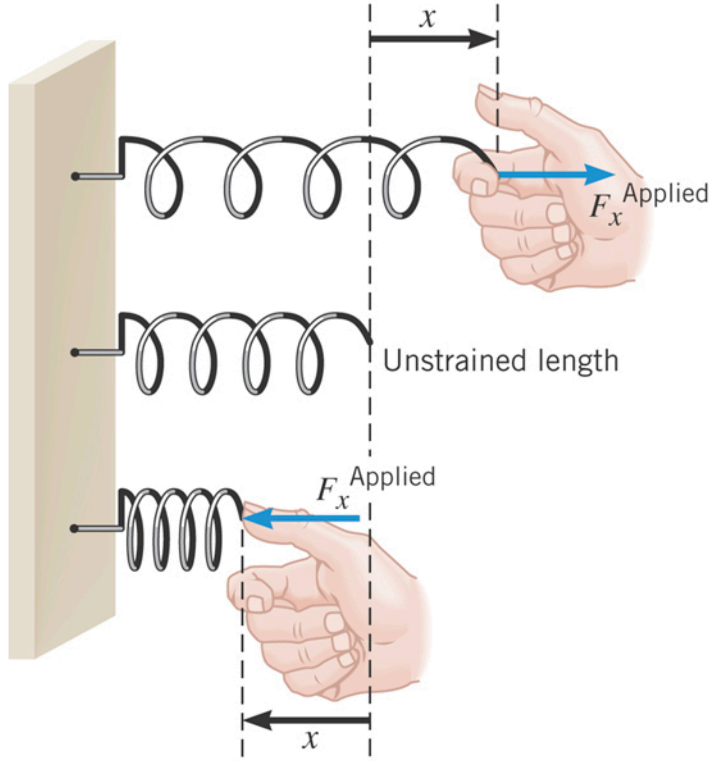
Chapter 7

Simple Harmonic Motion

5.2 The Ideal Spring and Simple Harmonic Motion



$$F_x^{Applied} = kx$$

spring constant

Units: N/m

$F_x^{Applied}$ is the applied force in the x direction

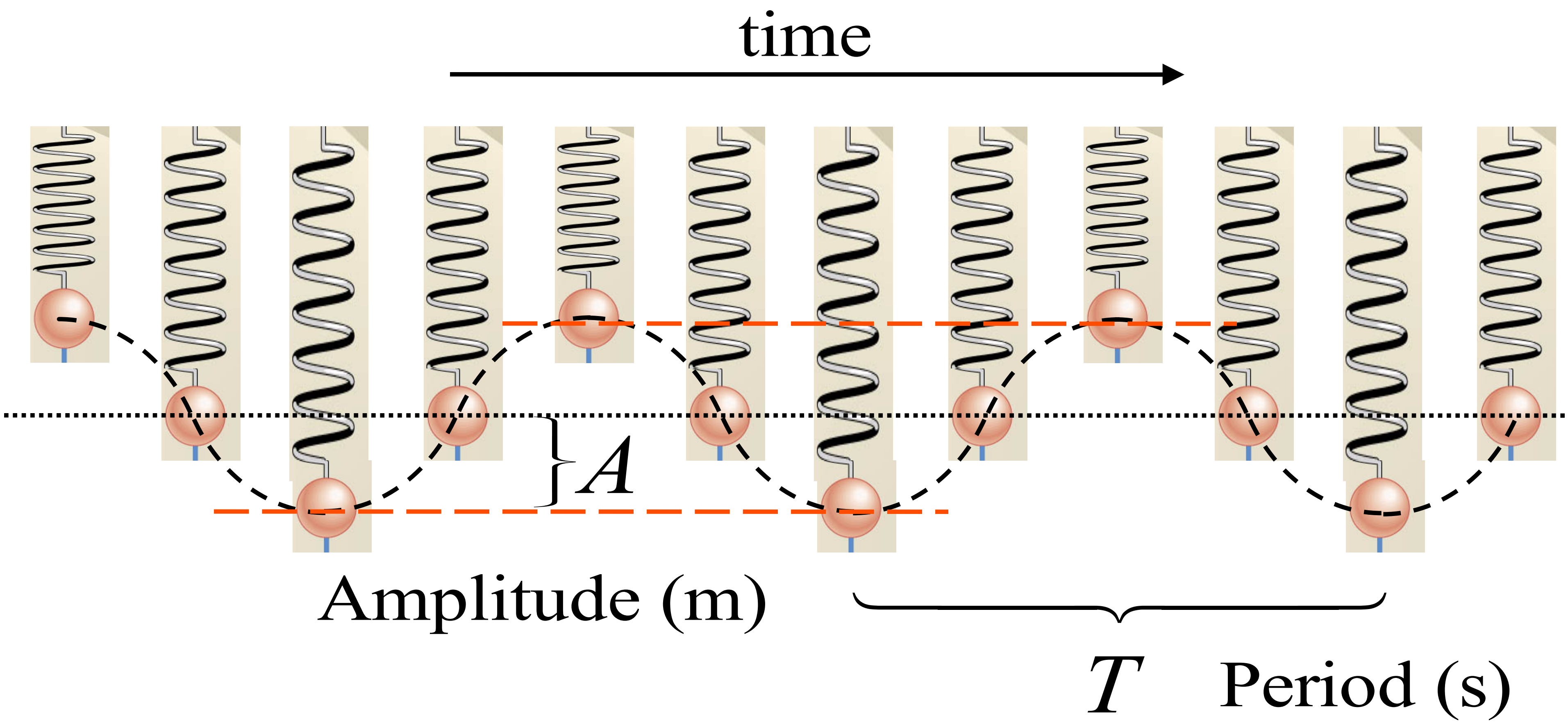
x is the spring displacement

k is the spring constant (strength of the spring)

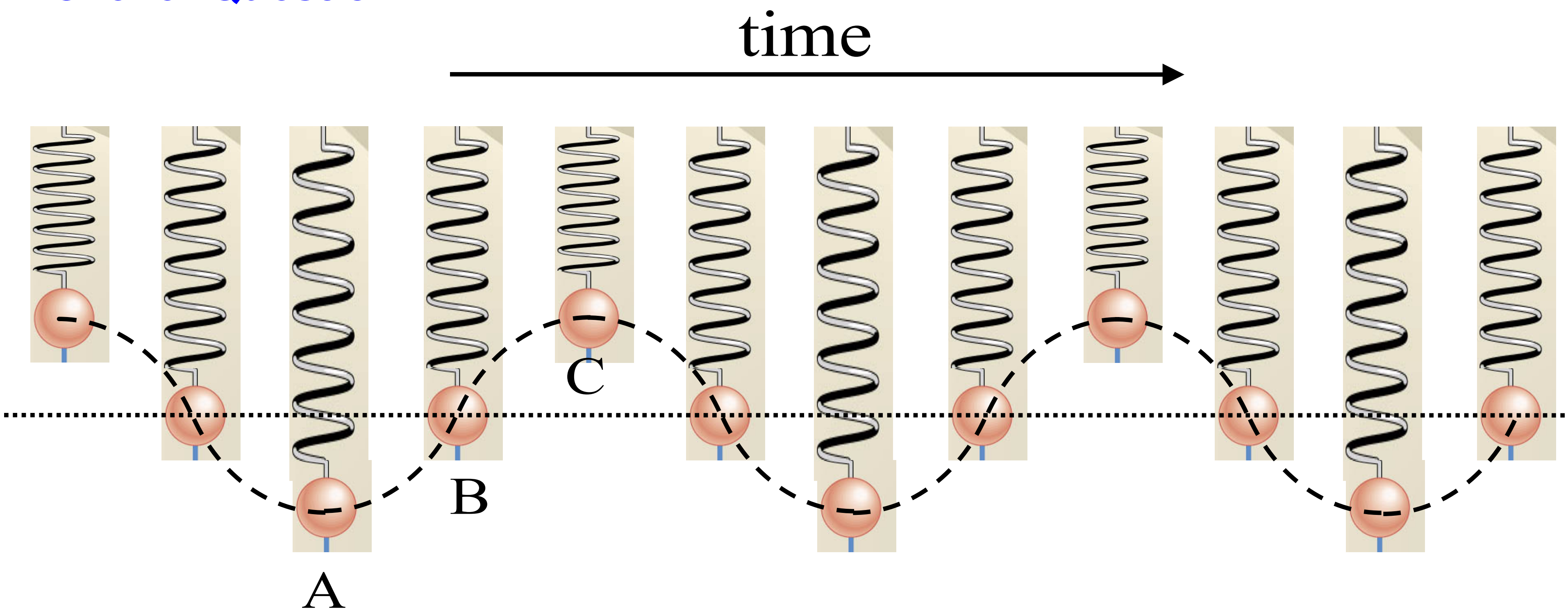
$$F_x^{Spring} = -kx$$

F_x^{Spring} is the spring's force in the x direction

Simple Harmonic Motion



Clicker Question 7.1



Where is the mass moving vertically at the highest speed?

A) only at A

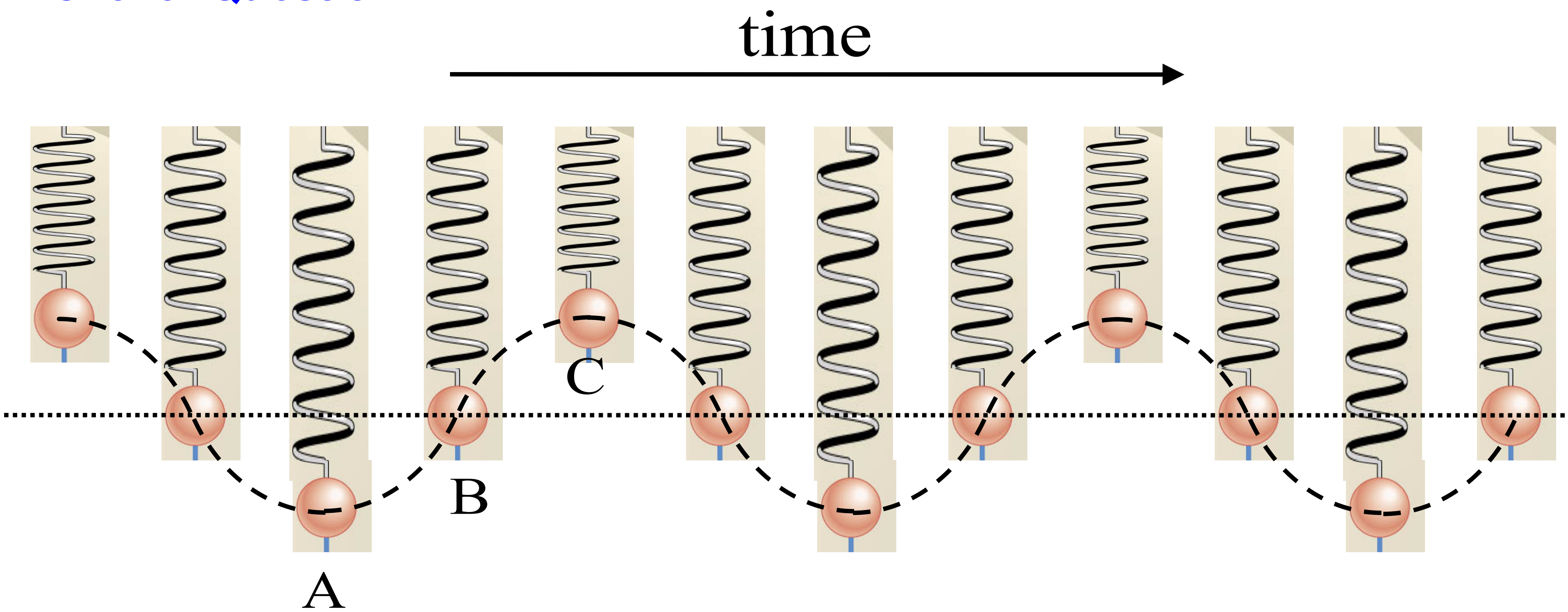
B) only at B

C) only at C

D) at both A and C

E) none of the above

Clicker Question 7.1



Where is the mass moving vertically at the highest speed?

A) only at A

B) only at B

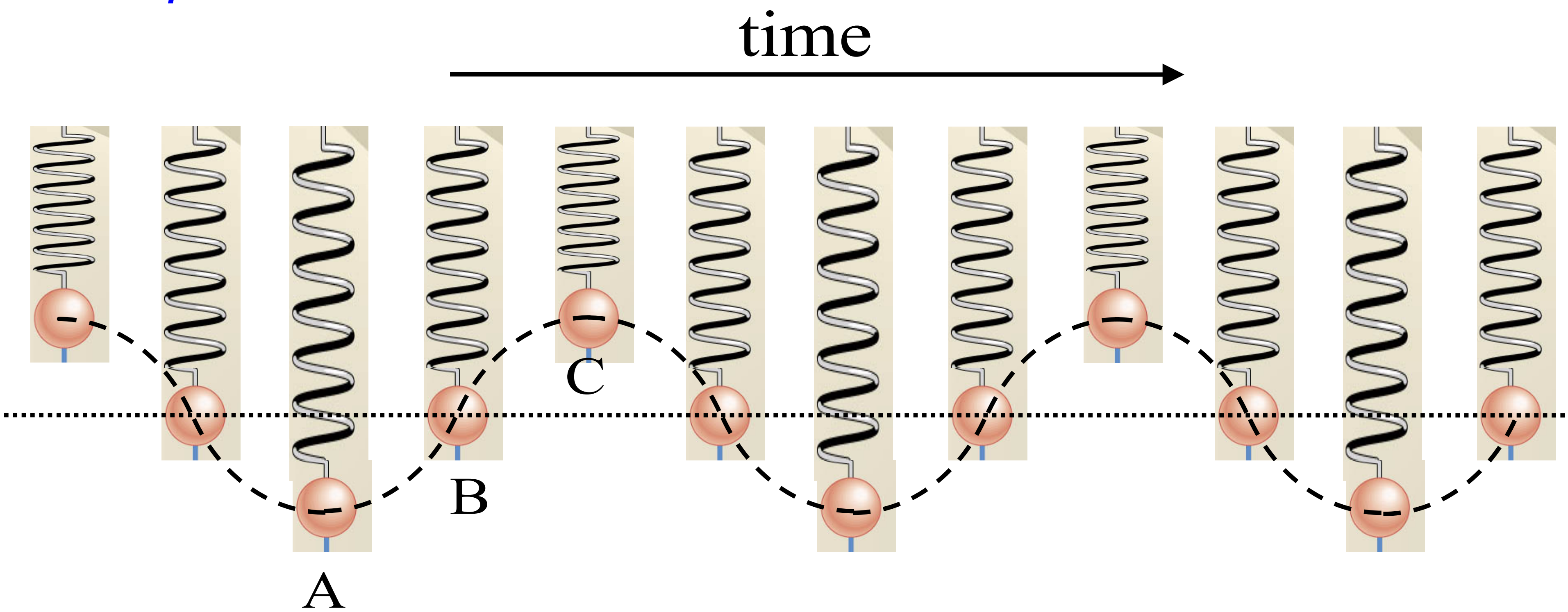
C) only at C

D) at both A and C

E) none of the above

at A and C mass is turning around
and must have a zero speed at the extreme point

7.1 Simple Harmonic Motion and the Reference Circle

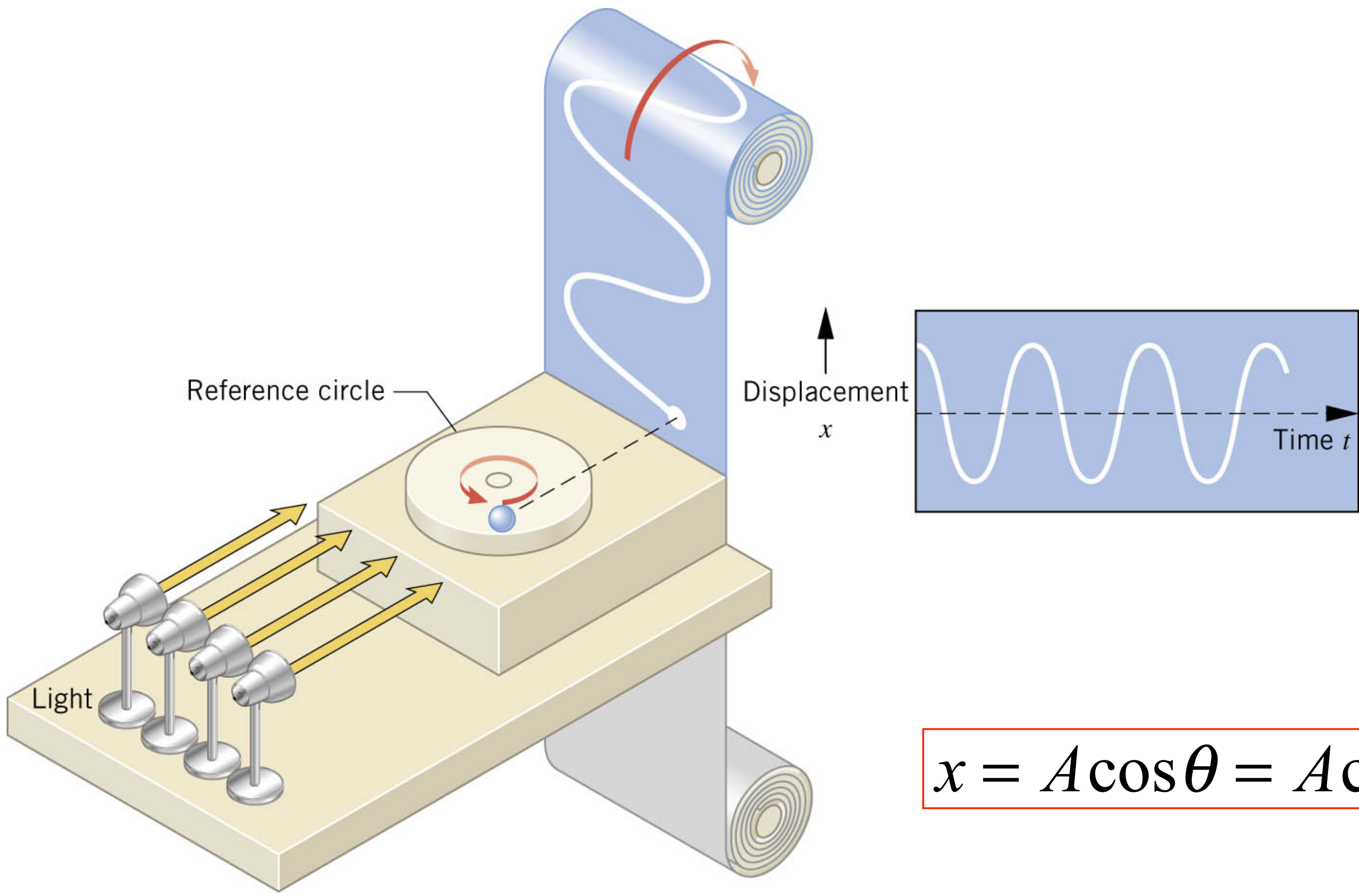


mass has the greatest magnitude of acceleration when
at points A and C - when it is turning around (speed is slowest)

acceleration vector is positive at point A and negative at C

mass has zero acceleration at point B - when speed is the greatest.

DISPLACEMENT

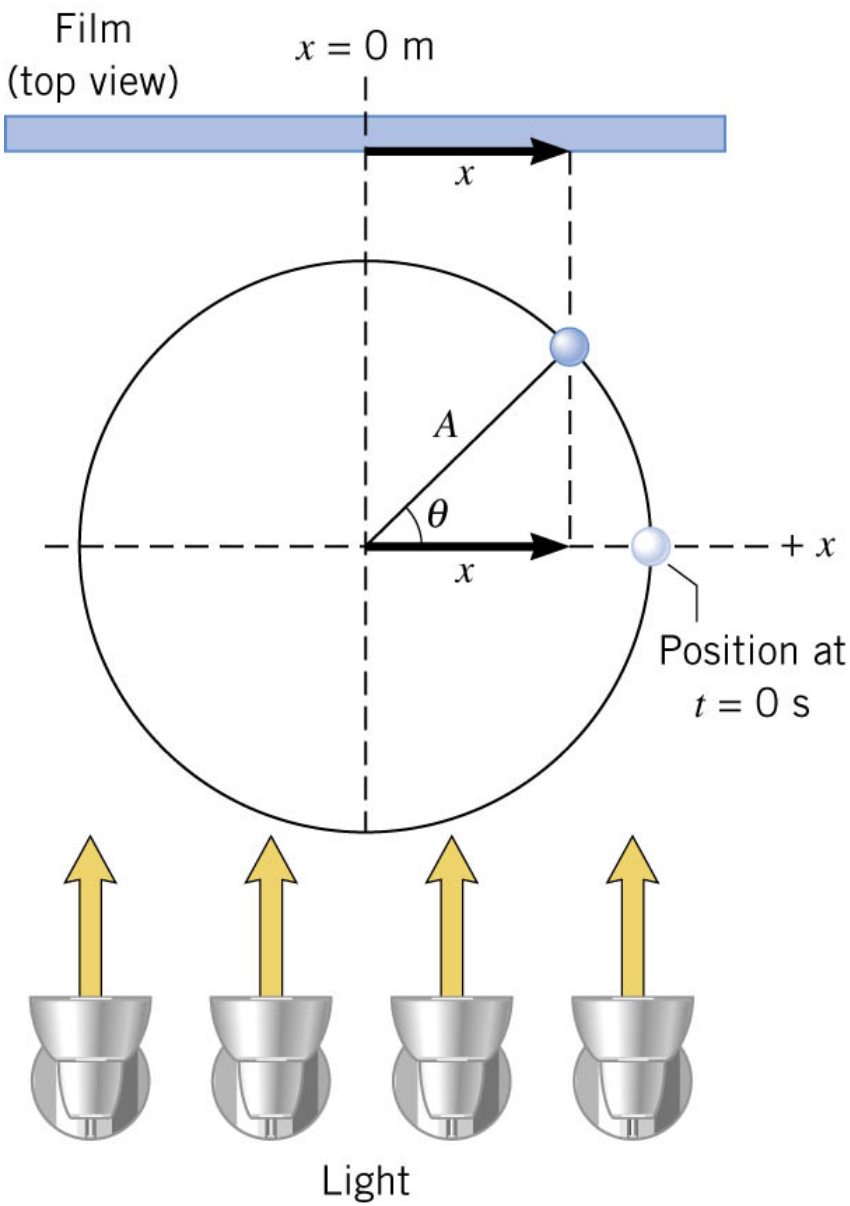


$$x = A\cos\theta = A\cos\omega t$$

Angular velocity, ω (unit: rad/s)

Angular displacement, $\theta = \omega t$ (unit: radians)

7.2 Simple Harmonic Motion and the Reference Circle



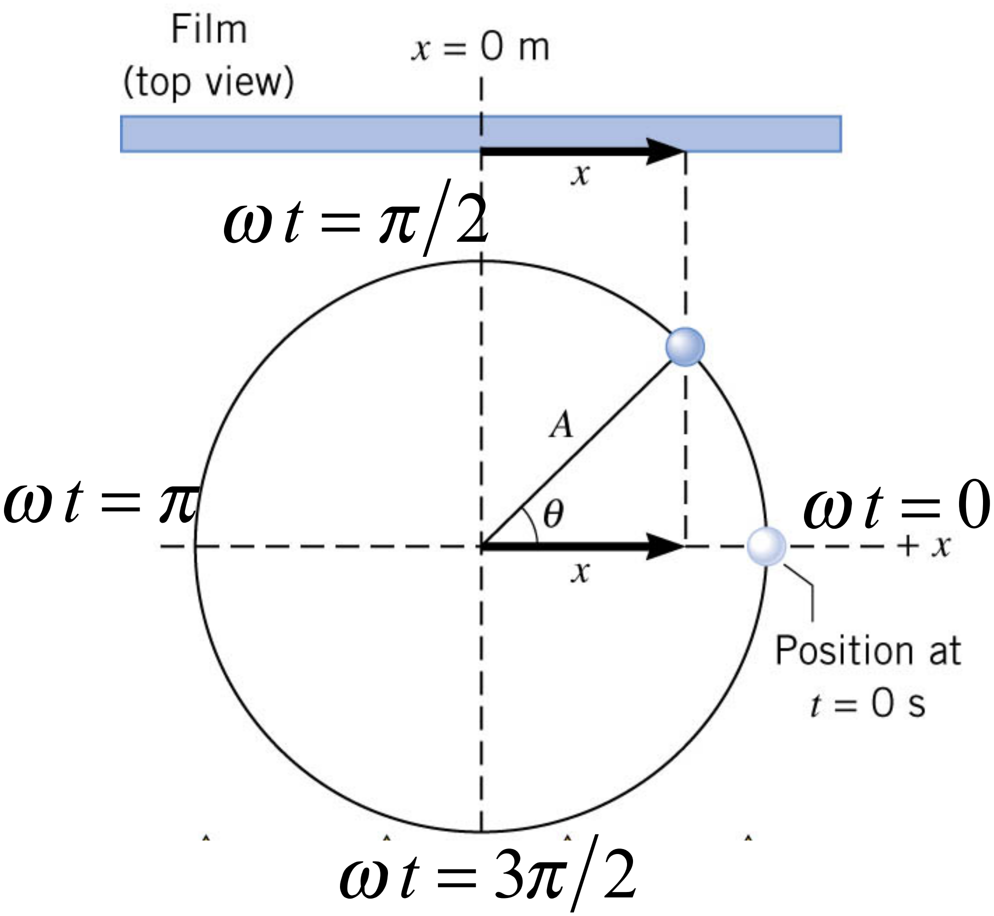
<u>uniform circular motion</u>

$$\theta = \omega t + \tfrac{1}{2}\alpha t^2 \text{ with } \alpha = 0$$

$$\theta = \omega t$$

$$x = A\cos\theta$$
$$= A\cos(\omega t)$$

7.2 Simple Harmonic Motion and the Reference Circle



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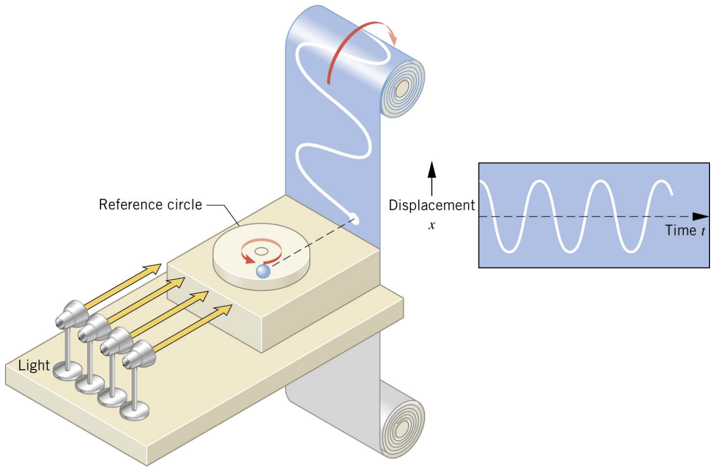
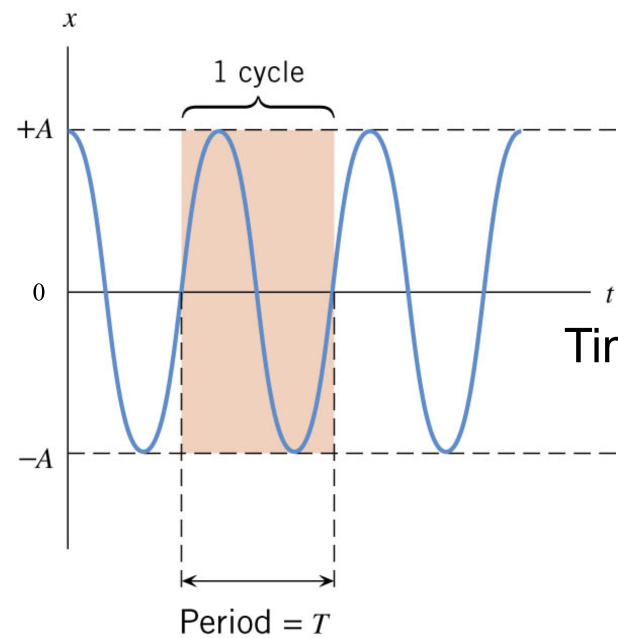


amplitude A:



Time

amplitude A: the maximum displacement

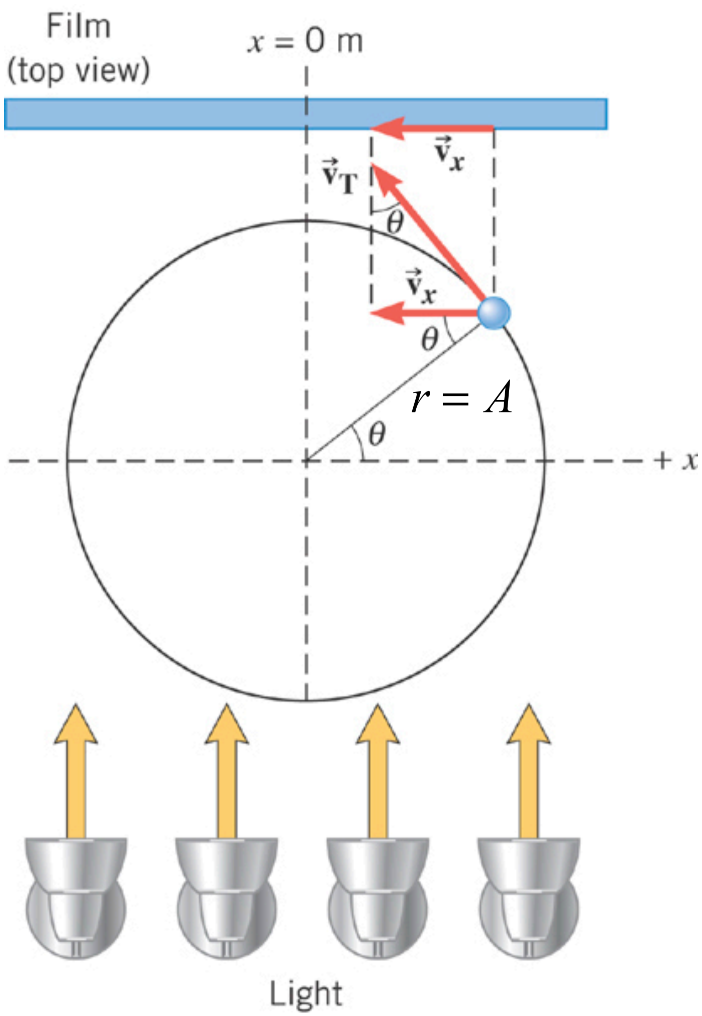
period T: the time required to complete one “cycle”

frequency f: the number of “cycles” per second (measured in Hz = 1/s)

frequency f: $$f = \frac{1}{T}$$

angular frequency: $$\omega = 2\pi f = \frac{2\pi}{T}$$ (Radians per second)

VELOCITY



Note: $\sin(\omega t)$

$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{max}} \sin(\omega t)$$

Maximum velocity: $\mp A\omega$ (units, m/s)

$$v_x = -A\omega \sin(\omega t) = \mp A\omega$$

when $\omega t = \pi/2,\ 3\pi/2$ radians

Maximum velocity occurs at

$$x = A\cos(\omega t)$$
$$= A\cos(\pi/2) = 0$$

Example The Maximum Speed of a Loudspeaker Diaphragm

The frequency of motion is 1.0 KHz and the amplitude is 0.20 mm.
(a) What is the maximum speed of the diaphragm?
(b) Where in the motion does this maximum speed occur?

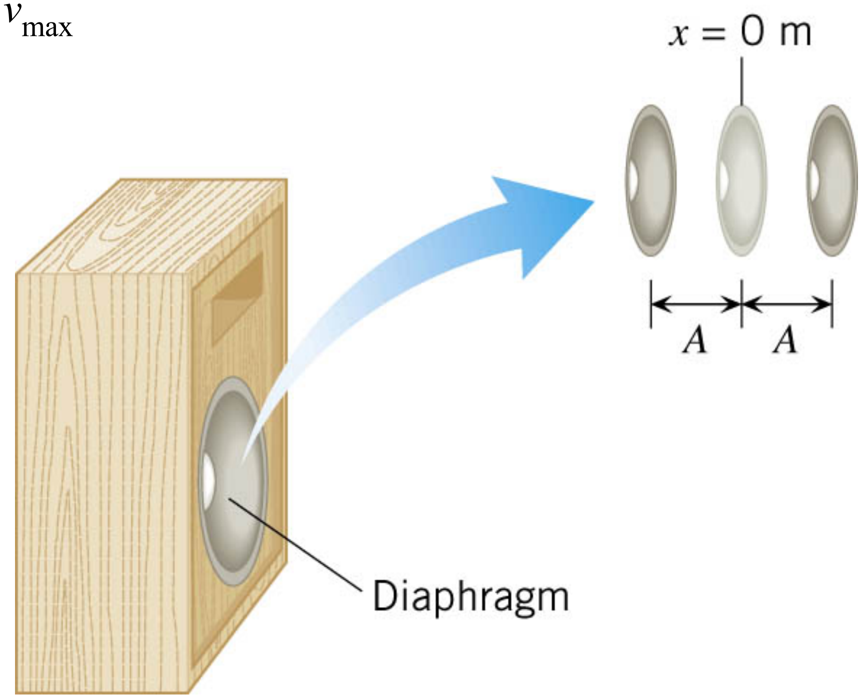
$$v_x = -v_T \sin\theta = -\underbrace{A\omega}_{v_{\max}} \sin\omega t$$

a) $v_{\max} = A\omega = A(2\pi f)$

$= (0.20\times10^{-3}\,\text{m})(2\pi)(1.0\times10^{3}\,\text{Hz})$

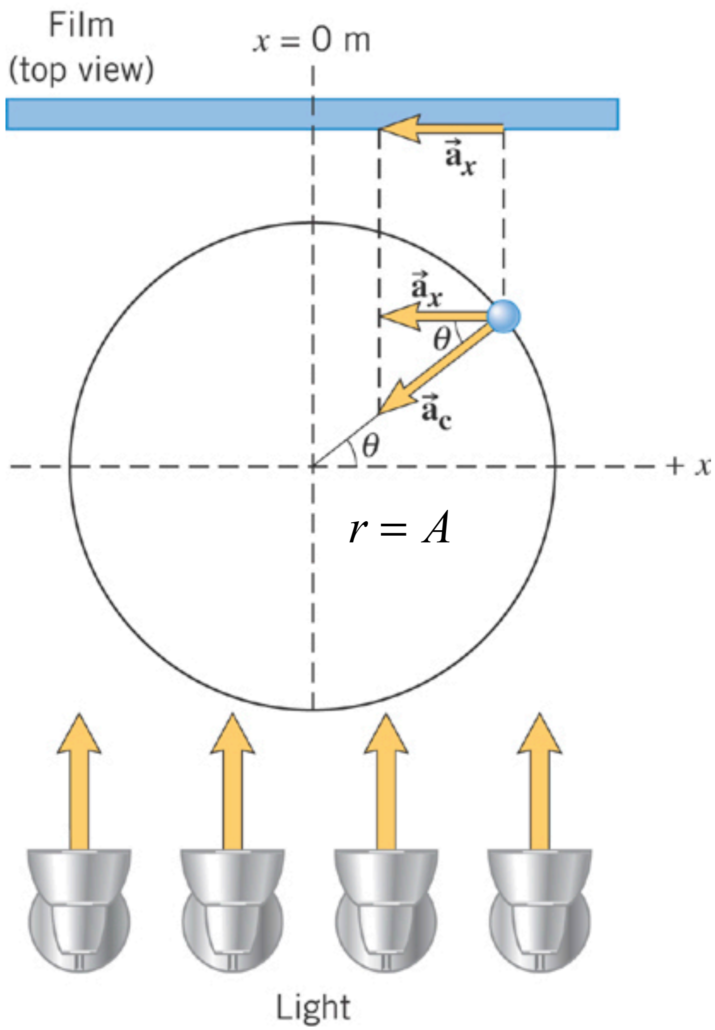
$= 1.3\,\text{m/s}$

b) The maximum speed occurs midway between the ends of its motion.



ACCELERATION

$$a_c = \frac{v^2}{r} = r\omega^2$$



$$a_x = -a_c \cos\theta = -\underbrace{A\omega^2}_{a_{max}} \cos\omega t$$

Maximum a_x: $\mp A\omega^2$ (units, m/s^2)

$$a_x = -A\omega^2 \cos(\omega t) = \mp A\omega^2$$

when $\omega t = 0, \pi$ radians

Maximum a_x occurs at

$$x = A\cos(\omega t)$$
$$= A\cos(0) = A$$
$$= A\cos(\pi) = -A$$

FREQUENCY OF VIBRATION

$$\sum F_x = ma_x$$

$$-kx = ma_x$$

$$x = A\cos\omega t$$

$$a_x = -A\omega^2\cos\omega t$$

$$-Ak\cos\omega t = -Am\omega^2\cos\omega t$$

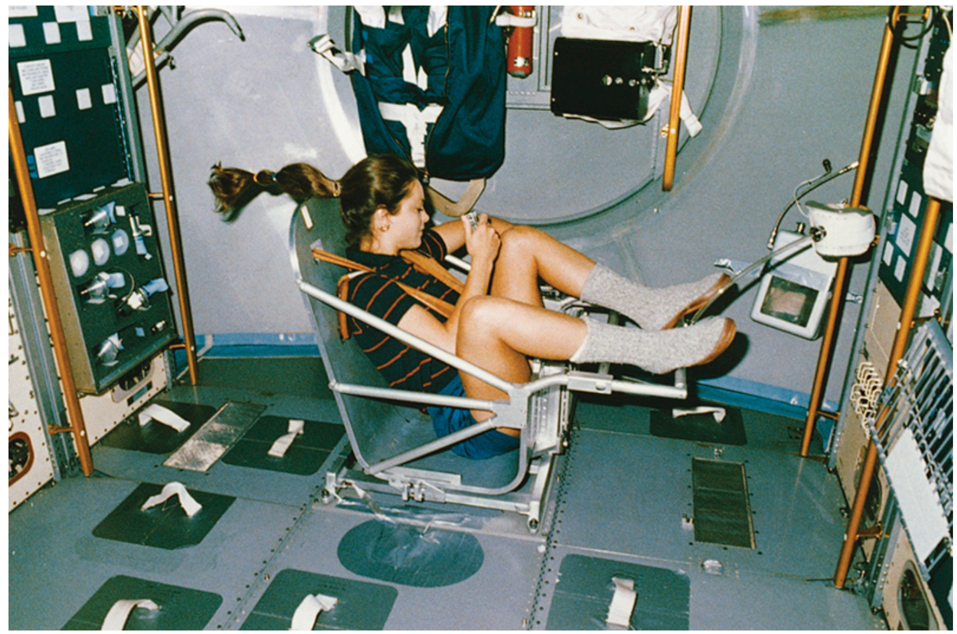
$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

angular frequency for oscillations of a mass (m) on a spring (k)

Example A Body Mass Measurement Device

The device consists of a spring-mounted chair in which the astronaut sits. The spring has a spring constant of 606 N/m, and the mass of the chair is 12.0 kg. The measured period is 2.41 s. Find the mass of the astronaut.



spring constant: $k = 606\,\text{N/m}$

chair mass: $m_{\text{chair}} = 12.0\,\text{kg}$

oscillation period: $T = 2.41\,\text{s}$

$$\omega = \sqrt{\frac{k}{m_{\text{total}}}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$m_{\text{total}} = \frac{k}{\omega^2} = \frac{kT^2}{4\pi^2} = 89.2\text{ kg}$$

$$m_{\text{astro}} = m_{\text{total}} - m_{\text{chair}} = 77.2\,\text{kg}$$

Summary: spring constants & oscillations

Hooke's Law $F_A = kx$ Displacement proportional to applied force

Oscillations $\omega = \sqrt{\frac{k}{m}}$ Angular frequency $(\omega = 2\pi f = 2\pi / T)$

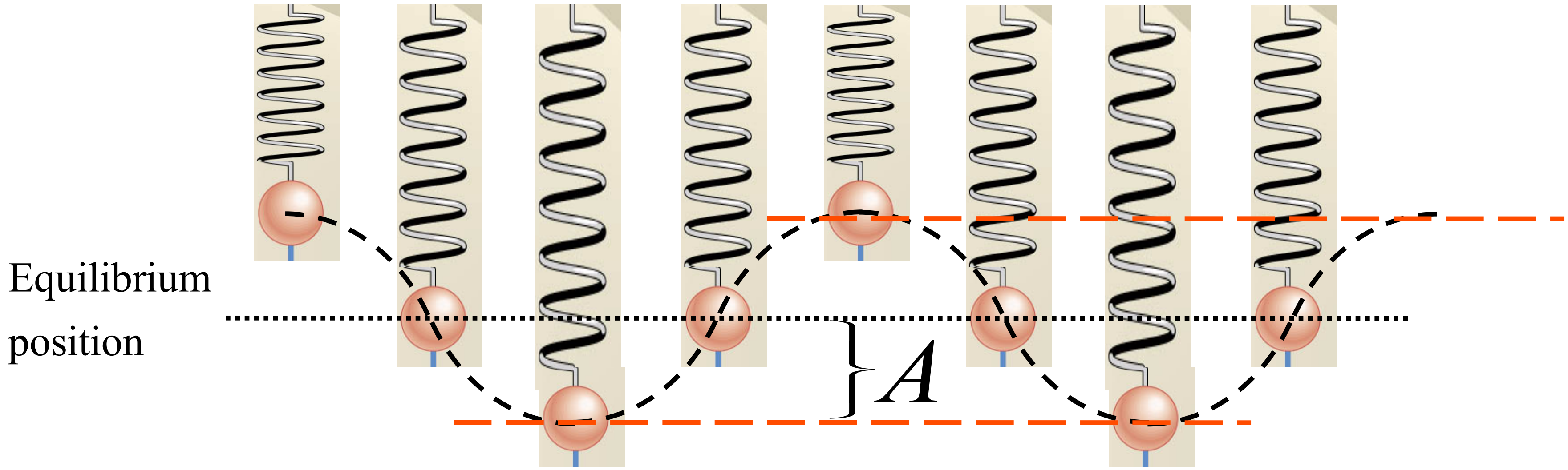
position: $x = A\cos(\omega t)$

velocity: $v_x = -\underbrace{A\omega}_{v_{\max}}\sin(\omega t)$

acceleration: $a_x = -\underbrace{A\omega^2}_{a_{\max}}\cos\omega t$

7.3 Energy in Simple Harmonic Motion

Consider this motion taking place far from the Earth



Speed maximum at equilibrium position

Energy all in kinetic energy: $E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}mA^2\omega^2$

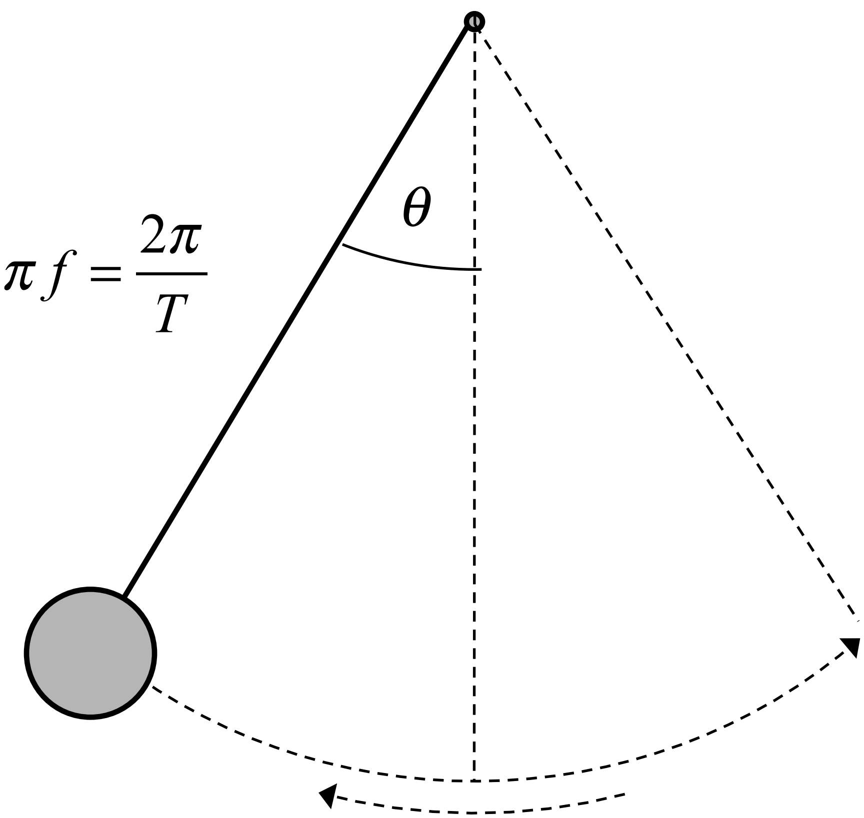
At highest and lowest point energy is

all in spring potential energy: $U_S = \frac{1}{2}kA^2 = E_{Total}$

At intermediate points total energy

$E_{Total} = \frac{1}{2}kA^2 = K + U_S = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

A ***simple pendulum*** consists of a particle attached to a frictionless pivot by a cable of negligible mass.



$$\omega = 2\pi f = \frac{2\pi}{T}$$

<u>Angular frequency</u>

$$\omega = \sqrt{\frac{g}{L}} \quad \text{(small angles only)}$$

$$I = mL^2$$

$$\frac{1}{L} = \frac{mL}{I}$$

$$\omega = \sqrt{\frac{mgL}{I}} \quad \text{(small angles only)}$$

Works for objects with moment of inertia, I and distance to center of mass, L_{CM}

Clicker Question 7.2

At the surface of Mars, the acceleration due to gravity is 3.71 m/s^2. What is the length of a pendulum on Mars that oscillates with a period of one second?

a) 0.0940 m

$$\omega_{\text{pendulum}} = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

b) 0.143 m

c) 0.248 m

d) 0.296 m

e) 0.655 m

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$$\omega_{\text{pendulum}} = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$\frac{(2\pi)^2}{T^2} = \frac{g_{\text{Mars}}}{L}$$

$$L = \frac{g_{\text{Mars}} T^2}{(2\pi)^2} = \frac{(3.71\ \text{m/s}^2)(1\ \text{s})^2}{(2\pi)^2}$$

$$= 0.094\ \text{m}$$

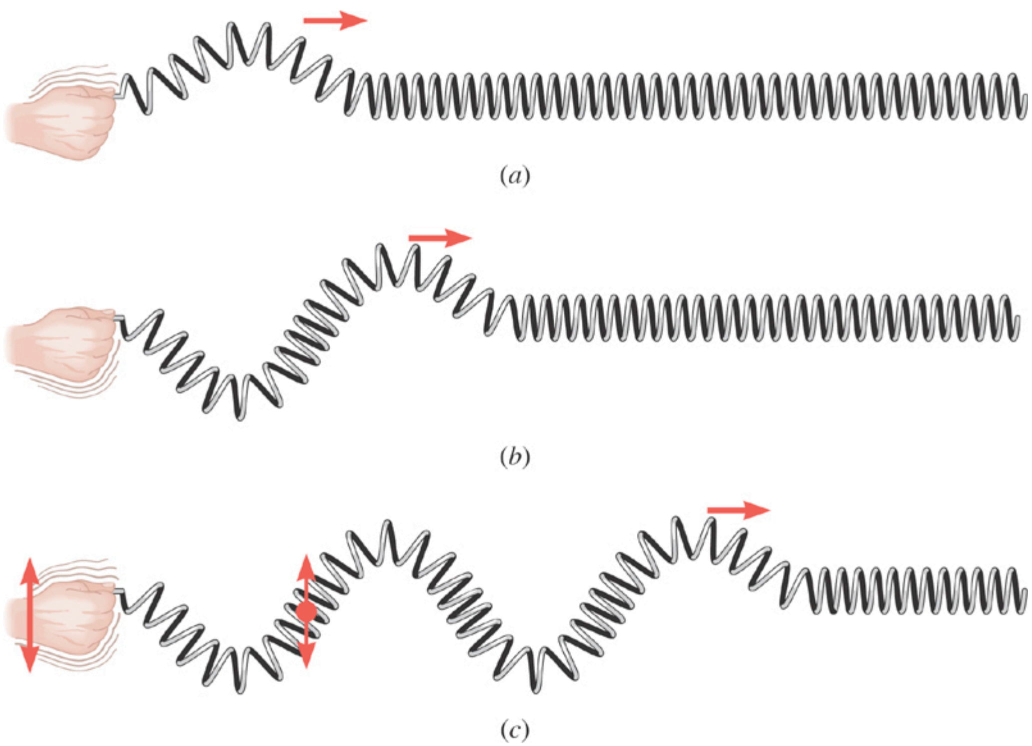
Chapter 11

Waves & Sound

11.1 The Nature of Waves

1. A wave is a traveling disturbance.

2. A wave carries energy from place to place.

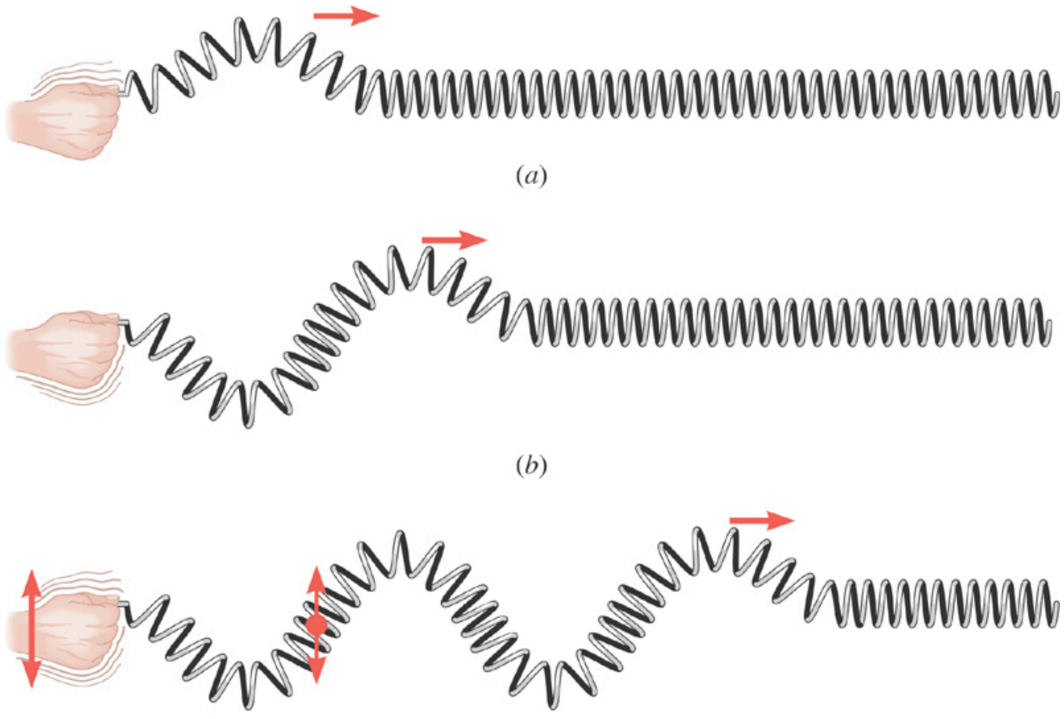


(a)

(b)

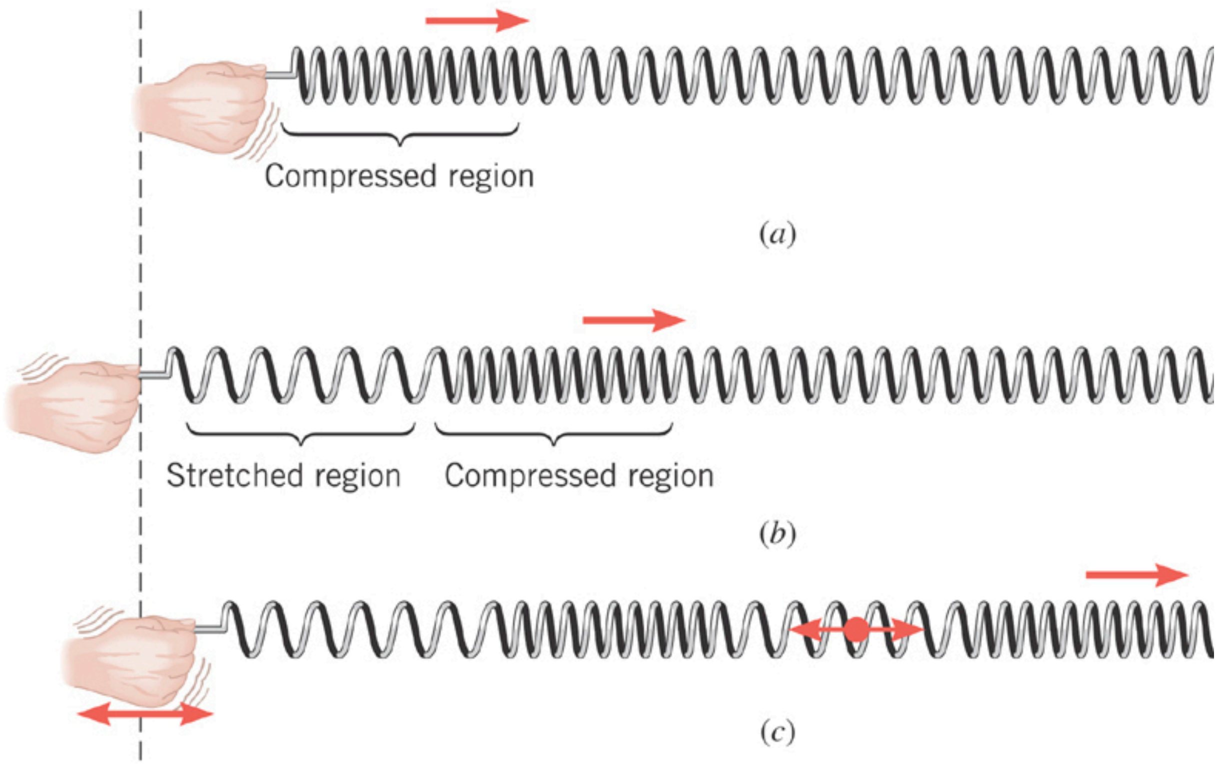
(c)

Transverse Wave



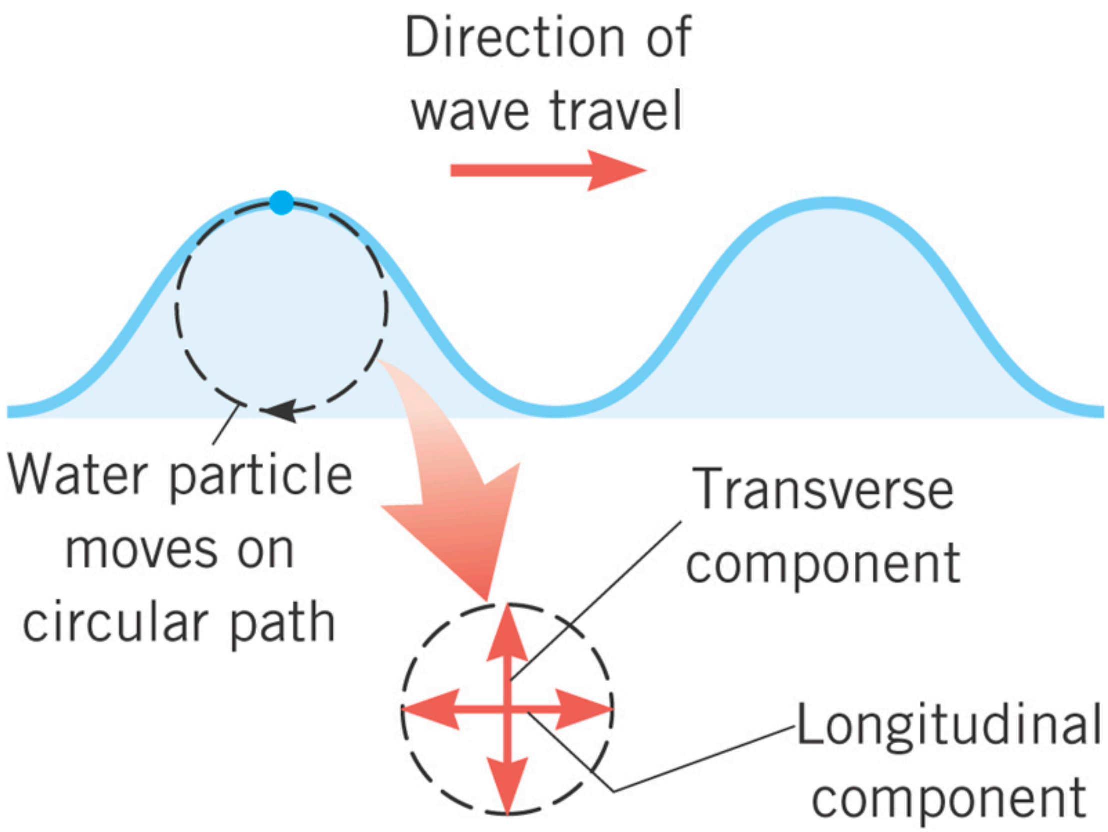
(a) (b) (c)

Longitudinal Wave



11.1 The Nature of Waves

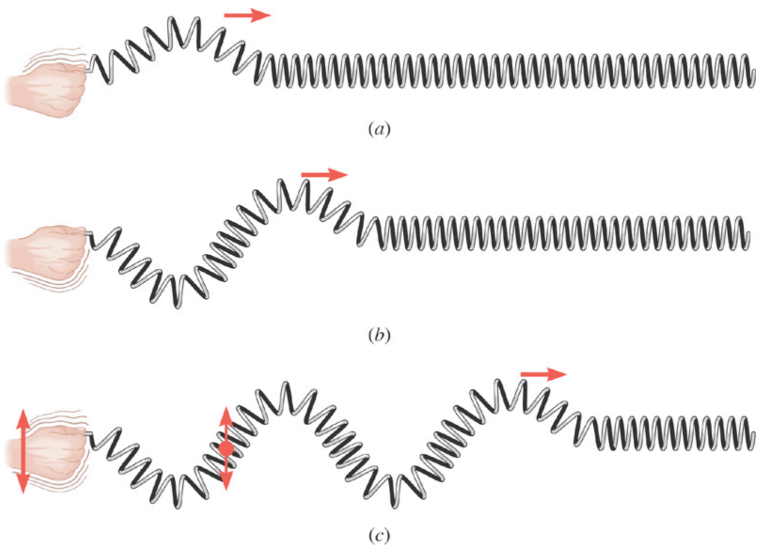
Water waves are partially transverse and partially longitudinal.

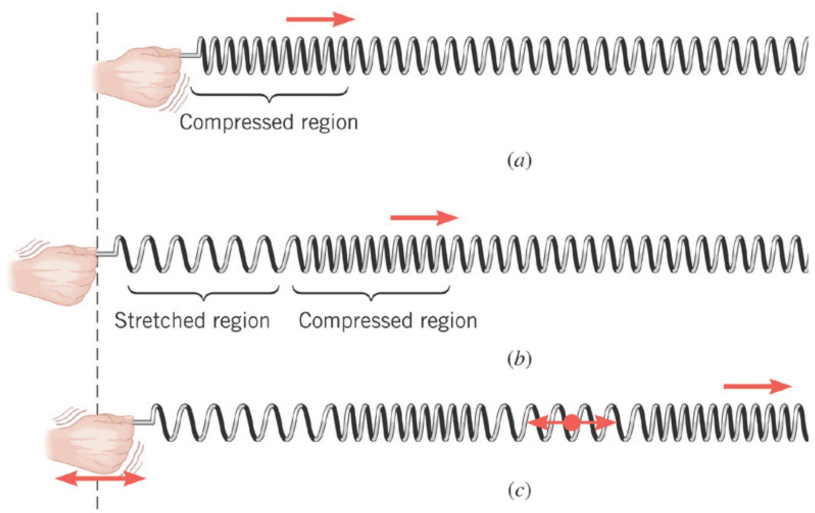


11.2 Periodic Waves

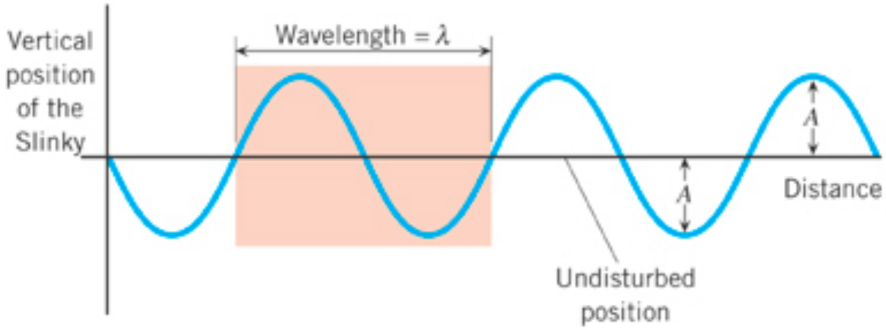
Periodic waves consist of cycles or patterns that are produced over and over again by the source.

In the figures, every segment of the slinky vibrates in simple harmonic motion, provided the end of the slinky is moved in simple harmonic motion.

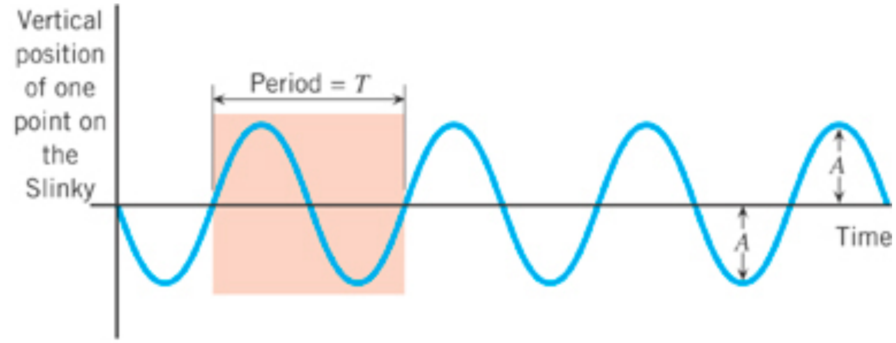




11.2 Periodic Waves



(*a*) At a particular time



(*b*) At a particular location

In the drawing, one ***cycle*** is shaded in color.

The ***amplitude*** A is the maximum excursion of a particle of the medium from the particles undisturbed position.

The ***wavelength*** is the horizontal length of one cycle of the wave.

The ***period*** is the time required for one complete cycle.

The ***frequency*** is related to the period and has units of Hz, or s^{-1}.

$$f = \frac{1}{T}$$