Experiment 9

The Rydberg Constant

9.1   Objectives

- Observe, measure, and analyze the emission spectrum of a hydrogen gas discharge tube.

- Use the data from the hydrogen emission spectrum to calculate the Rydberg constant.

9.2   Introduction

White light is made up of all the colors in the rainbow. Light that is emitted by excited gases is made of very particular colors which reveal information about the underlying structure of the atoms in the gas.

For example, the light you see when you plug in a hydrogen gas discharge tube is a shade of lavender, with some pinkish tint at a higher current. If you observe the light through a spectroscope, you can identify four distinct colors in the visible light range.

9.3   Key Concepts

As always, you can find a summary on-line at Hyperphysics[1]. Look for keywords: "emission, quantum", electromagnetic spectrum, hydrogen spec-

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[1]http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

trum[2]

9.4 Theory

The history of the study of the distinct colors emitted from excited gases, called spectral lines, dates back to the late 19th century, where we meet a high school mathematics teacher from Basel, Switzerland, named Johann Balmer. Balmer created an equation describing the wavelengths of the visible hydrogen emission lines. However, he did not support his equation with a physical explanation. In a paper written in 1885, Balmer proposed that his equation could be used to predict the entire spectrum of hydrogen, including the ultraviolet and infrared spectral lines. To find the nth wavelength, λ_n, he said to use this formula, the Balmer formula:

$$\lambda_n = B \left(\frac{n^2}{(n^2 - 4)} \right), \tag{9.1}$$

where n is an integer greater than 2 (e.g. 3, 4, 5, etc.), and B is Balmer's constant, a number found to make the equation fit the observed spectrum. When one solves the equation, the calculated wavelengths are very close to the four emission lines in the visible light range for a hydrogen gas discharge tube. Balmer apparently derived his equation by trial and error. Sadly, he would not live to see Niels Bohr and Johannes Rydberg prove the validity of his equation.

Johannes Rydberg was a mathematics teacher like Balmer (he also taught a bit of physics). In 1890, Rydberg's research of spectroscopy (inspired, it is said, by the work of Dmitri Mendeleev) led to his discovery that Balmer's equation was a specific case of a more general principle that worked for many elements, not just hydrogen. Rydberg substituted the wavenumber, 1/wavelength, for wavelength and, by applying appropriate constants, developed a variation of Balmer's equation. The Rydberg equation is given as

$$\frac{1}{\lambda_{if}} = R_{\mathrm{H}} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right). \tag{9.2}$$

Here, n_i and n_f are again integers. R_{H} is the *Rydberg constant*. Relating it to the Balmer equation, $R_{\mathrm{H}} = 4/B$. For the hydrogen atom, $n_f = 2$, as

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[2]http://hyperphysics.phy-astr.gsu.edu/hbase/hyde.html#c4

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shown in the first equation. The term $1/\lambda$ is the wavenumber, as expressed by Rydberg in his version of the Balmer equation. Niels Bohr used this equation to show that each line in the hydrogen spectrum corresponded to the release of energy by an electron as it passed from a higher to a lower energy level.

9.5 In today's lab

In this experiment you will measure the emissions from a hydrogen gas discharge tube and analyze the emission data to calculate the Rydberg constant.

9.6 Equipment

- diffraction grating (Fig. 9.1)

- digital spectrometer with optical fiber (Ocean Optics USB650 Red Tide) (Fig. 9.2)

- Logger Pro (software) (Fig. 9.3)

- gas discharge tubes (Fig. 9.4)

Safety Tips

- Gas discharge tubes get **very hot**. Be very careful to let them cool down before touching them.
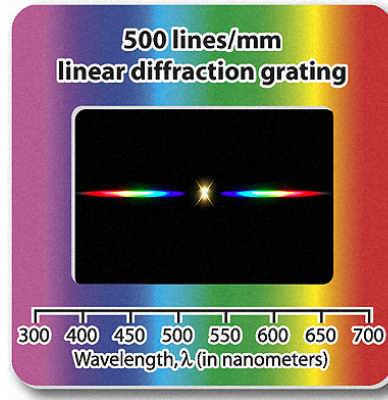
Figure 9.1: The diffraction grating.[2]

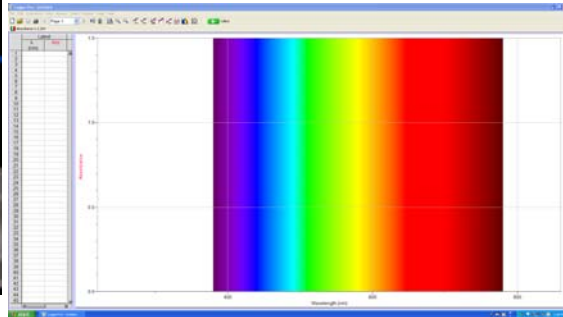Figure 9.2: Digital spectrometer. Note the white USB cable and blue optical fiber.

Figure 9.3: Screenshot of Logger *Pro*.
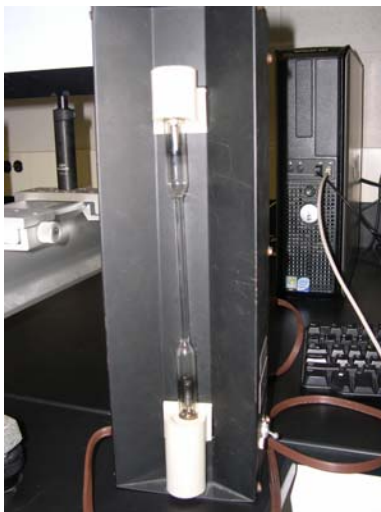
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Figure 9.4: Gas discharge tube mounted in box.

9.7 Procedure

1. Connect the spectrometer to the USB port of a computer. Start the data-collection program, and then choose `New` from the `File` menu.

2. Connect a fiber optic cable to the threaded detector housing of the spectrometer.

3. To prepare the spectrometer for measuring light emissions: In Logger *Pro*, open the `Experiment` menu and select `Change Units` ▶ `Spectrometer:  1` ▶ `Intensity`.

4. To set an appropriate sampling time for collecting emission data: In Logger *Pro*, open the `Experiment` menu and choose `SetUp Sensors` ▶ `Spectrometer:1`. In the small dialog that appears, change the `Sample Time` to 60 ms, change the `Wavelength Smoothing` to 0, and change the `Samples to Average` to 1.

5. Turn on the hydrogen gas discharge tube. Aim the tip of the fiber optic cable at the tube.

6. Observe the gas discharge tube by looking through the diffraction grating. Note how it spreads out the light into different distinct colors.

7. Start data collection. An emission spectrum will be graphed. Note how the diffraction grating observation relates to the emission spectrum. Set the distance between the discharge tube and the tip of the fiber optic cable so that the peak intensity on the graph stays below 1.0. When you achieve a satisfactory graph, stop data collection. The graph is satisfactory when the spectrum is stable and peaks are generally very distinct.

8. To analyze your emission spectrum graph click the `Examine` icon, ⬚, on the toolbar in Logger *Pro*. Identify as precisely as possible each of the four wavelengths of hydrogen's Balmer series. The third and fourth peaks are very small but they may be identifiable. (If you are unable to measure the two weaker peaks, measure the peaks you can and then repeat the run, scaling the more intense of the two weaker peaks to 1.) Enter the wavelength of the four peaks of the graph in your data table and print the graph to turn in.

9. Find the uncertainty in each peak wavelength. The uncertainty is related to the width of the peak, since the wider the peak is, the more uncertain where the highest point is. Use the "half width at half maximum" (HWHM) of the peak as a measure of this. To assist you, label these points as seen in Fig. 9.5.

   a) The point of peak intensity. Label with the wavelength (x) and intensity (y) at that point.

   b) The two points on either side of the peak where the intensity is half of the peak intensity. Find the wavelength (x) and intensity (y) at those points and label them.

   Then, the HWHM, which is the uncertainty, is found by

   $$\delta\lambda = \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{2}. \tag{9.3}$$

   Calculate and record these on your data table.

10. A method of determining the Rydberg constant is to analyze a graph of the values of n in the Balmer Series vs. the wavenumber, $1/\lambda$. Import your data into a plotting program and **prepare a plot of wavenumber vs.** $\mathbf{1/n_i^2}$, including error bars for the wavenumber. The uncertainty in the wavenumber, $\delta(1/\lambda) = \delta\lambda/\lambda^2$
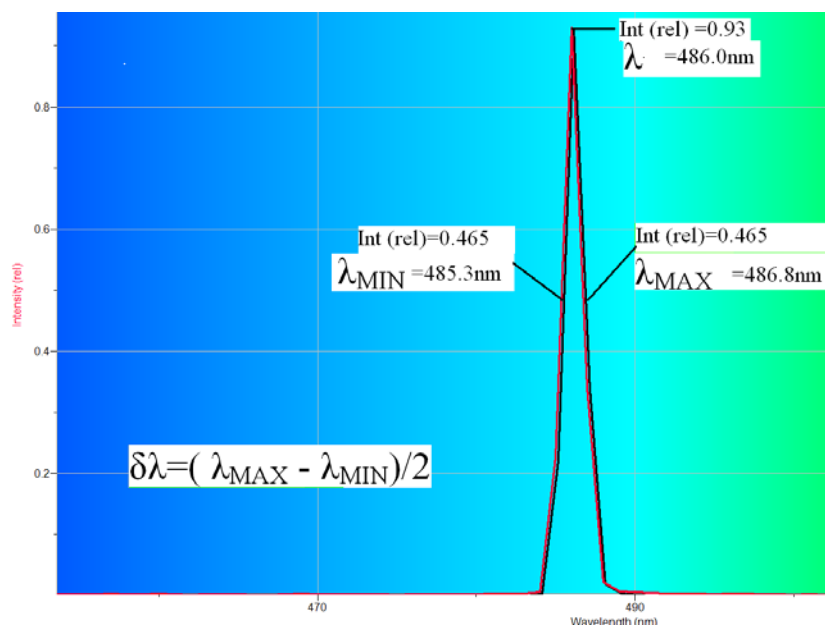
Figure 9.5: Example of how to label the graph of the emission spectra to find the HWHM.

11. Calculate the best-fit line equation for the plot, then answer Question 2.

9.8 Questions

1. Use Eq. 9.2 to calculate the Rydberg constant for the four lines in Balmer Series that you identified in your spreadsheet. What is the average value for the Rydberg constant, based on your data? What is the standard deviation?

2. Comparing the x- and y-axis variables of your plot, Eq. 9.2, and the equation of a straight line, how does the slope of this line relate to R_H? Use this to find R_H and its uncertainty.

3. Compare your value of the Rydberg constant from the previous question to the reference value, $R_\mathrm{H} = 1.097 \times 10^7 \, \mathrm{m}^{-1}$. (more precise value listed in Appendix B)

4. Use the R_H that you calculated in Question 2 to predict the wavelength of the fifth line in the Balmer Series ($n = 7$). Examine your graph of the hydrogen discharge tube emissions. Does the fifth Balmer line appear as a peak in your graph? Explain why.

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