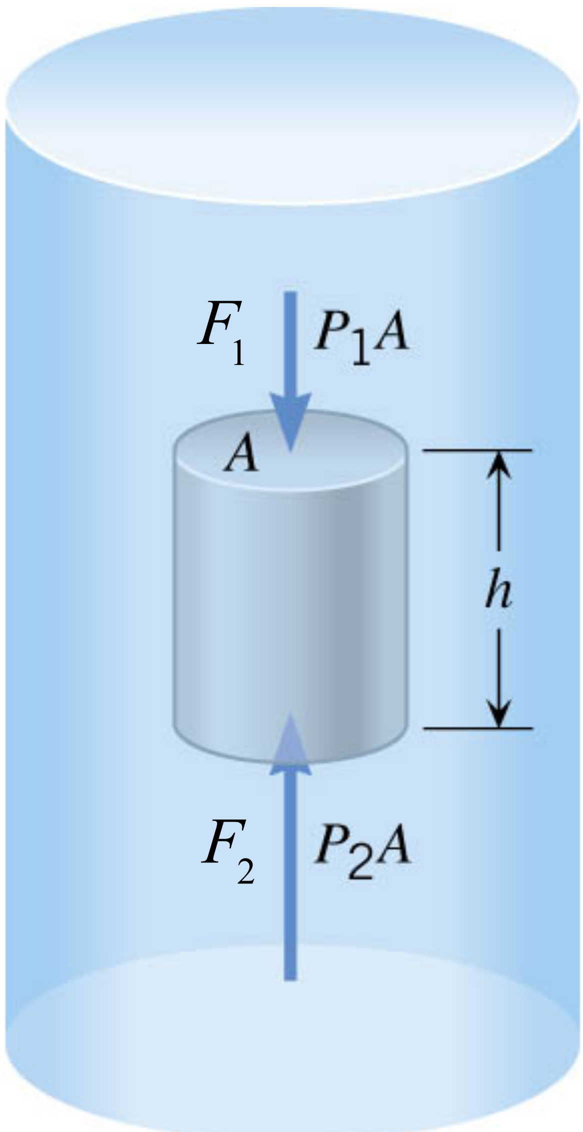
$$F_1 = F_2\left(\frac{A_1}{A_2}\right)$$

$$= (20500\ \text{N})\frac{\pi(0.0120\ \text{m})^2}{\pi(0.150\ \text{m})^2} = 131\ \text{N}$$



(*a*)





Buoyant Force

$$F_B = F_2 + (-F_1)$$

$$= P_2A - P_1A = (P_2 - P_1)A$$

$$= \rho ghA \qquad \text{since } P_2 = P_1 + \rho gh$$

$$= \underbrace{\rho V}_{\text{mass of displaced fluid}} g \qquad \text{and } V = hA$$

Buoyant force = Weight of displaced fluid

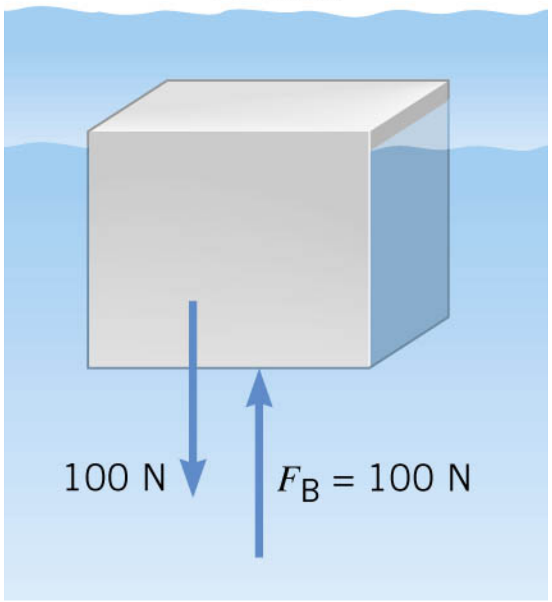
ARCHIMEDES' PRINCIPLE

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces:

$$\underbrace{F_B}_{\text{Magnitude of buoyant force}} = \underbrace{W_{\text{fluid}}}_{\text{Weight of displaced fluid}}$$

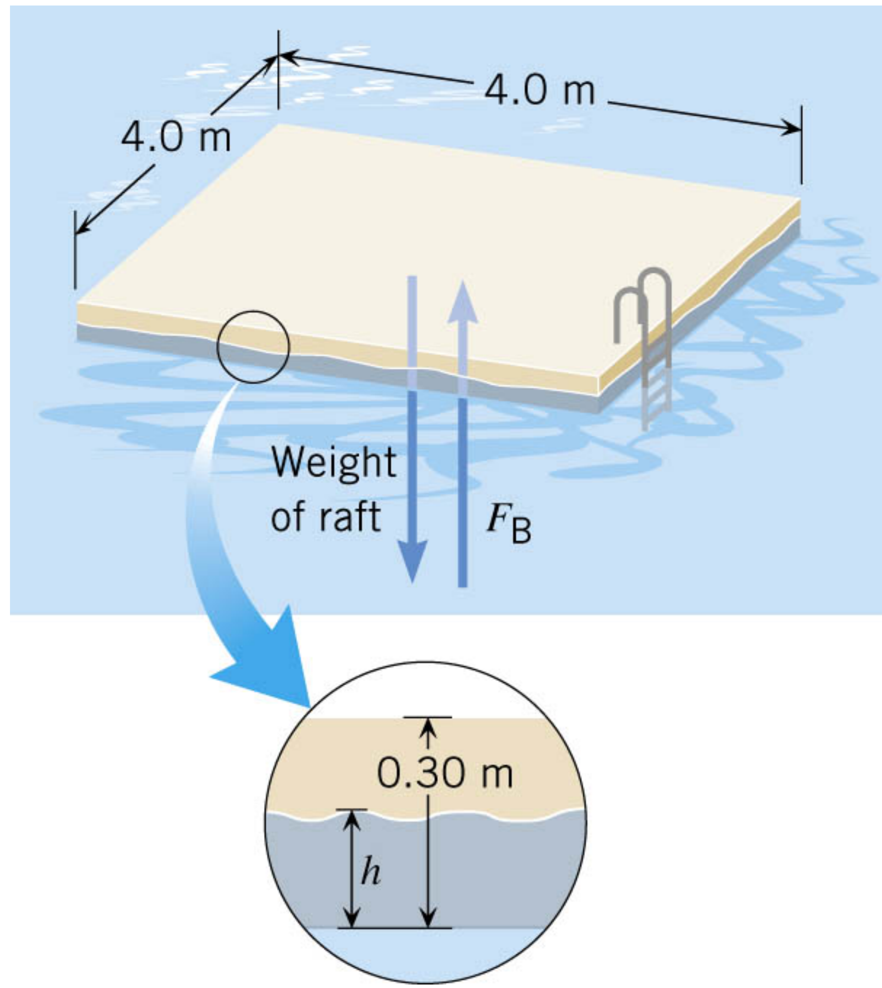
CORROLARY

If an object is floating then the magnitude of the buoyant force is equal to the magnitude of its weight.



Example: A Swimming Raft

The raft is made of solid square pinewood. Determine whether the raft floats in water and if so, how much of the raft is beneath the surface.



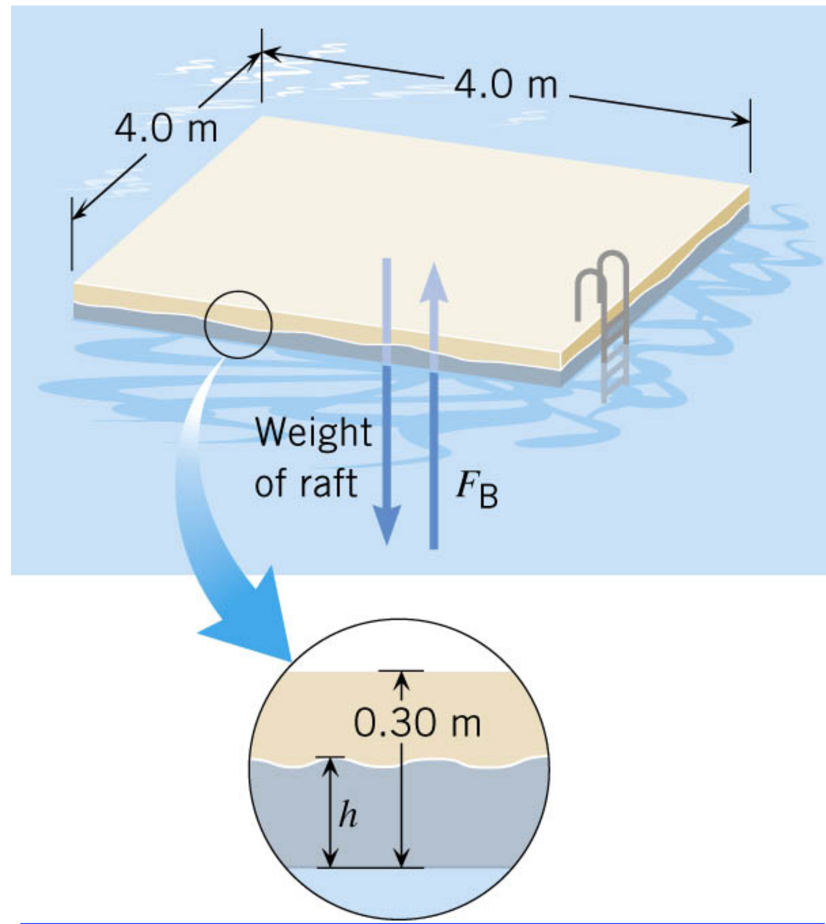
$$W_{raft} = m_{raft} g = \rho_{pine} V_{raft} g$$
$$= \left(550\,\mathrm{kg/m^3}\right)\left(4.8\,\mathrm{m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)$$
$$= 26000\ \mathrm{N}$$

Raft properties

$$V_{raft} = (4.0)(4.0)(0.30)\,\mathrm{m^3} = 4.8\ \mathrm{m^3}$$
$$\rho_{pine} = 550\ \mathrm{kg/m^3}$$

If $W_{\mathrm{raft}} < F_{\mathrm{B}}^{\max}$, raft floats

$$F_{\mathrm{B}}^{\max} = W_{\mathrm{fluid}} \quad \text{(full volume)}$$



$$F_{B}^{\max} = \rho V g = \rho_{water} V_{water} g$$
$$= \left(1000\,\mathrm{kg/m^3}\right)\left(4.8\,\mathrm{m^3}\right)\left(9.80\,\mathrm{m/s^2}\right)$$
$$= 47000\ \mathrm{N}$$

$W_{raft} < F_{B}^{\max}$ Raft floats

Part of the raft is above water

How much of raft below water?

Floating object

$$F_B = W_{\text{raft}}$$

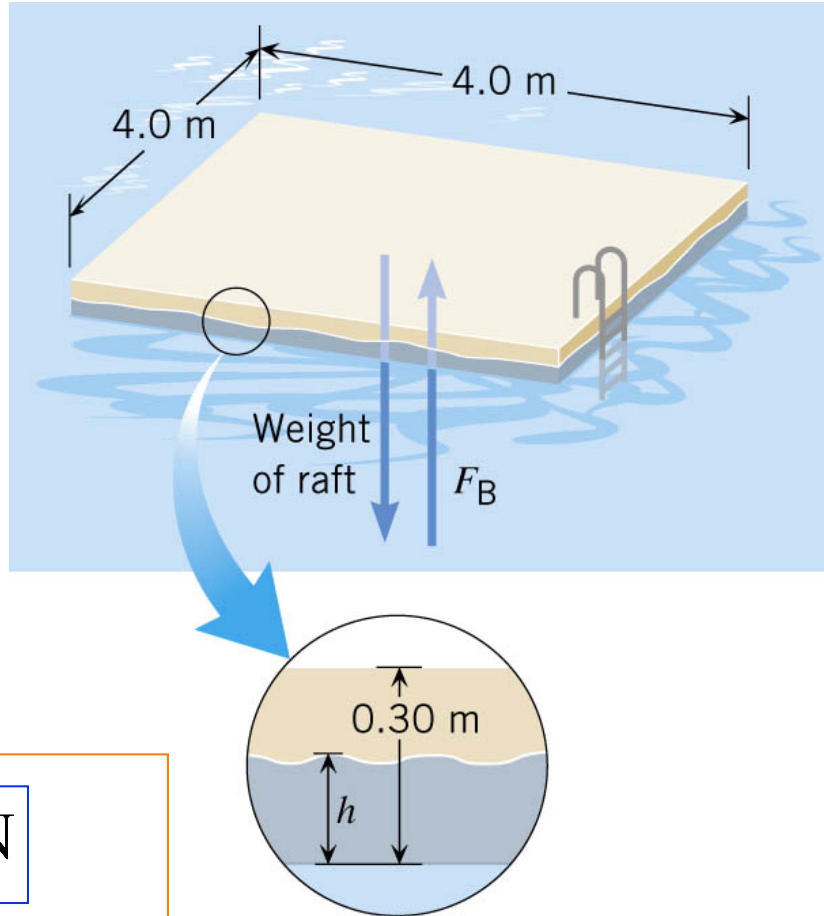
$$F_B = \rho_{\text{water}} g V_{\text{water}}$$
$$= \rho_{\text{water}} g (A_{\text{water}} h)$$

$$h = \frac{W_{\text{raft}}}{\rho_{\text{water}} g A_{\text{water}}}$$

$$W_{raft} = 26000 \text{ N}$$

$$= \frac{26000\text{N}}{\left(1000\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(16.0\ \text{m}^2\right)}$$

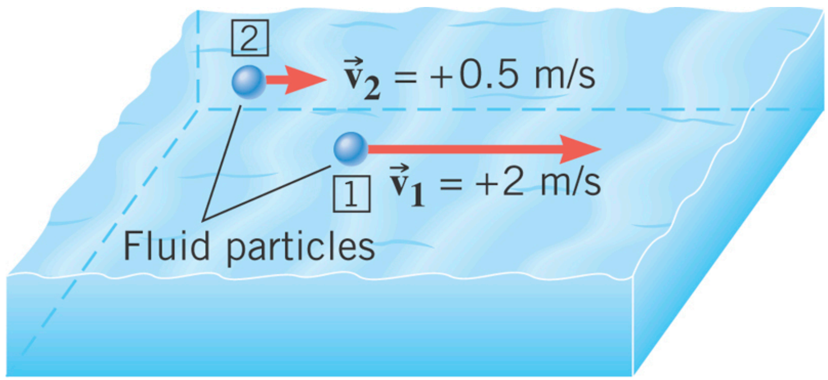
$$= 0.17 \text{ m}$$



10.5 ***Fluids in Motion***

In ***steady flow*** the velocity of the fluid particles at any point is constant as time passes.

Unsteady flow exists whenever the velocity of the fluid particles at a point changes as time passes.



Turbulent flow is an extreme kind of unsteady flow in which the velocity of the fluid particles at a point change erratically in both magnitude and direction.

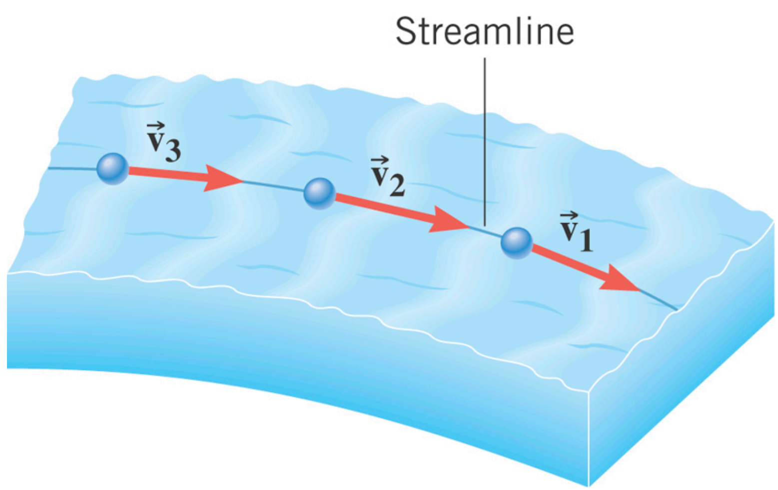
Fluid flow can be ***compressible*** or ***incompressible***. Most liquids are nearly incompressible.

Fluid flow can be ***viscous*** or ***nonviscous***.

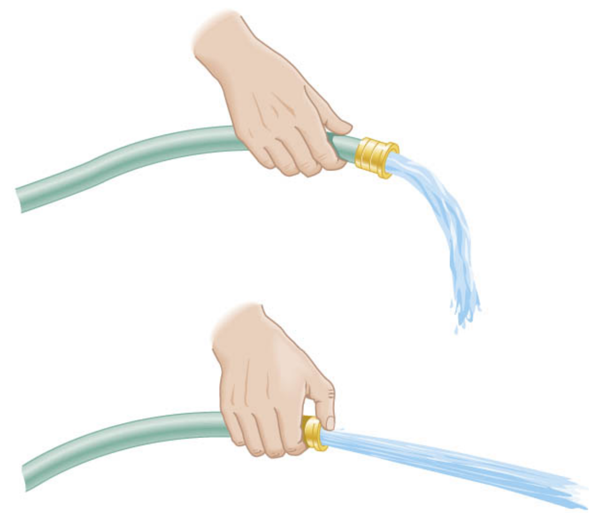
An incompressible, nonviscous fluid is called an ***ideal fluid***.

10.5 Fluids in Motion

When the flow is steady, ***streamlines*** are often used to represent the trajectories of the fluid particles.



The mass of fluid per second that flows through a tube is called the ***mass flow rate***.



EQUATION OF CONTINUITY

The mass flow rate has the same value at every position along a tube that has a single entry and a single exit for fluid flow.

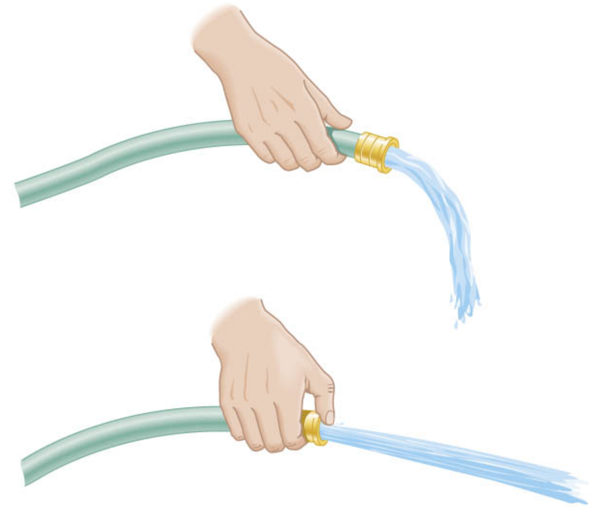
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

SI Unit of Mass Flow Rate: kg/s

Incompressible fluid: $\rho_1 = \rho_2$ $\quad A_1 v_1 = A_2 v_2$

Volume flow rate Q: $Q = Av$

Example: A Garden Hose



A garden hose has an unobstructed opening with a cross sectional area of $2.85\text{x}10^{-4}\text{m}^2$. It fills a bucket with a volume of $8.00\text{x}10^{-3}\text{m}^3$ in 30 seconds.

Find the speed of the water that leaves the hose through (a) the unobstructed opening and (b) an obstructed opening with half as much area.

a) $Q = Av$

$$v = \frac{Q}{A} = \frac{\left(8.00\times10^{-3}\,\text{m}^3\right)/\left(30.0\text{ s}\right)}{2.85\times10^{-4}\,\text{m}^2} = 0.936\,\text{m/s}$$

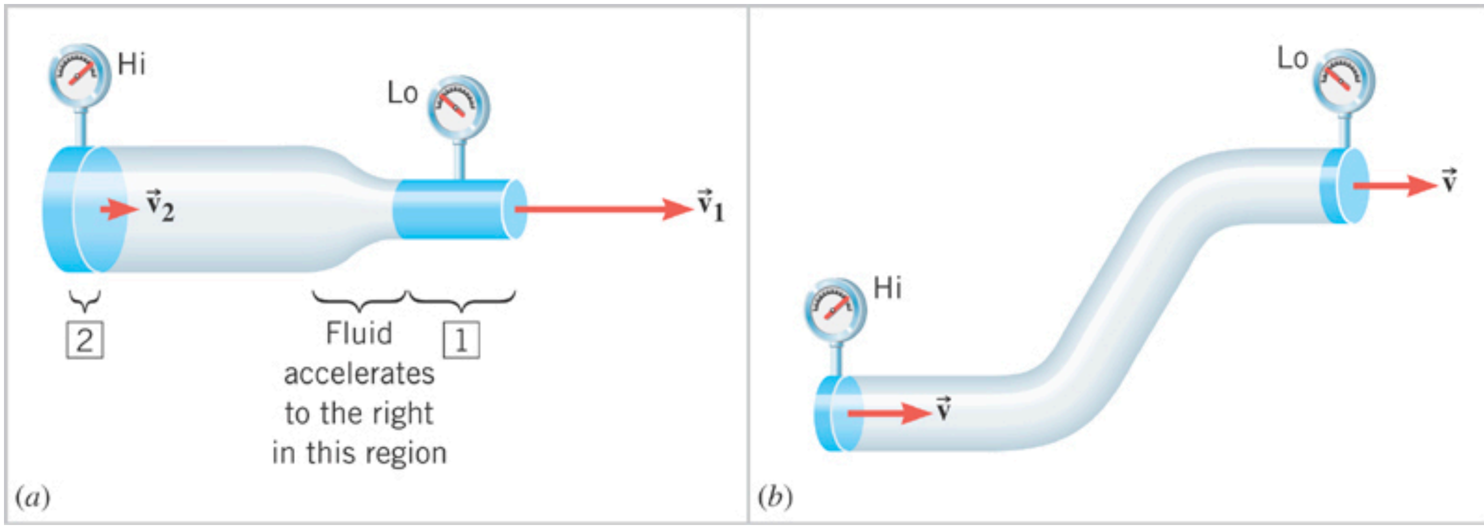
b) $A_1 v_1 = A_2 v_2$

$$v_2 = \frac{A_1}{A_2}v_1 = \left(2\right)\left(0.936\,\text{m/s}\right) = 1.87\,\text{m/s}$$

10.5 Bernoulli's Equation

The fluid accelerates toward the lower pressure regions.

According to the pressure-depth relationship, the pressure is lower at higher levels, provided the area of the pipe does not change.

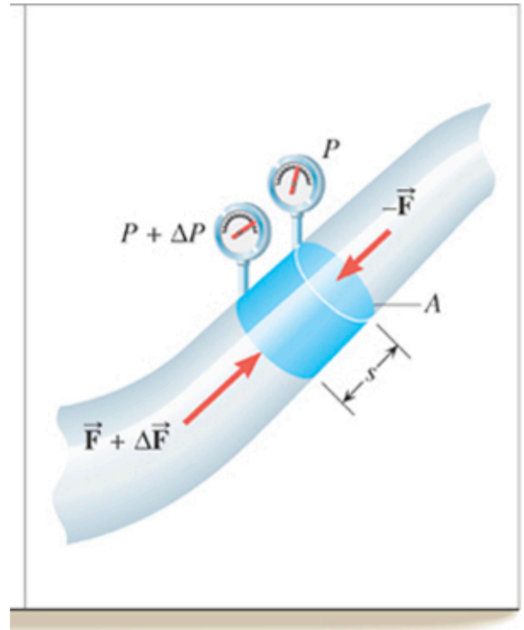


Apply Work-Energy theorem
to determine relationship between
pressure, height, velocity, of the fluid.

10.5 *Bernoulli's Equation*

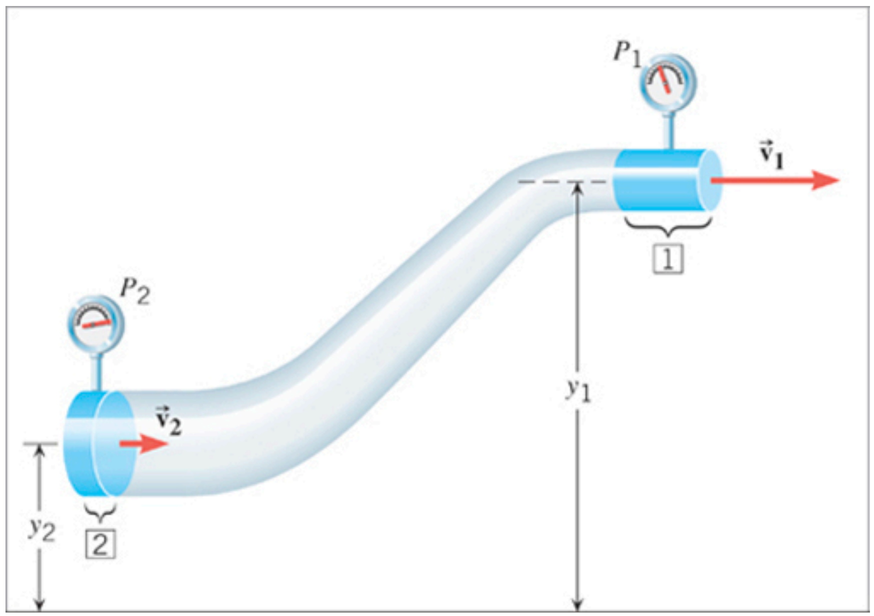
Work done by tiny pressure "piston"

$$W_{\Delta P} = \left(\sum F\right)s = \left(\Delta F\right)s = \left(\Delta PA\right)s; \quad V = As$$



Work (NC) done by pressure difference from 2 to 1

$$W_{NC} = \left(P_2 - P_1\right)V$$



$$E_1 = \tfrac{1}{2}mv_1^2 + mgy_1$$

$$E_2 = \tfrac{1}{2}mv_2^2 + mgy_2$$

$$W_{NC} = E_1 - E_2 = \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_2^2 + mgy_2\right)$$

10.5 Bernoulli's Equation

$$W_{NC} = (P_2 - P_1)V$$

$$W_{NC} = E_1 - E_2 = \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_2^2 + mgy_2\right)$$

NC Work yields a total Energy change.

Equating the two expressions for the work done,

$$(P_2 - P_1)V = \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_2^2 + mgy_2\right)$$

$$m = \rho V$$

$$(P_2 - P_1) = \left(\tfrac{1}{2}\rho v_1^2 + \rho gy_1\right) - \left(\tfrac{1}{2}\rho v_2^2 + \rho gy_2\right)$$

Rearrange to obtain Bernoulli's Equation

BERNOULLI'S EQUATION

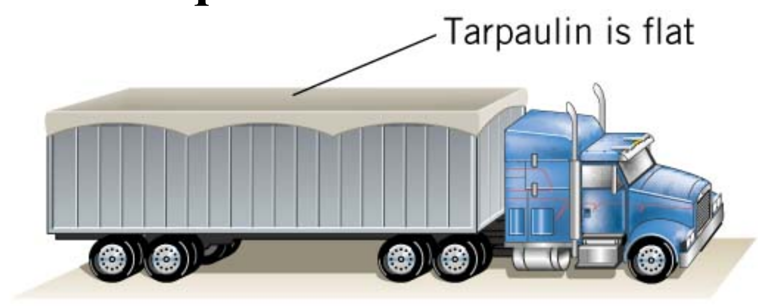
In steady flow of a nonviscous, incompressible fluid, the pressure, the fluid speed, and the elevation at two points are related by:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho gy_2$$

Conceptual Example: **Tarpaulins and Bernoulli's Equation**

When the truck is stationary, the tarpaulin lies flat, but it bulges outward when the truck is speeding down the highway.

Account for this behavior.

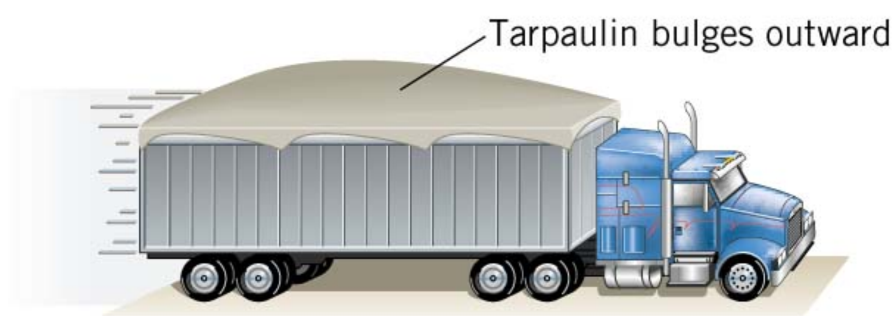


Stationary

Relative to moving truck

$v_1 = 0$ under the tarp

v_2 air flow over top



Moving

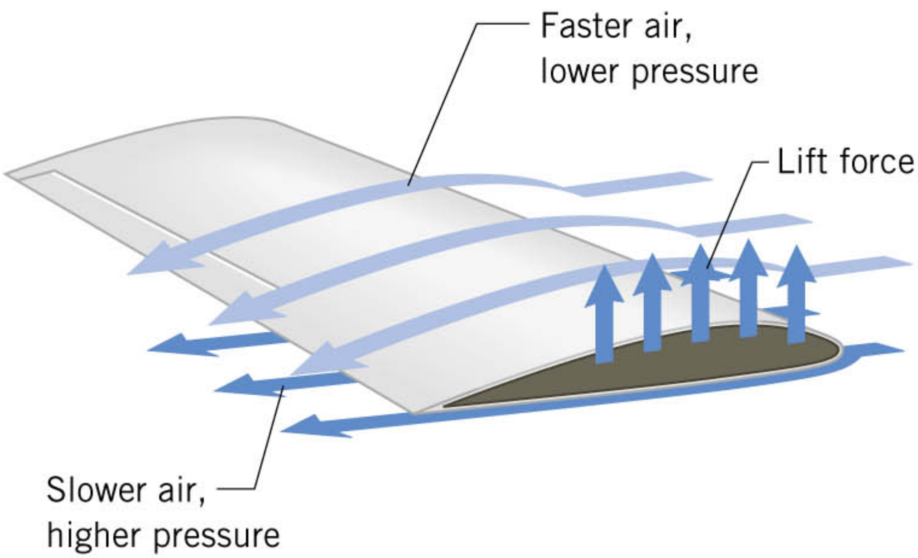
<u>Bernoulli's Equation</u>

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

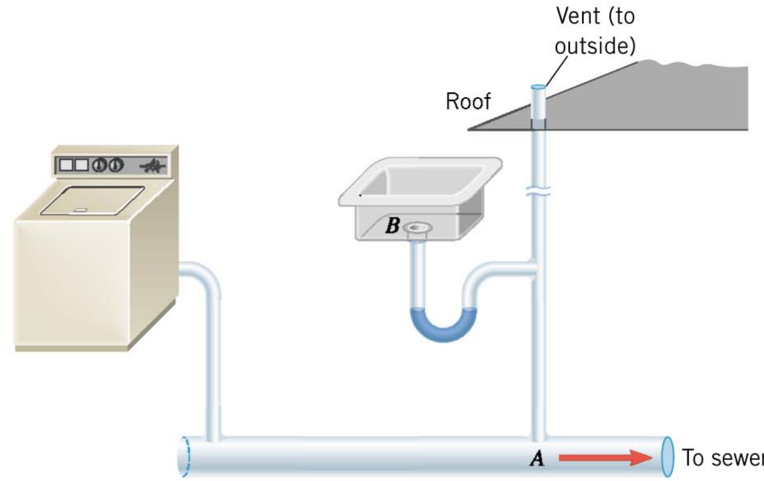
$$P_1 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$\boxed{P_1 > P_2}$$

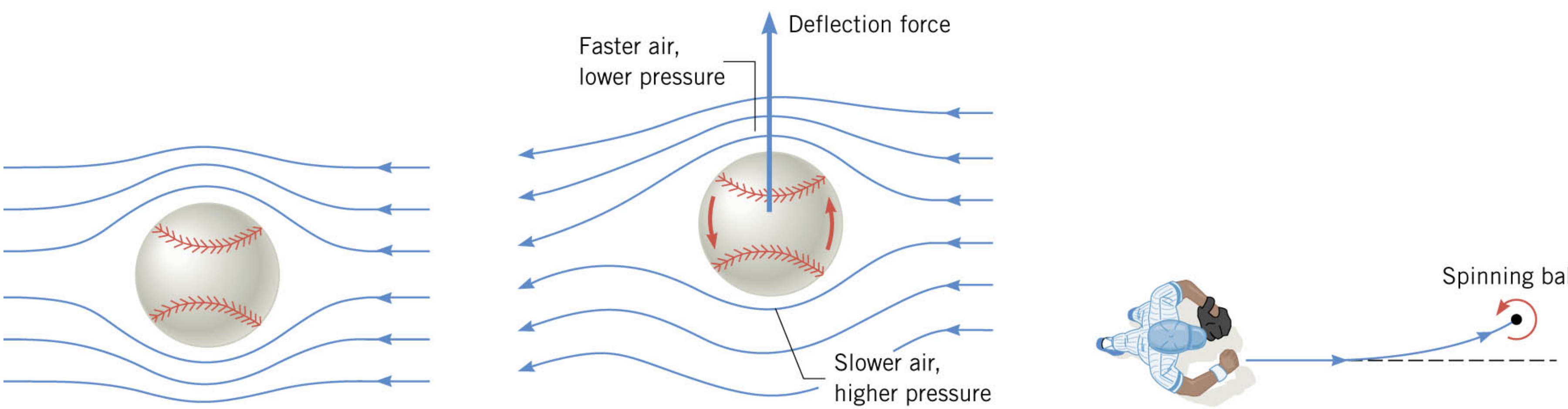
10.5 Applications of Bernoulli's Equation



(*a*)



(*b*) With vent



(*a*) Without spin (*b*) With spin (*c*)

Example: **Efflux Speed**

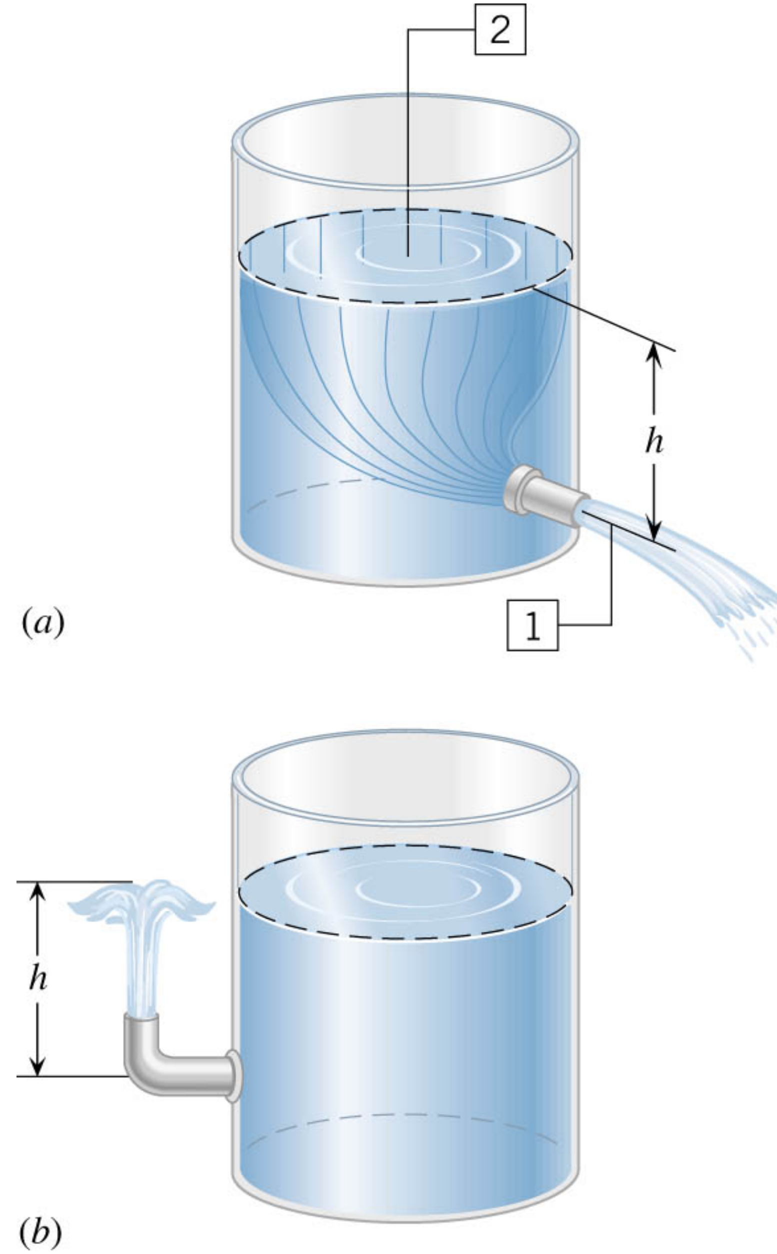
The tank is open to the atmosphere at the top. Find and expression for the speed of the liquid leaving the pipe at the bottom.

$$P_1 = P_2 = P_{atmosphere} \quad (1\times 10^5 \text{ N/m}^2)$$
$$v_2 = 0, \quad y_2 = h, \quad y_1 = 0$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

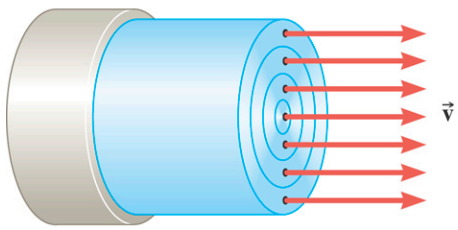
$$\tfrac{1}{2}\rho v_1^2 = \rho g h$$

$$v_1 = \sqrt{2gh}$$



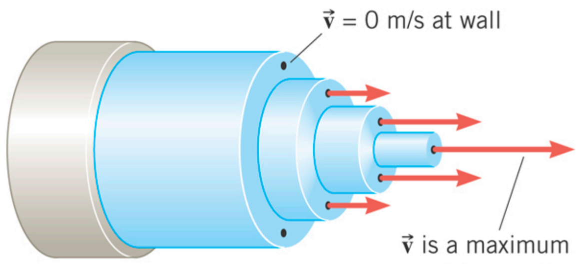
10.6 Viscous Flow

Flow of an ideal fluid.



(a)

Flow of a viscous fluid.



(b)

FORCE NEEDED TO MOVE A LAYER OF VISCOUS FLUID WITH CONSTANT VELOCITY

The magnitude of the tangential force required to move a fluid layer at a constant speed is given by:

$$F = \frac{\eta A v}{y}$$

η, is the coefficient of viscosity

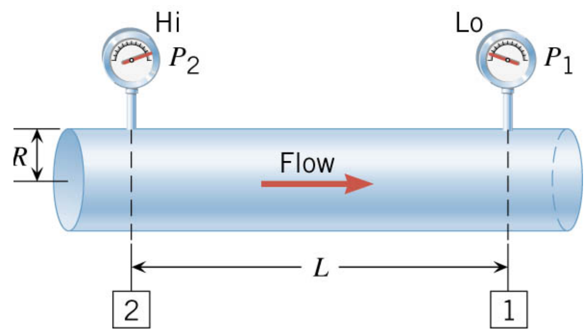
SI Unit: $\text{Pa}\cdot\text{s}$; 1 poise (P) = 0.1 $\text{Pa}\cdot\text{s}$

POISEUILLE'S LAW (flow of viscous fluid)

The volume flow rate is given by:

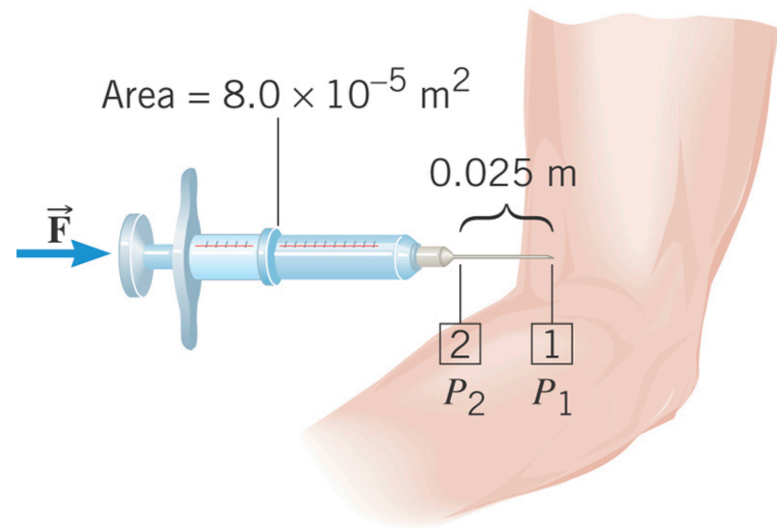
$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta L}$$

Pressure drop in a straight uniform diamater pipe.



Example: **Giving and Injection**

A syringe is filled with a solution whose viscosity is $1.5\text{x}10^{-3}$ Pa·s. The internal radius of the needle is $4.0\text{x}10^{-4}$m.



The gauge pressure in the vein is 1900 Pa. What force must be applied to the plunger, so that $1.0\text{x}10^{-6}\text{m}^3$ of fluid can be injected in 3.0 s?

$$P_2 - P_1 = \frac{8\eta L Q}{\pi R^4}$$

$$= \frac{8\left(1.5\times10^{-3}\,\text{Pa}\cdot\text{s}\right)\left(0.025\text{ m}\right)\left(1.0\times10^{-6}\,\text{m}^3/3.0\text{ s}\right)}{\pi\left(4.0\times10^{-4}\text{m}\right)^4} = 1200\text{ Pa}$$

$$P_2 = \left(1200 + P_1\right)\text{Pa} = \left(1200+1900\right)\text{Pa} = 3100\text{ Pa}$$

$$F = P_2 A = \left(3100\text{ Pa}\right)\left(8.0\times10^{-5}\text{m}^2\right) = 0.25\text{N}$$