

Exam 3 Solutions

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$$\text{Centripetal acceleration: } a_c = \frac{v^2}{r} \Rightarrow v = \sqrt{a_c r}$$

$$\text{Period of motion: } T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi r}{\sqrt{a_c r}} = \frac{2\pi\sqrt{r}}{\sqrt{a_c}} = \frac{2\pi\sqrt{50\text{m}}}{\sqrt{0.548\text{m/s}^2}} = 60.0\text{s}$$

1. A car travels at a constant speed around a flat circular track with a radius of 50.0 m and takes 1.00 minute to complete one revolution. What is the centripetal acceleration of the car?

$$\begin{aligned} a_c &= \frac{v^2}{r} \quad T = \frac{2\pi r}{v} \Rightarrow v = \frac{2\pi r}{T} \\ &= \left(\frac{2\pi}{T} \right)^2 r = \left(\frac{2\pi}{60\text{s}} \right)^2 (50.0\text{m}) = 0.548\text{m/s}^2 \end{aligned}$$

2. The mass and radius of the moon are 7.4×10^{22} kg and 1.7×10^6 m, respectively. What is the mass of an object with a weight of 1.71 N on the surface of the moon?

$$\begin{aligned} W &= mg_{\text{moon}} \quad g_{\text{moon}} = \frac{GM_{\text{moon}}}{R_{\text{moon}}^2} \\ m &= \frac{W}{g_{\text{moon}}} = \frac{(1.71\text{N})(1.70 \times 10^6\text{m})^2}{(6.67 \times 10^{-11}\text{Nm}^2/\text{kg}^2)(7.40 \times 10^{22}\text{kg})} = 1.00\text{kg} \end{aligned}$$

Clicker Question 13.3

3. A spaceship is in orbit around the earth at an altitude of 19,290 km. Which one of the following statements best explains why an astronaut experiences "weightlessness"?

- A) The centripetal force of the earth on the astronaut in orbit is zero newtons.
- B) The pull of the earth on the spaceship is canceled by the pull of the other planets.
- C) The spaceship is in free fall so its floor cannot press upward on the astronaut.
- D) The force decreases as the inverse square of the distance from the earth's center.
- E) The force of the earth on the spaceship and the force of the spaceship on the earth cancel because they are equal in magnitude but opposite in direction.

4. An airplane engine starts from rest; and 2 seconds later, it is rotating with an angular speed of 420 rev/min. If the angular acceleration is constant, how many revolutions does the propeller undergo during this time?

$2\text{ s} = \frac{1}{30}\text{ min}$ (OR convert all angular velocities to rev/s or rad/s)

$$\begin{aligned}\theta &= \frac{1}{2}(\omega + \omega_0)t \\ &= \frac{1}{2}(420\text{ rev/min})(\frac{1}{30}\text{ min}) \\ &= 7.0\end{aligned}$$

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$$\begin{aligned}\Delta\theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \quad \text{but what is } \alpha? \\ \omega &= \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{\omega - \omega_0}{t} = \frac{420\text{ rev/min}}{(\frac{1}{30}\text{ min})} = 24,000\text{ rev/min}^2 \\ &\text{Get the same answer !}\end{aligned}$$

5. A hollow sphere of radius 0.25 m is rotating at 13 rad/s about an axis that passes through its center. The mass of the sphere is 3.8 kg. Assuming a constant net torque is applied to the sphere, how much work is required to bring the sphere to a stop?

Note: moment of inertia of a hollow sphere, $I_S = \frac{2}{3} MR^2$.

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$$\begin{aligned} W &= \Delta K & K_0 &= \frac{1}{2} I \omega_0^2, \quad K = 0 \text{ (final kinetic energy)} \\ &= \frac{1}{2} I \omega_0^2 & (W \text{ and } \Delta K \text{ are negative)} \\ &= \frac{1}{3} MR^2 \omega_0^2 = \frac{1}{3} (3.8 \text{ kg})(0.25 \text{ m})^2 (13.0 \text{ rad/s})^2 \\ &= 13.4 \text{ J} \end{aligned}$$

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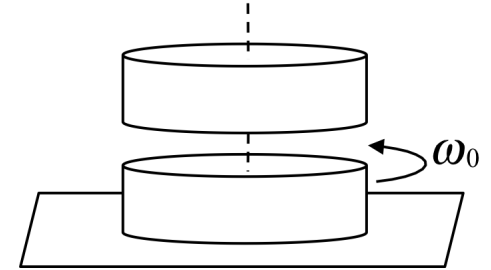
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6. A string is wrapped around a pulley of radius 0.20 m and moment of inertia 0.40 kg · m². The string is pulled generating an angular acceleration of 14.0 rad/s². With what magnitude of force was the string pulled?

$$\begin{aligned} \tau &= I\alpha & \tau &= FR \\ F &= \frac{\tau}{R} = \frac{I\alpha}{R} = \frac{0.40 \text{ kg} \cdot \text{m}^2 (14 \text{ rad/s}^2)}{(0.20 \text{ m})} = 28 \text{ N} \end{aligned}$$

7. A solid disk with a mass of 0.50 kg is rotating on a frictionless surface with an angular speed of 15.0 rad/s. Another disk just above the first with the same radius is dropped onto the lower disk. Kinetic friction between the disks brings both disks to a common angular speed of 3.00 rad/s. What is the mass of the top disk?

No external torques – angular momentum conserved

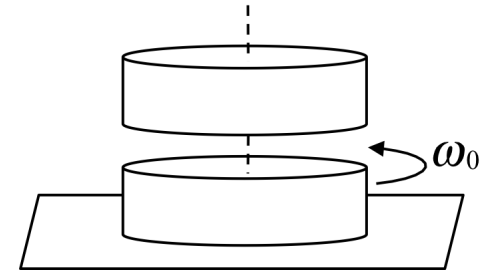


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Only mass changes, not the shape.

Moment of Inertia changes proportional to mass

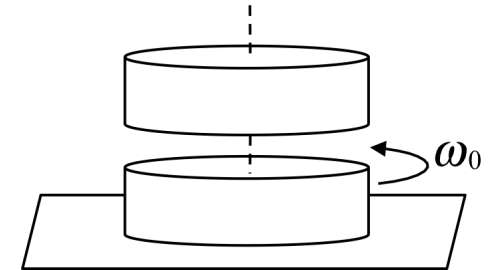


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$$\text{initial: } L_0 = I_0 \omega_0 \quad \text{final: } L = (I_0 + 2I_0) \omega$$

$$\text{angular momentum conservation: } L_0 = L$$

$$I_0 \omega_0 = (I_0 + I_{\text{Top}}) \omega$$

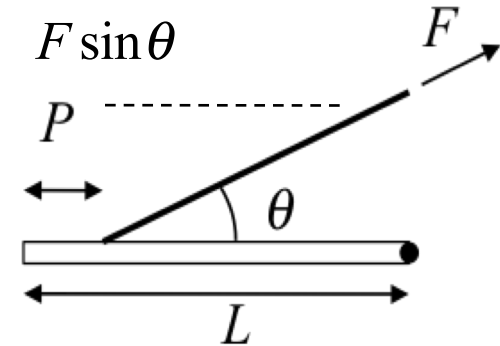
$$I_{\text{Top}} = \frac{I_0(\omega_0 - \omega)}{\omega} = I_0 \frac{15 - 3}{3} = 4I_0$$

$$M_{\text{Top}} = 4M_0 = 2.00 \text{ kg}$$

8. A board in equilibrium has a mass of 23.0 kg, length 25.0 m is pivoted about one end. A rope tied a distance of 5.00 m from the other end makes an angle of 15° with the board. What is the magnitude of the force F ?

Equilibrium – torques must sum to zero

$$\tau = rF \sin \theta \quad (+ \text{ cntr-clockwise})(- \text{ clockwise})$$



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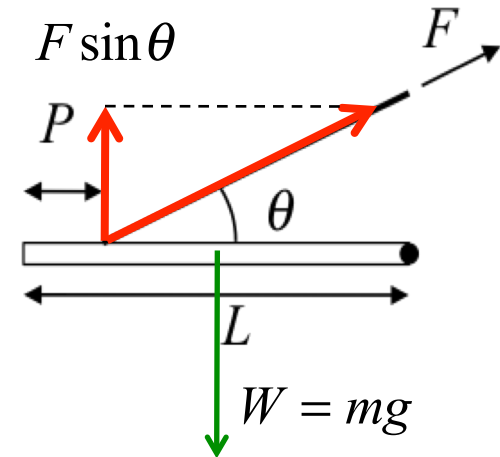
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$$\vec{\tau}_{\text{Net}} = \vec{\tau}_1 + \vec{\tau}_2 = 0 \quad \Rightarrow \quad \vec{\tau}_1 = -\vec{\tau}_2$$

$$\vec{\tau}_1 = (L/2)mg \quad (+ \text{ cntr-clockwise})$$

$$\vec{\tau}_2 = -(L - P)F \sin \theta \quad (- \text{ clockwise})$$



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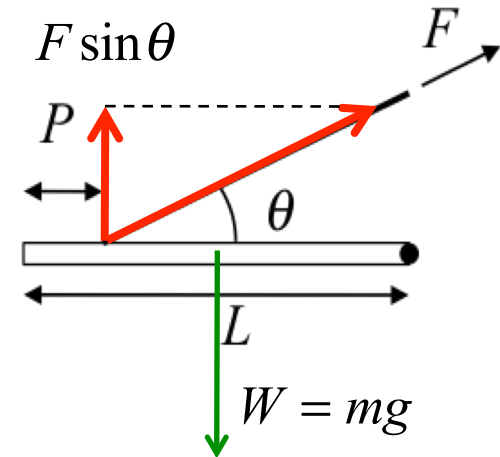
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$$\vec{\tau}_1 = -\vec{\tau}_2$$

$$= (L - P)F \sin \theta$$

$$F = \frac{(L/2)mg}{(L - P)\sin \theta} = \frac{(12.5\text{m})23.0 \text{ kg}(9.81\text{N/kg})}{(20\text{m})\sin(15^\circ)} = 545\text{N}$$

9. The maximum compression stress that a bone can withstand is $1.60 \times 10^8 \text{ N/m}^2$ before it breaks. A thighbone (femur), which is the largest and longest bone in the human body, has a cross sectional area of $7.70 \times 10^{-4} \text{ m}^2$. What is the maximum compression force that can be applied to the thighbone?

$$\frac{F}{A} < 1.6 \times 10^8 \text{ N/m}^2$$

$$F < (1.6 \times 10^8 \text{ N/m}^2)(7.70 \times 10^{-4} \text{ m}^2) = 1.23 \times 10^5 \text{ N}$$

10. A force of 250 N is applied to a hydraulic jack piston that is 0.02 m in diameter. A mass of 1400 kg can be lifted by the jack. Ignoring any difference in height between the pistons, the piston that supports the load has what diameter?

Hydraulic pressure
on pistons the same

Area in terms
of diameter

$$A_1 = \frac{\pi}{4} d_1^2$$

$$A_2 = \frac{\pi}{4} d_2^2$$

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = \left(\frac{A_2}{A_1} \right) F_1$$

9. Young's modulus of nylon is $3.70 \times 10^9 \text{ N/m}^2$. A force of $6.00 \times 10^5 \text{ N}$ is applied to a 1.50-m length of nylon of cross sectional area 0.250 m^2 . By what amount does the nylon stretch?

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$

$$\Delta L = \frac{FL}{YA} = \frac{(6.00 \times 10^5 \text{ N})(1.5 \text{ m})}{(3.70 \times 10^9 \text{ Pa})(0.25 \text{ m}^2)} = 0.000973 \text{ m}$$

$$= 0.973 \text{ mm}$$

10. A force of 250 N is applied to a hydraulic jack piston that is 0.02 m in diameter. If the piston that supports the load has a diameter of 0.15 m, approximately how much mass can be lifted by the jack? Ignore any difference in height between the pistons.

Hydraulic pressure
on pistons the same

$$P_1 = P_2$$

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Area in terms
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$$A_1 = \frac{\pi}{4} d_1^2$$

$$A_2 = \frac{\pi}{4} d_2^2$$

$$d_2^2 = \left(\frac{F_2}{F_1} \right) d_1^2$$

$$m = \frac{F_2}{g} = \frac{d_2^2}{d_1^2} \frac{F_1}{g} = \frac{(0.15)^2}{(0.02)^2} \frac{250 \text{ N}}{9.81 \text{ N/kg}} = 1400 \text{ kg}$$

11. A balloon inflated with a gas (density = 0.5 kg/m^3) has a volume of $6.00 \times 10^{-3} \text{ m}^3$. If the buoyant force (F_B) exerted on the balloon is 0.0765 N , what is the density of the air?

$$\begin{aligned} F_B &= \rho_f V g \\ \rho_f &= \frac{F_B}{V g} = \frac{7.65 \times 10^{-2} \text{ N}}{(6.00 \times 10^{-3} \text{ m}^3)(9.81 \text{ N/kg})} \\ &= 1.30 \text{ kg/m}^3 \end{aligned}$$

12. You want to fill a 0.0189-m^3 bucket with water to wash your car. Water exits a hose with a speed of 0.61 m/s . The radius of the faucet is 0.0078 m . How long does it take to fill the bucket completely?

$$\begin{aligned} V &= A v t = \pi r^2 v t \\ t &= \frac{V}{\pi r^2 v} = \frac{0.0189 \text{ m}^3}{\pi (0.0078 \text{ m})^2 (0.61 \text{ m/s})} = 162 \text{ s} \end{aligned}$$

13. In level flight of an airplane weighing $1.92 \times 10^4 \text{ N}$, the air speed over the top of each wing is 62.0 m/s and the air speed beneath each wing is 54.0 m/s . If the density of the air at this altitude is 1.29 kg/m^3 , what is the surface area of *each* wing of the airplane?

Bernoulli's Equation: $P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$

$$\frac{F_{\text{Net}}}{A_{2\text{wings}}} = P_2 - P_1 = \frac{1}{2} \rho (v_1^2 - v_2^2)$$

$$\frac{1}{2} A_{2\text{wings}} = A_{\text{wing}} = \frac{F_{\text{Net}}}{\rho (v_1^2 - v_2^2)} = \frac{1.92 \times 10^4 \text{ N}}{(1.29 \text{ kg/m}^3)(62^2 - 54^2)(\text{m}^2/\text{s}^2)} = 16 \text{ m}^2$$

14. Steel has a Young's modulus $2.00 \times 10^{11} \text{ N/m}^2$ and a coefficient of thermal expansion that is $12.0 \times 10^{-6} (\text{°C})^{-1}$. A steel beam at 10 °C is constrained to a length of 2.50 m . A pressure of $7.20 \times 10^7 \text{ N/m}^2$ is generated at each end of the beam at what temperature?

Stress-Strain: $F = YA \frac{\Delta L}{L}$ Thermal Expansion: $\Delta L = \alpha L \Delta T$

$$P = \frac{F}{A} = Y \alpha \Delta T$$

$$\Delta T = \frac{P}{Y \alpha} = \frac{7.20 \times 10^7 \text{ N/m}^2}{(2.00 \times 10^{11} \text{ N/m}^2)(12.0 \times 10^{-6} \text{ °C}^{-1})}$$

$$= 30 \text{ °C} \quad T = T_0 + \Delta T = (10 + 30) \text{ °C} = 40 \text{ °C}$$

15. How many molecules are in 0.088 kg of carbon dioxide, CO₂? (atomic masses: C = 12 u; O = 16 u).

$$\begin{aligned}
 N &= N_A n = N_A \frac{m(\text{in g})}{m(\text{in u})} \quad \text{u = atomic mass unit} \\
 &= (6.02 \times 10^{23})(88) / (44\text{u}) \\
 &= 1.20 \times 10^{24}
 \end{aligned}$$

6. Helium atoms at 450 K have an RMS speed of 1675 m/s. What is the speed of the helium atoms if the temperature is raised to 900 K?

Average Kinetic Energy: $\bar{K} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T$

$\frac{1}{2} m \overline{v_1^2} = \frac{3}{2} k_B T_1 \quad \frac{1}{2} m \overline{v_2^2} = \frac{3}{2} k_B T_2 \quad v_{RMS} = \sqrt{\overline{v^2}}$

$v_{RMS2} = v_{RMS1} \sqrt{\frac{T_2}{T_1}} = 1675 \text{ m/s} \sqrt{\frac{900}{450}} = 2370 \text{ m/s}$

Chapter 13

The Transfer of Heat

continued
RADIATION

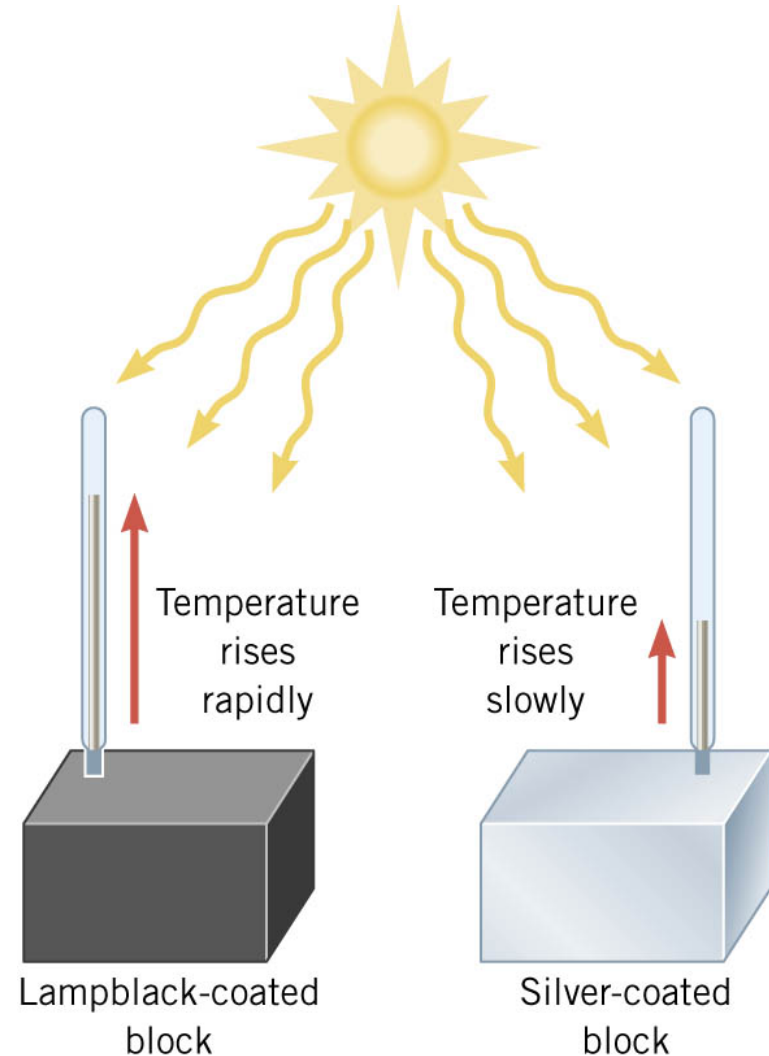
13.3 Radiation

RADIATION

Radiation is the process in which energy is transferred by means of electromagnetic waves.

A material that is a good absorber is also a good emitter.

A material that absorbs completely is called a ***perfect blackbody***.



13.3 Radiation

THE STEFAN-BOLTZMANN LAW OF RADIATION

The radiant energy Q , emitted in a time t by an object that has a Kelvin temperature T , a surface area A , and an emissivity e , is given by

$$Q = e\sigma T^4 At$$

emissivity $e = \text{constant between } 0 \text{ to } 1$
 $e = 1$ (perfect black body emitter)

Stefan-Boltzmann constant
 $\sigma = 5.67 \times 10^{-8} \text{ J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4)$

Example A Supergiant Star

The supergiant star Betelgeuse has a surface temperature of about 2900 K and emits a power of approximately $4 \times 10^{30} \text{ W}$. Assuming Betelgeuse is a perfect emitter and spherical, find its radius.

$$\text{power, } P = \frac{Q}{t}$$

with $A = 4\pi r^2$ (surface area of sphere with radius r)

$$\begin{aligned} r &= \sqrt{\frac{Q/t}{4\pi e\sigma T^4}} = \sqrt{\frac{4 \times 10^{30} \text{ W}}{4\pi(1)[5.67 \times 10^{-8} \text{ J}/(\text{s} \cdot \text{m}^2 \cdot \text{K}^4)](2900 \text{ K})^4}} \\ &= 3 \times 10^{11} \text{ m} \end{aligned}$$

Chapter 14

Thermodynamics

14.1 The First Law of Thermodynamics

THE FIRST LAW OF THERMODYNAMICS

The internal energy of a system changes due to heat and work:

$$\Delta U = U_f - U_i = Q + W$$

$Q > 0$ system gains heat

$W > 0$ if work done on the system

The internal energy (U) of an Ideal Gas depends only on the temperature:

$$\text{Ideal Gas (only): } U = \frac{3}{2} nRT \text{ or } U = \frac{3}{2} Nk_B T$$

$$\begin{aligned} \Delta U &= U_f - U_i \\ &= \frac{3}{2} nR(T_f - T_i) \end{aligned}$$

Otherwise, values for both Q and W are needed to determine ΔU

Clicker Question 14.1

An insulated container is filled with a mixture of water and ice at zero °C. An electric heating element inside the container is used to add 1680 J of heat to the system while a paddle does 450 J of work by stirring. What is the increase in the internal energy of the ice-water system?

$$\Delta U = Q + W$$

- a) 450 J
- b) 1230 J
- c) 1680 J
- d) 2130 J
- e) zero J

14.1 Thermal Processes

Work done on a gas

$$(\Delta P = 0) \text{ *isobaric*: constant pressure: } W = -P\Delta V$$

$$(\Delta V = 0) \text{ *isochoric*: constant volume: } W = -P\Delta V = 0$$

For an Ideal Gas only

$$(\Delta T = 0) \text{ *isothermal*: constant temperature: } W = nRT \ln(V_i/V_f)$$

$$(Q = 0) \text{ *adiabatic*: no transfer of heat: } W = \frac{3}{2} nR(T_f - T_i)$$

14.2 Thermal Processes

An **isobaric** process is one that occurs slowly at **constant pressure**.

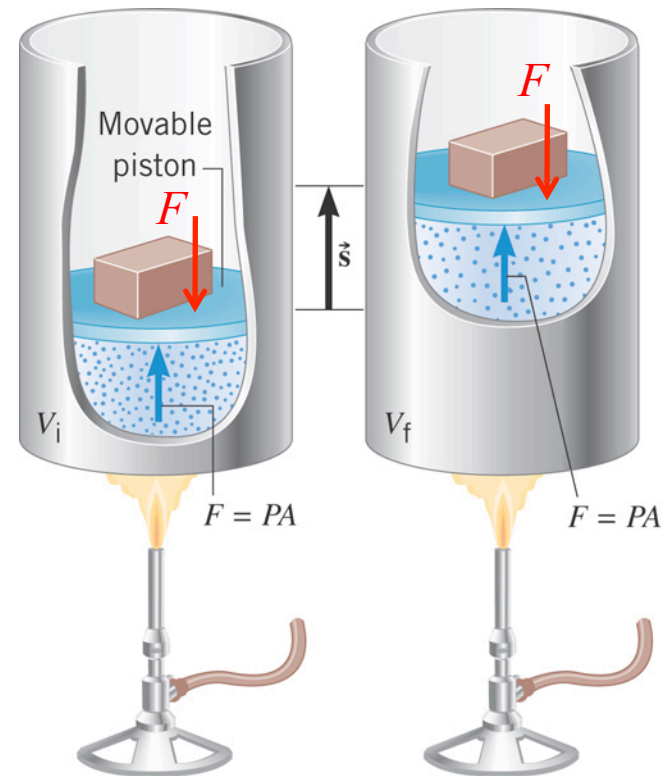
$$\cos\theta = +1$$

$$\cos\theta = -1$$

If piston is pushed down by mass, $W_{\text{on gas}} > 0$.

If piston is pushed upward by pressure, $W_{\text{on gas}} < 0$

$$\begin{aligned} W &= Fs \cos\theta = -P(As) \\ &= -P\Delta V \\ &= -P(V_f - V_i) \end{aligned}$$

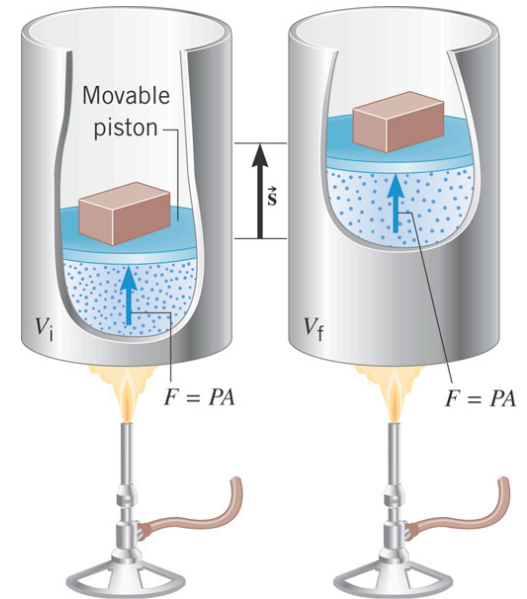


14.2 Thermal Processes

Example Isobaric Expansion of Water (Liquid)

One gram of water is placed in the cylinder and the pressure is maintained at $2.0 \times 10^5 \text{ Pa}$. The temperature of the water is raised by 31°C . The water is in the liquid phase and expands by a very small amount, $1.0 \times 10^{-8} \text{ m}^3$.

Find the work done and the change in internal energy.



$$W = -P\Delta V$$
$$= -(2.0 \times 10^5 \text{ Pa})(1.0 \times 10^{-8} \text{ m}^3) = -0.0020 \text{ J}$$

$$\text{Liquid water } \Delta V \sim 0$$

$$Q = mc\Delta T$$
$$= (0.0010 \text{ kg})[4186 \text{ J}/(\text{kg} \cdot ^\circ\text{C})](31 ^\circ\text{C}) = 130 \text{ J}$$

$$c_{\text{water}} = 4186 \text{ J}/(\text{kg} \cdot ^\circ\text{C})$$

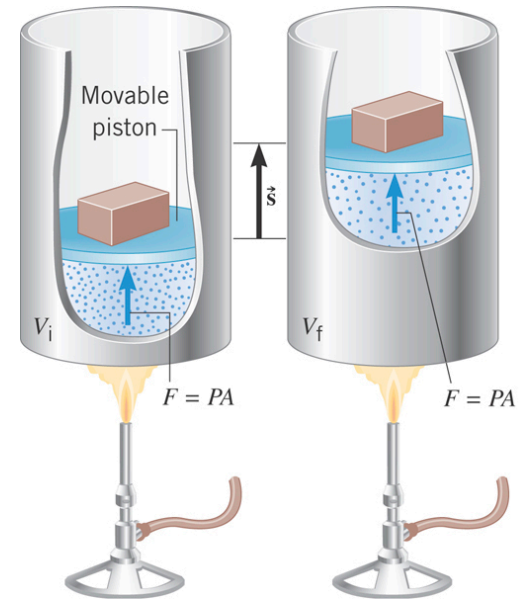
$$\Delta U = Q - W = 130 \text{ J} - 0.0020 \text{ J} = 130 \text{ J}$$

14.2 Thermal Processes

Example Isobaric Expansion of Water (Vapor)

One gram of water vapor is placed in the cylinder and the pressure is maintained at $2.0 \times 10^5 \text{ Pa}$. The **temperature of the vapor is raised by 31°C** , and the gas expands by $7.1 \times 10^{-5} \text{ m}^3$. Heat capacity of the gas is $2020 \text{ J}/(\text{kg}\cdot^\circ\text{C})$.

Find the work done and the change in internal energy.



$$W = -P\Delta V = -(2.0 \times 10^5 \text{ Pa})(7.1 \times 10^{-5} \text{ m}^3) \\ = -14.2 \text{ J}$$

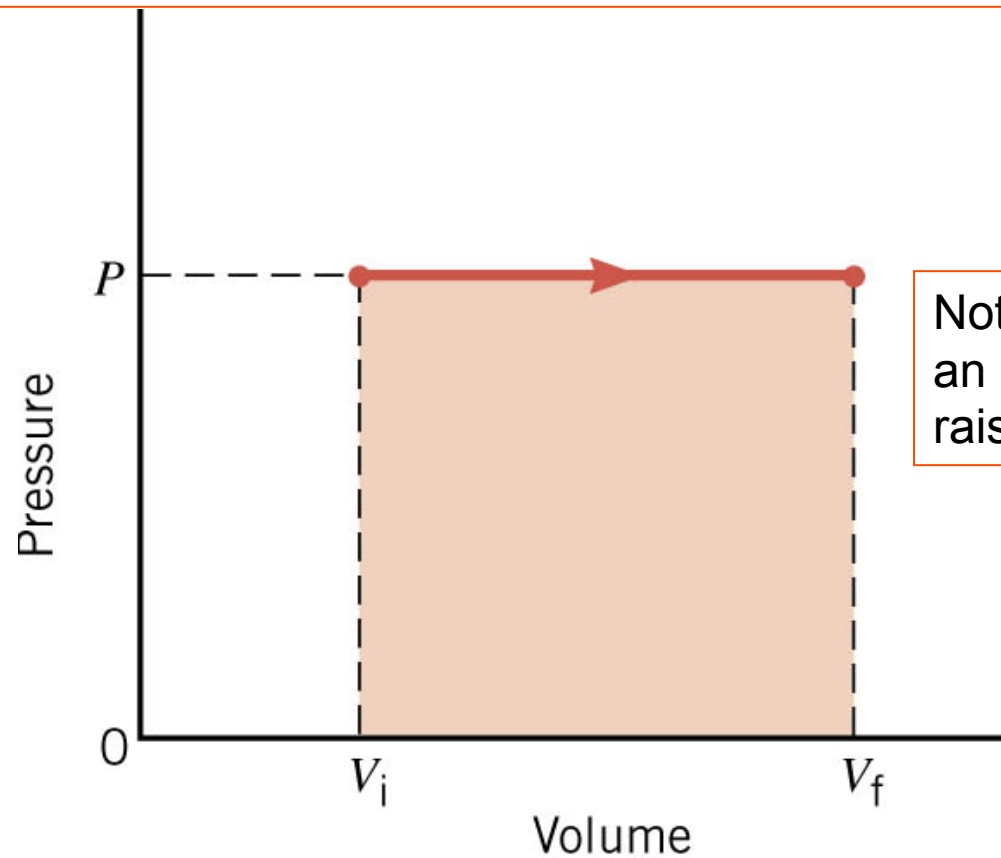
$$Q = mc\Delta T \\ = (0.0010 \text{ kg}) \left[2020 \text{ J}/(\text{kg} \cdot ^\circ\text{C}) \right] (31 ^\circ\text{C}) = 63 \text{ J}$$

$$\Delta U = Q + W = 63 \text{ J} + (-14 \text{ J}) = 49 \text{ J}$$

14.2 Thermal Processes

$$W = -P\Delta V = -P(V_f - V_i)$$

The work done on a gas at constant pressure - the work done is (minus) the area under a P-V diagram.



Note: the temperature of an ideal gas must be raised to do this.

Clicker Question 14.2

An ideal gas at a constant pressure of 1×10^5 Pa is reduced in volume from 1.00 m^3 to 0.25 m^3 . What work was done on the gas?

$$W = -P\Delta V$$

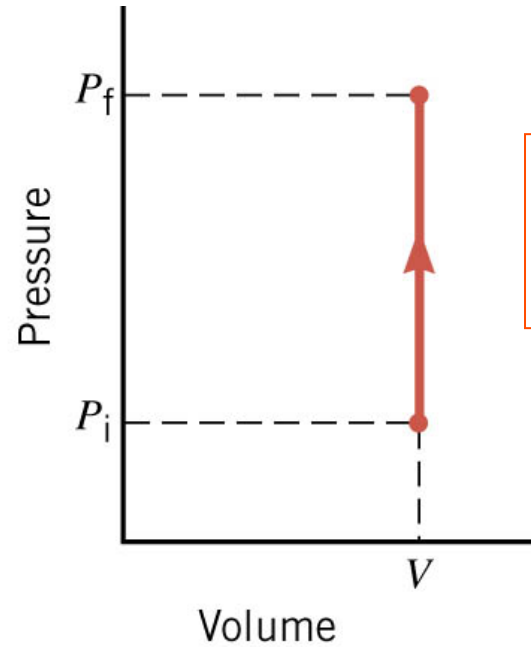
- a) zero J
- b) 0.25×10^5 J
- c) 0.50×10^5 J
- d) 0.75×10^5 J
- e) 4.00×10^5 J

14.2 Thermal Processes

isochoric: constant volume



(a)



(b)

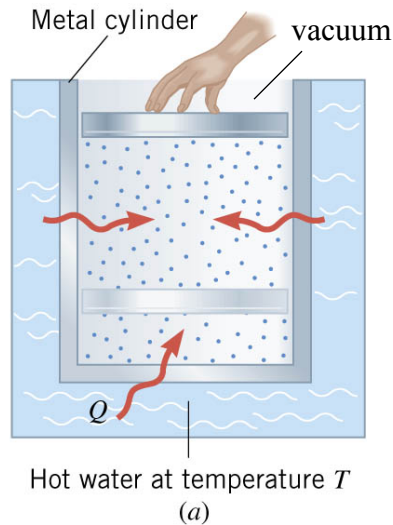
The work done at constant volume is the area under a P-V diagram. The area is **zero!**

$$W = 0$$

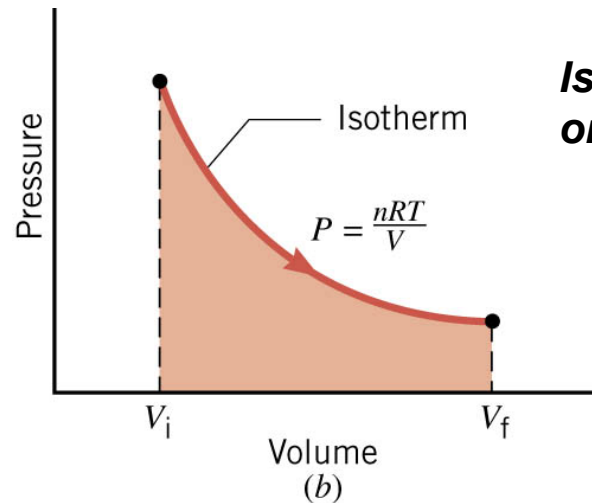
$$\Delta U = Q + W = Q$$

Change in internal energy is equal to the heat added.

14.2 Thermal Processes Using and Ideal Gas



ISOTHERMAL EXPANSION OR COMPRESSION



***Isothermal expansion
or compression of an ideal gas***

$$W_{\text{on gas}} = nRT \ln\left(\frac{V_i}{V_f}\right)$$

Example 5 Isothermal Expansion of an Ideal Gas

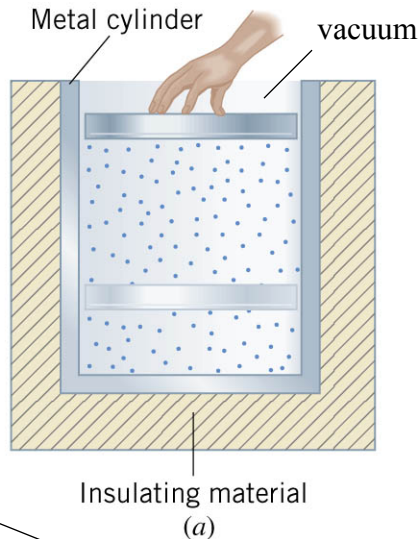
Two moles of argon (ideal gas) expand isothermally at 298K, from initial volume of 0.025m³ to a final volume of 0.050m³. Find (a) the work done by the gas, (b) change in gas internal energy, and (c) the heat supplied.

$$\begin{aligned} \text{a) } W_{\text{on gas}} &= nRT \ln(V_i/V_f) \\ &= (2.0 \text{ mol})(8.31 \text{ J}/(\text{mol} \cdot \text{K}))(298 \text{ K}) \ln\left(\frac{0.025}{0.050}\right) \\ &= -3400 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{b) } \Delta U &= U_f - U_i = \frac{3}{2} nR\Delta T \\ \Delta T &= 0 \text{ therefore } \Delta U = 0 \end{aligned}$$

$$\begin{aligned} \text{c) } \Delta U &= Q + W = 0 \\ Q &= -W = 3400 \text{ J} \end{aligned}$$

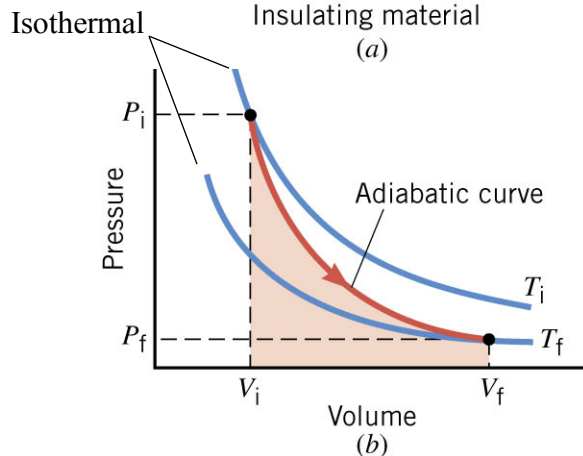
14.2 Thermal Processes Using and Ideal Gas



ADIABATIC EXPANSION OR COMPRESSION

Adiabatic expansion or compression of a monatomic ideal gas

$$W_{\text{on gas}} = \frac{3}{2} nR(T_i - T_f)$$



Adiabatic expansion or compression of a monatomic ideal gas

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$\gamma = c_P / c_V$$

Ratio of heat capacity at constant P over heat capacity at constant V .

These are needed to understand basic operation of refrigerators and engines

ADIABATIC EXPANSION OR COMPRESSION

ISOTHERMAL EXPANSION OR COMPRESSION

14.2 Specific Heat Capacities

To relate heat and temperature change in **solids and liquids (mass in kg)**, use:

$$\boxed{Q = mc\Delta T} \quad \text{specific heat capacity, } c \quad \left[\text{J}/(\text{kg} \cdot ^\circ\text{C}) \right]$$

For gases, the amount of gas is given in moles, use molar heat capacities:

$$\boxed{Q = nC\Delta T} \quad \text{molar heat capacity, } C \quad \left[\text{J}/(\text{mole} \cdot ^\circ\text{C}) \right]$$

$$\boxed{C = (m/n)c = m_u c; \quad m_u = \text{mass/mole (kg)}}$$

ALSO, for gases it is necessary to distinguish between the molar specific heat capacities at constant pressure and at constant volume:

$$C_P, C_V$$

14.2 Specific Heat Capacities

$$\text{Ideal Gas: } PV = nRT; \quad \Delta U = \frac{3}{2} nR\Delta T$$

$$\text{1st Law of Thermodynamics: } \Delta U = Q + W_{\text{on gas}}$$

**Constant pressure
for a monatomic ideal gas**

$$\text{Constant Pressure } (\Delta P = 0)$$

$$W_P = -P\Delta V = -nR\Delta T$$

$$Q_P = \Delta U - W = \frac{3}{2} nR\Delta T + nR\Delta T = \frac{5}{2} nR\Delta T$$

$$Q_P = nC_P\Delta T$$

$$C_P = \frac{5}{2} R$$

**Constant volume
for a monatomic ideal gas**

$$\text{Constant Volume } (\Delta V = 0)$$

$$W_V = -P\Delta V = 0$$

$$Q_V = \Delta U - W = \frac{3}{2} nR\Delta T = \frac{3}{2} nR\Delta T$$

$$Q_V = nC_V\Delta T$$

$$C_V = \frac{3}{2} R$$

**monatomic
ideal gas**

$$\gamma = C_P / C_V = \frac{5}{2} R / \frac{3}{2} R = 5/3$$

any ideal gas

$$C_P - C_V = R$$

14.3 *The Second Law of Thermodynamics*

The second law is a statement about the natural tendency of heat to flow from hot to cold, whereas the first law deals with energy conservation and focuses on both heat and work.

THE SECOND LAW OF THERMODYNAMICS: THE HEAT FLOW STATEMENT

Heat flows spontaneously from a substance at a higher temperature to a substance at a lower temperature and does not flow spontaneously in the reverse direction.

14.3 Heat Engines

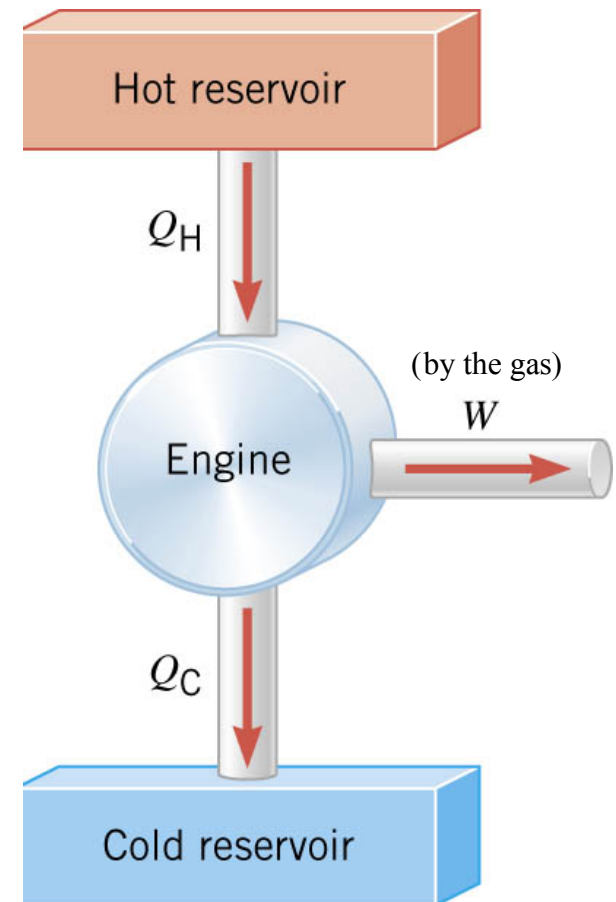
A **heat engine** is any device that uses heat to perform work. It has three essential features.

1. Heat is supplied to the engine at a relatively high temperature from a place called the *hot reservoir*.
2. Part of the input heat is used to perform work by the *working substance* of the engine.
3. The remainder of the input heat is rejected to a place called the *cold reservoir*.

$|Q_H|$ = magnitude of input heat

$|Q_C|$ = magnitude of rejected heat

$|W|$ = magnitude of the work done



14.3 Heat Engines

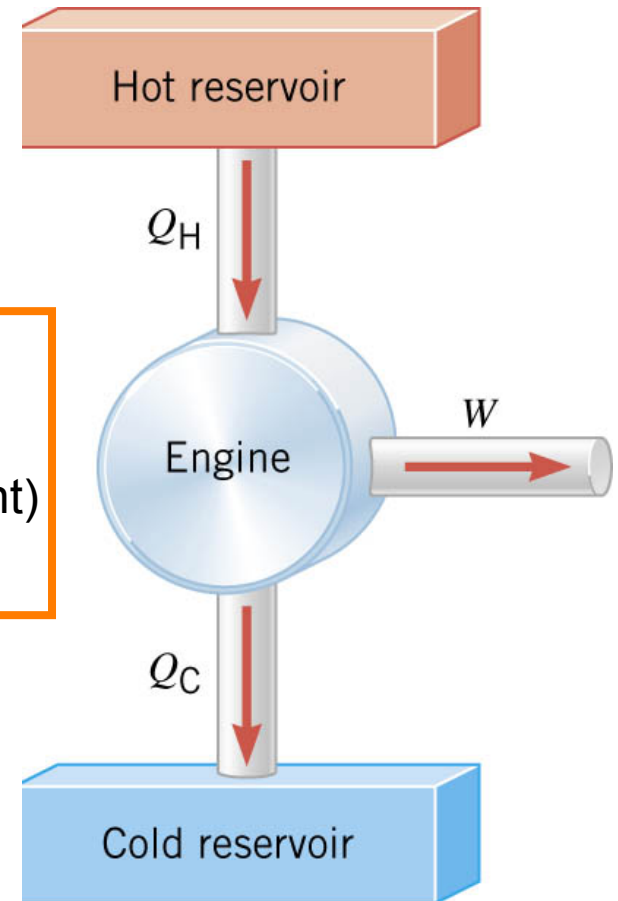
Carnot Engine Working with an Ideal Gas

1. **ISOTHERMAL** EXPANSION ($Q_{in}=Q_H$, T_{Hot} constant)
2. **ADIABATIC** EXPANSION ($Q=0$, T drops to T_{Cold})
3. **ISOTHERMAL** COMPRESSION ($Q_{out}=Q_C$, T_{Cold} constant)
4. **ADIABATIC** COMPRESSION ($Q=0$, T rises to T_{Hot})

$|Q_H|$ = magnitude of input heat

$|Q_C|$ = magnitude of rejected heat

$|W|$ = magnitude of the work done



14.3 Heat Engines

The **efficiency** of a heat engine is defined as the ratio of the work done to the input heat:

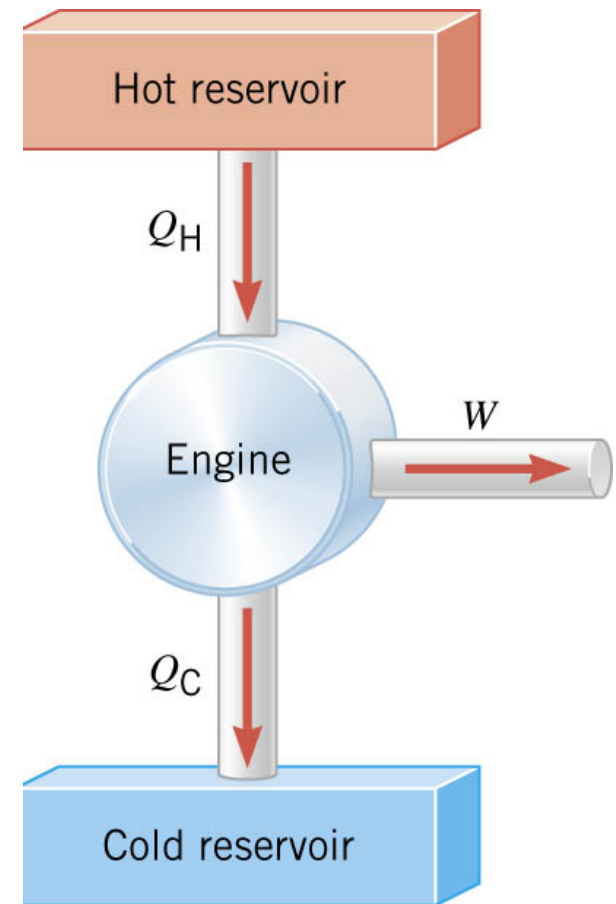
$$e = \frac{|W|}{|Q_H|}$$

If there are no other losses, then

$$|Q_H| = |W| + |Q_C|$$



$$e = 1 - \frac{|Q_C|}{|Q_H|}$$



14.3 Heat Engines

Example An Automobile Engine

An automobile engine has an efficiency of 22.0% and produces 2510 J of work. How much heat is rejected by the engine?

$$e = \frac{|W|}{|Q_H|}$$
$$= \frac{|W|}{|Q_C| + |W|} \Rightarrow e(|Q_C| + |W|) = |W|$$

$$|Q_C| = \frac{|W| - e|W|}{e} = |W| \left(\frac{1}{e} - 1 \right) = 2510 \text{ J} \left(\frac{1}{0.22} - 1 \right)$$
$$= 8900 \text{ J}$$

14.3 Carnot's Principle and the Carnot Engine

A reversible process is one in which both the system and the environment can be returned to exactly the states they were in before the process occurred.

CARNOT'S PRINCIPLE: AN ALTERNATIVE STATEMENT OF THE SECOND LAW OF THERMODYNAMICS

No irreversible engine operating between two reservoirs at constant temperatures can have a greater efficiency than a reversible engine operating between the same temperatures. Furthermore, all reversible engines operating between the same temperatures have the same efficiency.

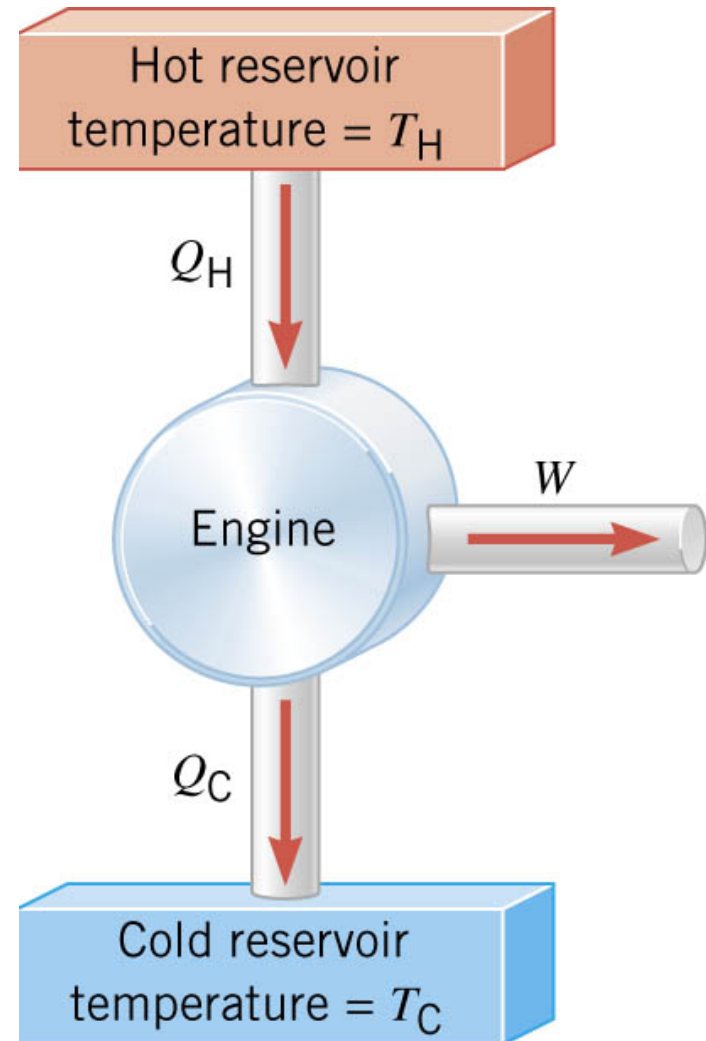
14.3 Carnot's Principle and the Carnot Engine

The **Carnot engine** is useful as an idealized model.

All of the heat input originates from a single temperature, and all the rejected heat goes into a cold reservoir at a single temperature.

Since the efficiency can only depend on the reservoir temperatures, the ratio of heats can only depend on those temperatures.

$$e = 1 - \frac{|Q_C|}{|Q_H|} = 1 - \frac{T_C}{T_H}$$



14.3 Carnot's Principle and the Carnot Engine

Example A Tropical Ocean as a Heat Engine

Surface temperature is 298.2 K, whereas 700 meters deep, the temperature is 280.2 K. Find the maximum efficiency for an engine operating between these two temperatures.

$$e_{\text{carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{280.2 \text{ K}}{298.2 \text{ K}} = 0.060$$

Maximum of only 6% efficiency.
Real life will be worse.

Conceptual Example 8 Natural Limits on the Efficiency of a Heat Engine

Consider a hypothetical engine that receives 1000 J of heat as input from a hot reservoir and delivers 1000J of work, rejecting no heat to a cold reservoir whose temperature is above 0 K. Decide whether this engine violates the first or second law of thermodynamics.

$$\text{If } T_H > T_C > 0$$

$$e_{\text{carnot}} = 1 - \frac{T_C}{T_H} \text{ must be less than } 1$$

$$e_{\text{hypothetical}} = \frac{|W|}{|Q_H|} = \frac{1000 \text{ J}}{1000 \text{ J}} = 1$$

Violates 2nd law of thermodynamics

14.3 Entropy

In general, irreversible processes cause us to lose some, but not necessarily all, of the ability to do work. This partial loss can be expressed in terms of a concept called **entropy**.

**Carnot
engine**

$$\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H} \quad \Rightarrow \quad \frac{|Q_C|}{T_C} = \frac{|Q_H|}{T_H}$$

**entropy
change**

$$\Delta S = \left(\frac{Q}{T} \right)_R$$

reversible

14.3 Entropy

Entropy, like internal energy, is a function of the state of the system.

$$\Delta S = \left(\frac{Q}{T} \right)_R$$

Consider the entropy change of a Carnot engine. The entropy of the hot reservoir decreases and the entropy of the cold reservoir increases.

$$\Delta S = +\frac{|Q_C|}{T_C} - \frac{|Q_H|}{T_H} = 0$$

Reversible processes do not alter the entropy of the universe.