PHY 251 Introductory Physics Laboratory I

Fall 2014

Experiments

Experiment 1

Introduction to Computer Tools and Uncertainties

1.1 Objectives

- To become familiar with the computer programs and utilities that will be used throughout the semester.
- To become familiar with experimental uncertainties.

1.2 Introduction

Microsoft Excel is a spreadsheet program that allows you to manipulate text as well as data. Most importantly for the labs you will be doing, Excel can perform calculations quickly that would otherwise be very time consuming. Learning a few basic commands and skills in Excel now will save you a considerable amount of calculation time the rest of the semester.

Once you have the data and computations from Excel, you can make a graph that will quickly and easily show what trends or relations the data exhibits. Kaleidagraph is a versatile graphing program and allows you complete control over how the data will be presented. It is up to you to decide as to what kind of graph will be best, though you will often be given guidance, especially for the first few experiments.

1.3 Miscellaneous comments

- Always bring a flash drive with you to class and save your work often. When the computer restarts, it erases all changes to the hard drive.
- The Excel spreadsheet is made up of rectangles called *cells*.
- Kaleidagraph is a graphing program that you will use to analyze the data we compute in the Excel spreadsheet.

1.4 Theory

Uncertainties

When we make a measurement, we need to know how precise that measurement is. The amount of precision of the measurement is called the *uncertainty* — we need to be able to report how uncertain we are about a measurement that we have taken. For example, if I measured how long my finger was with a ruler, I might say that my finger was measured to be 9.5 ± 0.5 cm — that is, I'm confident that it is actually between $(9.5 - 0.5)$ cm and $(9.5 + 0.5)$ cm.[1] This is a lot more meaningful of a statement than if I measured 9.5 ± 5.0 cm. I wouldn't know much about my finger length at all, and finding gloves that fit would be a nightmare!

An oft-used synonym for "uncertainty" is "error". Here, "error" does not mean a mistake, but rather a physical inability to make perfect measurements. All measurements are to some extent imperfect, and therefore the results obtained are always subject to some uncertainty. The scientist must indicate the magnitude of these uncertainties.

We express the uncertainty of a quantity x by writing $x \pm \delta x$, where δx is the uncertainty of x. Note that even though "δx" has two characters, we treat it as one variable, not "δ" multiplied by "x". Uncertainties are always positive numbers, and they always have the same units as the quantity in the equation (2 meters + 2 seconds doesn't make much sense!).

[1] There is a more rigorous definition of uncertainty and confidence, but we will not use it in this course.

Random Errors

When you make a series of measurements of the same quantity using the same measuring instruments, you often find that you do not obtain exactly the same answer each time. Your measurements are said to be affected by *random errors*. Random errors arise from small, uncontrollable differences in the way each measurement was made, and the differences make the measurement fall either above or below the true value, with equal probability. Random errors determine the uncertainty in the value of a directly measured or calculated quantity.

Systematic Errors

Unlike random errors, *systematic errors* tend to make each of your measurements be off in the same direction. For example, if you weighed a series of rocks on a scale, and put them on a plate each time, then you'd be measuring the weight of the plate as well each time, making the scale read higher than the actual weight of the rocks every time.

Such errors can result from either improper calibration of the equipment or from a failure to account properly for some unexpected perturbation such as friction. These errors are generally harder to estimate than random errors, though they can be predicted more easily, as in the case of the plate.

Some general rules about uncertainties

Appendix B contains a more detailed reference guide to uncertainties, but here is a summary:

- An uncertainty is always a positive number, $\delta x > 0$.
- If the uncertainty of x is δx, then the *fractional uncertainty* of x is $\delta x/x$.
- If the fractional uncertainty of x is 5%, then $\delta x = 0.05x$.
- If you measure x with a device that has a precision of u, then δx is at smallest as large as u (you might make your reported uncertainty larger to account for some other difficulty in measurement).

- If you add or subtract two quantities with uncertainties, the uncertainties add to give the uncertainty of the result (since they could both be wrong in the same direction lower or higher than the true value). So if $z = x + y$ or if $z = x - y$, then $\delta z = \delta x + \delta y$.
- if d is your mesaured value ("data") and e is the expected value,
  - The difference is $D = d - e$.
  - % difference is $D/e \times 100\%$.
  - They are *compatible* if $|D| < \delta d + \delta e$.
- If you multiply or divide two quantities with uncertainties, the fractional uncertainties add to give the fractional uncertainty of the result (contrast with adding or subtracting above). So if $z = xy$ or $z = x/y$, then $\delta z/z = \delta x/x + \delta y/y$.

Graphs

It's hard for people to read a table of numbers and see a pattern or trend. Graphs allow mathematical relationships to be visualized and consequently more clearly understood. Graphs also help in determining the mathematical relationships between variables.

1.5 In today's lab

You will learn to use Microsoft Excel and Kaleidagraph to perform some simple tasks.

1.6 Equipment

- **Data sheet.** Before making your graph, record your data in a systematic form, showing the units and uncertainties for each measurement. In this course, you will use an Excel spreadsheet to document your data. This way there will be no confusion in your mind about what point you are graphing.
- **Graphing software.** You will use Kaleidagraph to graph and analyze your data.

- **Choosing axes.** If you are asked to graph a vs. b, the variable before the "vs.", a, goes on the vertical axis, and the variable after the "vs.", b, goes on the horizontal axis. Label both your axes (showing units) and title the entire graph (at the top), so readers can identify what you are plotting.
- **Choosing scales.** The range of the scales should be chosen so that you can easily see any meaningful variation in the data, but random errors are not magnified out of proportion to their significance. Your axes need not always begin at zero, but consider carefully whether they should (Is zero a physically relevant point for the experiment?).
- **Error bars.** Wherever possible, indicate the uncertainty of each point by using *error bars*. An error bar is a line passing through the data point and extending from the smallest value which that point could reasonably have, up to the largest value it could have. An error bar parallel to the vertical axis shows the uncertainty in the variable on that axis, and an error bar parallel to the horizontal axis shows the uncertainty in the variable on that axis. In most cases you will have uncertainties in only one of your variables. Your instructor will tell you when error bars can be omitted in a variable.
- **Finding the best straight line through a set of data points.** KaleidaGraph can be used to fit a straight line to your data, complete with the equation of this best fit line. In addition, KaleidaGraph will provide an estimate in the uncertainty in the slope and y-intercept of your best fit line.

1.7 Procedure

Part 1: Graphing data points

In this part of the experiment, you will use Excel and Kaleidagraph to graph the x and y positions of a projectile as they change with time.

Entering data and formulas into Excel

1. Open the lab folder titled `Introduction to Computers`, which is in the `251 Lab` folder on the computer's desktop. Remember, you can open the folder by double-clicking on the icon with your mouse.

2. Double-click on the Excel file.

3. In the lower left hand corner of the spreadsheet window, click on the tab labeled `PART 1`.

   You should see two data tables on the spreadsheet. The first data table is used to define the acceleration due to gravity, g, the initial horizontal and vertical coordinates of the object's position (x_0 and y_0), and the corresponding initial velocities ($v_{x,0}$ and $v_{y,0}$). The second data table will be used to display the x and y locations of the object at various times.

4. Fill in the first column of the second data table with times $t = 0.0\,\mathrm{s}$ through $t = 3.0\,\mathrm{s}$ in increments of $0.1\,\mathrm{s}$. Do not waste time filling in each of these values by hand — instead, let Excel do the work for you. Here's how:

   a) In the first cell of the time column (cell `B11`), enter 0.

   b) In the second cell of the column (`B12`), you can give Excel the formula you want it to follow. In each cell in the column, we would like Excel to add 0.1 seconds to the cell immediately above it. So, in cell `B12`, enter `=B11+0.1`. The `=` lets Excel know the cell contains a mathematical or logical operation. After entering the formula, cell `B12` should contain `0.1`.

   c) Click and drag the mouse to highlight the entire column of the data table, starting with the cell that has the formula in it (`B12`), open the `Edit` menu at the top of the screen, scroll down to `fill` and select `down`. The entire column should now be filled with numbers from 0 to 3.0 in increments of 0.1. Alternatively, left click the bottom right corner of a cell and fill down by dragging the mouse down to lower cells.

5. If you are viewing a spreadsheet and you are not sure what formula Excel is using for some calculation, you can click on the cell and the equation will be displayed in the formula bar near the top of the screen. Try it by clicking on cell `B23` which should contain `1.2`. In the formula bar, `=B22+0.1` should now be displayed.

   The location of the projectile depends on the initial conditions and the acceleration due to gravity. The object's position in the horizontal direction at time t is given by

$$x = x_0 + v_{x,0}t \tag{1.1}$$

and in the vertical by

$$y = y_0 + v_{y,0}t - \frac{1}{2}gt^2, \tag{1.2}$$

where g is the acceleration due to gravity, 9.8 m/s^2. We would like to use the same method here to calculate the values for the location of the object (x and y values), as we did for the time values in the second data table. However, you need to either redefine the names of the cells containing g, x_0, y_0, $v_{x,0}$, and $v_{y,0}$ before using them in an equation with the fill-down method, or explicitly reference these cells in an equation. The method presented and used in this lab will be to redefine the cells.[2]

6. To start with, let's redefine the name of the cell which will contain the acceleration due to gravity. Right click on cell `B4`, and select 'define name' from the window that appears. A `New name` window should open up.

7. Enter `g` into the top line in the new name window and select `OK`. Now, when referring to this cell in an Excel formula, you can just enter `g`, and when the fill-down option is used, Excel will not change the referenced cell like it did with the cells for time.

8. Redefine the cell names for `C4`, `D4`, `E4`, and `F4` to `x0`, `y0`, `vx0`, and `vy0` respectively.

9. We need to give numerical values to the acceleration due to gravity, as well as the initial position and velocity. The acceleration due to gravity is 9.8 m/s^2. Let's put our object 20 m away from us and 15 m above the ground, with an initial horizontal velocity of 12 m/s away and initial vertical velocity of 10 m/s upwards. Enter each numerical value (not including the unit) into its corresponding cell.

[2] An alternative way to tell Excel that you want to use a particular cell and not increment down the column is to use the $ symbol before the row number. For example, when typing equation 1.1 instead of using the defined cell name x_0 you could use C$4.

10. The next step is to enter the correct Excel formula to calculate the positions x and y as a function of time. In order to do this, you need to enter Eqs. 1.1 and 1.2 in Excel's programming language. The Excel formula for the position x, which should be entered in cell `C11`, is `=x0+vx0*B11`.

11. Use the fill-down method to calculate the rest of the x positions.

12. The Excel formula for the position y, which should be entered into cell `D11`, is `=y0+vy0*B11-0.5*g*B11^2`. Use the fill-down method to calculate the other y positions. A reference to some useful Excel equations can be found in Appendix C.

13. Print out the Excel spreadsheet with your data. Also print out the formula view which is gotten by pushing the `Ctrl+~` keys.[3] This will display the formulas for the entire spreadsheet. Pressing these two keys again reverts back to the calculated numbers. Make sure none of the formulas in the formula view are cut-off, you may need to resize some columns. When printing it is a good idea to fit the spreadsheet to a single page as long as it is still legible, changing the orientation to landscape often helps.

Transferring data to Kaleidagraph

Once your data table is complete, you are ready to transfer your data into Kaleidagraph using the cut-and-paste method. **If you cannot fix something that broke in Kaleidagraph, sometimes you will need to close and restart the program.**

Here are the steps to transfer your data:

1. Highlight the area you want to move. Highlight only the data values. Do not include any cells containing text, such as column headers. The program will not make a graph for you if you do.

2. From the drop-down menu, choose `Edit` ▶ `Copy`, or press `Ctrl+C` to copy the highlighted text to the computer's clipboard.

3. Open Kaleidagraph by double-clicking its icon on the desktop.

[3] The tilde (~) key is to the left of the number 1 on the US keyboard.

4. Click on the upper-left-most cell of the spreadsheet that appears.

5. Choose `Edit` ▶ `Paste` from the drop-down menu, or press `Ctrl+V` to paste in your data.

Graphing in Kaleidagraph

You are now ready to make a graph.

1. You can change the column names in Kaleidagraph by double-clicking them after you've transferred the data. In this case, you should name the first column "Time (s)", the second column "X (m)" and the third column "Y (m)".

2. To choose the graph type, choose `Gallery` ▶ `Linear` ▶ `Scatter`. This option is used to create a scatter-plot of the x and y coordinates of the projectile. A plot window should open.

   We want to plot both the x position and y position versus time. This means that we want both x and y to be on the `Y`-axis, and time to be on the `X`-axis.[4]

3. Click on the bubble under the `X` column for `Time (s)`, the `Y`-column for `X (m)` and the `Y`-column for `Y (m)`.

4. Click the `New Plot` button. This will create a scatter plot with time plotted on the horizontal axis and both the x and y coordinates plotted on the vertical axis.

   Graphs should always contain proper labels. Each axis should be labeled with the variable name and the units in parentheses, and the graph itself should have a title.

5. To change the label of the vertical axis, double click on it. An `Edit String` window should appear. Change the text in the window to `X (m) and Y(m)`. The same method is used to change the horizontal axis label and the graph's title. The appropriate way to title a graph is "what physical quantity is on the vertical axis" versus "what physical quantity on the horizontal axis". Include this graph with your lab report.

[4] Apologies for the overused variable y, as it is used for both the vertical direction in the graph and the vertical direction in the physical situation. Be careful about which you are referring to.

Part 2: Analyzing the graph

The slope and y-intercept often have physical meaning, and we can use the graphing software to calculate them. In this part of the experiment, you will use Excel and Kaleidagraph to graph and calculate the slope and intercept of a set of data. In addition, Kaleidagraph will provide an estimate of the uncertainty in the slope and y-intercept.

Adding error bars

1. Click on the `PART 2` tab near the bottom of your Excel spreadsheet. You should find a set of times and positions for a ball rolling across a horizontal surface whose motion is described by Equation 1.1. (You will also notice two empty data columns — you'll get to those shortly.)

2. Transfer the data in the first two columns into Kaleidagraph and make a position vs. time graph.

3. When graphing data which include experimental uncertainties, you should include error bars to help the reader understand how significant the trend shown is. The experimental uncertainty in the position of the object is given in your Excel spreadsheet. To add error bars, select `Plot` ▶ `Error Bars`. An `Error Bar Variables` window should appear.

4. Use your mouse to check the box under `Yerr`. An `Error Bar Settings` window should now appear. Make sure the `Link Error Bars` box is checked. Just above and just below the `Link Error Bars` box, you should see two identical pull-down selection boxes. These allow you to define the size of your error bars. Since you have the error bars linked, you only need to change one of these, and the other will follow. Clicking on one of these boxes will give you the choice of setting your error bars as a % of the value, a fixed value, a standard deviation, a standard error, or referencing them to a data column.

5. For this exercise, choose `Fixed Value` from the drop down menu and then enter the uncertainty given on your Excel spreadsheet in the `Fixed Error` box.

6. Click `OK` and then click `Plot`. You should now have error bars on all of your data points on your graph. For additional information on estimating uncertainties in measurements, see Appendix B.

> **Note: The numerical value of the uncertainty used here is for this example only and should not be used in subsequent labs requiring an uncertainty in a length or distance. You will find your own uncertainties in those experiments.**

Plotting a best-fit line

The next thing you will do is to have Kaleidagraph find and plot a best-fit line to your graph.

1. Select `Curve Fit` ▶ `General` ▶ `fit1`. A `Curve Fit Selections` window will open, check the box and click `OK`. Kaleidagraph will plot a best-fit line on your graph. Also, a small data table will appear on your graph. (Note that the table can be moved elsewhere on the graph by clicking and dragging if it is covering up your data points.)

   The equation of the line is represented as `y=m1+m2*M0` in this data table, where `y` is the variable plotted on the vertical axis, `M0` is the variable plotted on the horizontal axis, `m1` is the coordinate where the line crosses the vertical axis (also referred to as the y-intercept) and `m2` is the slope of the line. The data table will display numerical values for the slope and y-intercept, as well as their respective uncertainties, δ(slope) and δ(int). The bottom two lines `R` and `Chisq` are a measure of how well your data are represented by your best-fit line and will not be used in this course. Include this graph with your lab report.

   As mentioned before the slope and y-intercept often correspond to physical quantities. To understand what they mean for this graph, compare Kaleidagraphs's equation of the line (Eq. 1.3) with the equation that describes the motion of the ball (Eq. 1.4).

$$y = m1 + m2 * M0 \tag{1.3}$$

$$x = x_0 + v_{x,0} * t \tag{1.4}$$

The graph plots the position of the ball (x in Eq. 1.4) on the vertical axis (y in Eq. 1.3) versus time (t in Eq. 1.4) on the horizontal axis ($M0$ in Eq. 1.3). Matching up the remaining variables in Eq. 1.3 with Eq. 1.4 reveals that the slope ($m2$) corresponds to the initial velocity of the ball ($\mathrm{v}_{x,0}$) and that the y-intercept ($m1$) corresponds to the initial position of the ball (x_0).

2. We'd like to get a visual representation of what is meant by the slope and intercept uncertainties given by the Kaleidagraph straight line fit. With this in mind, we will use the slope, y-intercept (int) and their respective uncertainties, δ(slope) and δ(int), to plot the lines with the largest and smallest slope which could reasonably represent your data (by reasonable we mean one uncertainty unit away from the best fit).

   To do this, you will need to return to your Excel spreadsheet. The equation corresponding to the largest reasonable slope is

$$x_{\text{largest slope}} = (\text{slope} + \delta\text{slope}) \times t + (\text{int} - \delta\text{int})\,. \tag{1.5}$$

   The equation corresponding to the smallest reasonable slope is

$$x_{\text{smallest slope}} = (\text{slope} - \delta\text{slope}) \times t + (\text{int} + \delta\text{int})\,. \tag{1.6}$$

   Use these equations to generate data points in your Excel spreadsheet for the lines of largest and smallest reasonable slope.

3. Transfer your largest and smallest reasonable slope data to your Kaleidagraph data table. Make a new plot of your data. Your new plot should include all three data sets. Specifically, your original given data set, the data set for the largest reasonable slope and the data set for the smallest reasonable slope.

4. Include error bars on the given data set only — do not include error bars on your data points for the largest reasonable slope or smallest reasonable slope.

5. Calculate best-fit lines for all three lines using the Curve Fit `Linear` option, rather than `fit1` that you used earlier. Instead of displaying a box with parameters, it just displays the equations for each line. Include this graph with your lab report.

You now have your data and three graphs. You should print out all of your data tables and all of your graphs. These must be turned in to your instructor at the end of the class session for grading. In addition, you should include the answers to any required questions.

1.8 Checklist

Remember to turn in:

1. Part 1 data table and formula view
2. Graph for Part 1 including observations[5]
3. Part 2 data table and formula view
4. First graph for Part 2 including observations
5. Second graph for Part 2 including observations
6. Answers to Questions

[5]For help on what to include in the observations see Appendix A.

1.9 Questions

When answering the questions, a measured quantity must ALWAYS include: 1) the numerical value, 2) its units and 3) its uncertainty. Unless otherwise stated, missing any of these quantities means the measurement will be considered incomplete and will receive reduced credit.

1. What is the slope of the best-fit line of the dataset from the first graph of Part 2?

2. What physical quantity does the slope correspond to?

3. What is the y-intercept of the best-fit line from the first graph of Part 2?

4. What physical quantity does the y-intercept correspond to?

5. What is the equation of the line having the largest reasonable slope for this set of data (no uncertainties are necessary in the equation)?

6. What is the equation of the line having the smallest reasonable slope for this set of data (no uncertainties are necessary in the equation)?

7. Do the lines of largest reasonable slope and smallest reasonable slope fit the data well? Explain why or why not by comparing these lines to the original best-fit line including its error bars.

Experiment 2

Reaction Time

2.1 Objectives

- Make a series of measurements of your reaction time.
- Use statistics to analyze your reaction time.

2.2 Introduction

The purpose of this lab is to demonstrate repeated measurements that do not yeild identical results; but this variation can give uncertainties (δx). Sometimes throughout life, we are given numbers which can carry meaning. In science, we often take measurements of the same thing multiple times and want to know how these measurements relate to each other. Today, we will be looking at your reaction time, and will try to find your average reaction time. After finding your reaction time you will find a measure of how confident you are in this value and place your reaction times into a predictable model

2.3 Key Concepts

In case you don't remember your Physics I lecture material, you'll need to refer to the chapters in an introductory textbook to physics. Alternatively, you can find a summary on-line at Hyperphysics.[1] Look for keywords: mean,

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

standard deviation, gaussian distribution

2.4 Theory

Two of the main purposes of this experiment are to familiarize you with the taking of experimental data and with the reduction of such data into a useful and quantitative form.

In any experiment, one is concerned with the measurement of some physical quantity. In this particular experiment it will be your reaction time. When you make repeated measurements of a quantity you will find that your measurements are not all the same, but vary over some range of values. As the spread of the measurements increases, the reliability or precision of the measured quantity decreases. If the measured quantity is to be of any use in further work, or to other people, it must be capable of being described in simple terms. One method of picturing measured values of a single quantity is to create a histogram.

The histogram is a diagram drawn by dividing the original set of measurements into intervals or "bins" of predetermined size, and counting the number of measurements within each bin. One then plots the frequency (the number of times each value occurs) versus the values themselves. A histogram has the advantage of visually presenting the distribution of readings or measurements. Fig. 2.1 shows a typical histogram for a set of observations. The histogram displays the number of measurements. For example, the first bin has two measurements between 0.195 seconds and 0.200 seconds. When placing the values into bins, one systematically puts values that occur on the bin limits into the next higher bin.

When analyzing data with a histogram, the distribution often times suggests that there is a "best" or most likely value, around which the individual measurements are grouped. From an intuitive approach one might say that the best value is somehow related to the middle of the distribution, while the uncertainty is related to the spread of the distribution. The following formulas, which we will define, will in general only have significance for symmetrical distributions. Using mathematical statistical theory it turns out that the best value is nothing more than the arithmetic average or **mean** of our measurements, which we will denote with the symbol: $\overline{x}$.

$$\text{Best value} = \text{average} = \textbf{mean} = \overline{x} = \frac{\sum x_i}{N}$$

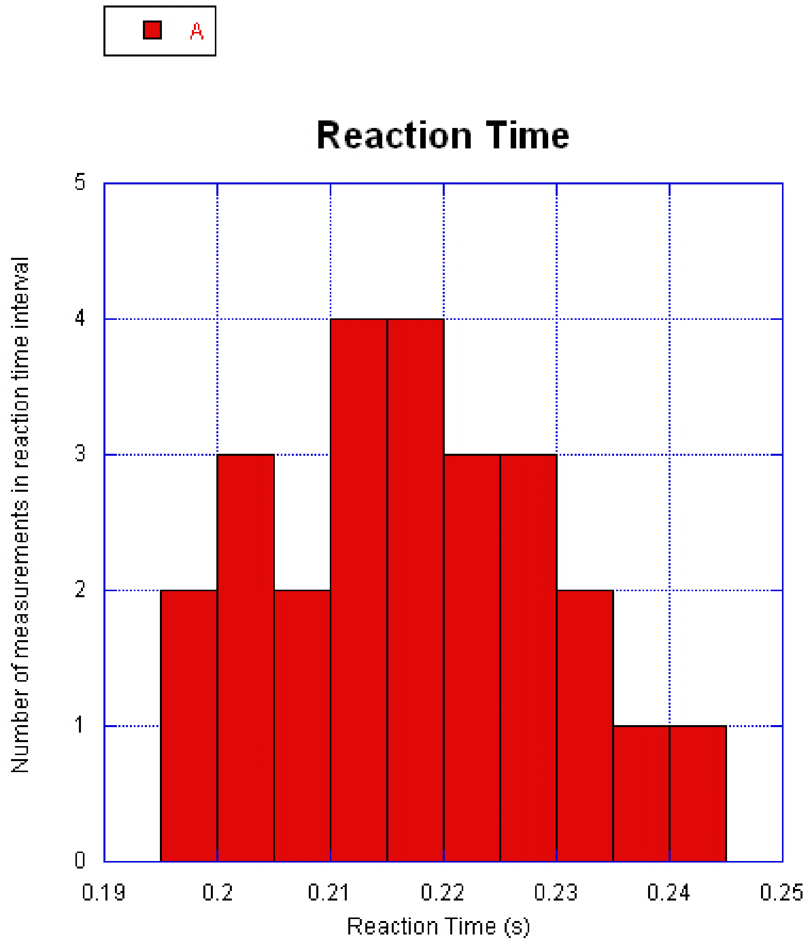


Figure 2.1: Typical histogram (bin size = 0.005 seconds).

where

$$\sum x_i = x_1 + x_2 + x_3 + ... + x_N$$

N is the total number of measurements and x_i are the values of individual measurements (i.e. x_1, x_2, x_3, etc.).

We now need to define a quantity that is connected with the width of the distribution curve. We use a quantity that tells us how the individual measurements deviate from the central (mean) value of the distribution. This is called "**standard deviation**", denoted by "s", and is defined as follows:

$$s = \sqrt{\frac{\sum(x_i - \overline{x})^2}{N-1}}$$

where

$$\sum(x_i - \overline{x})^2 = (x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + (x_3 - \overline{x})^2 + ... + (x_N - \overline{x})^2$$

We are also interested in the uncertainty of $\overline{x}$. That is, by how much $\overline{x}$, calculated for different sets of data, are likely to deviate from each other. This uncertainty is characterized by s_m, the width of the experimental distribution of values of $\overline{x}$ or "**standard deviation of the mean**" which is calculated by

$$s_m = \frac{s}{\sqrt{N}}$$

Note: the larger the number of measurements made of a quantity the smaller the random uncertainty associated with the mean value.

If the number of readings is very high and the bins are small, the histogram approaches a continuous curve and is called a "distribution curve". Many theoretical distribution curves have been defined and their properties evaluated, but the one that is most significant in the theory of measurement is the Gaussian or "Normal" distribution. If all of the experimental data that you have obtained correspond to one and the same physical quantity, then for very large number of measurements they will be described by the Gaussian distribution with its peak at the average value $\overline{x}$.

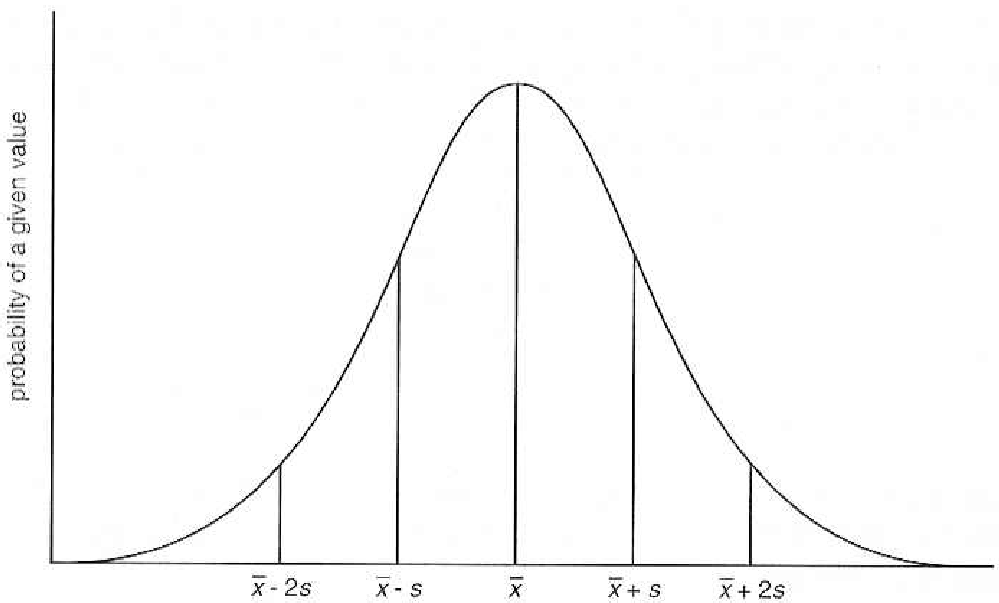


Figure 2.2: Gaussian distribution curve.

Some of the properties of this continuous distribution are that it is symmetric around a peak value and that it falls to zero on either side of the peak, giving it a "bell shaped" appearance (see Fig. 2.2). We use the Greek letter sigma "σ" to represent the standard deviation when referring to a Gaussian distribution and "s" for the standard deviation calculated from **finite** (limited) sets of observations ("s" is the best estimate of "σ" for a finite set of observations). When considering Gaussian distributions, the area enclosed by the range $\pm\sigma$ around the peak will contain 68% of the area of the curve (or 68% of the measurements). This means that an individual measurement has a 68% chance of falling within a region $\pm\sigma$ around the peak, or "mean" value, of the distribution. An area bounded by the range $\pm 2\sigma$ will contain 95% of the area of the curve and therefore represent a 95% chance that an individual measurement will fall within this region of the distribution. This is illustrated in Fig. 2.2.

2.5 In today's lab

In today's lab, you will be measuring your own reaction time and will use the above statistical formulae to hopefully create a Gaussian distribution of your reaction times. There should be sufficient time available to collect data and do the complete lab for your partner and for yourself.

2.6 Equipment

- Stop watch. - To run the stop watch, press `START` to start and press `STOP` or the red button to stop. In this lab, we will start the timer using the red remote start button. After getting your measurement, press the `RESET` button to return the timer to zero. It should be noted that you can increase the precision of the timer by holding the `STOP` button for 2 seconds. After increasing the precision of the timer, the smallest increment of measurement will go from 0.001 seconds to 0.0001 seconds. To go back, simply hold `STOP` again for 2 seconds.

Figure 2.3: The stopwatch used for this experiment.

2.7 Procedure

Note: Before starting, please practice steps 1–3 a few times before recording your data.

1. Put your finger on the `STOP` button while your partner takes the red `START` button in the wired remote.

2. The partner with the `START` button will secretly start the timer.

3. Try and stop the clock as quickly as possible.

4. Record your time in the Time column of the ".xls" spreadsheet in the Reaction folder and reset the timer.

5. Repeat steps 1–4 25 times.

6. On a separate sheet of paper, calculate (by hand) $\overline{x}$, s, and s_m for $N = 5$ trials. Be sure to show your work!

7. On the spreadsheet, calculate the mean by putting the equation "`=SUM(B12:B16)/5`" in cell `C19`. Note that the "SUM" function can be used to find the sum of a group of numbers, and that B12:B16 will evaluate the sum from cell B12 to B16 (B12, B13, B14, B15, B16). The mean is simply the sum divided by the number of values in that sum (in this case we have 5 values). Make sure this value matches the number you calculated by hand.

8. Now fill in column C using the formula "`=(B12-$C$19)`" in the cell C12 and fill down. The use of $ in front of C and in front of 19 "locks" in the cell that has the mean value so that when you fill down, that cell will not change in the formula. For example, when using the fill down feature in excel, the next cell would have the equation "`=(B13-$C$19)`", and so on.

9. Fill in column D by putting "`=C12^2`" in cell D12 and fill down again. Note how this calculated value is used in the formula for standard deviation "s".

10. On the spreadsheet, calculate the standard deviation in the appropriate cell by using the formula "`=SQRT(SUM(D12:D16)/4)`". Here we have that $N = 5$, so our denominator $N - 1 = 4$. Make sure this value matches the number you calculated by hand!

11. Now calculate the standard deviation of the mean in the appropriate cell by using the formula "`=C21/SQRT(5)`". Make sure this value matches the number you calculated by hand.

12. Using the methods above and the equations from the Theory section of this lab, fill in the remaining cells on the excel sheet. You **do not** need to do hand calculations for N=10 and N=25! Your mean and standard deviation of the mean should change as you add more samples to its calculation, but the standard deviation should remain about the same.

13. Record your standard deviation in the box below for future reference. You will need them in a later lab.

$s = \delta t =$ ______________

14. Transfer your data from column B into KaleidaGraph and plot a histogram. Do this by going to `Gallery` ▶ `Stat` and select `Histogram`.

15. Adjust the range of values shown on the x-axis such that the minimum is a few hundredths lower than your lowest measured time and the maximum is a few hundredths greater than your greatest measured time. Do this by going to `Plot` ▶ `Axis Options` ▶ `Limits`, and enter the correct values in their respective boxes.

16. Change the number of bins such that your histogram looks similar to the one shown in Fig. 2.1. Do this by going to `Plot` ▶ `Plot Options` ▶ `Histogram` ▶ `Specifying the Number of Bins`, select `Fixed`, input an appropriate integer number, and press `OK`. Make sure most of the bins are filled in so that there are not many gaps in your histogram.

17. Make sure your histogram is properly labeled and print.

18. Please label by hand the positions of $\overline{x}$, $\overline{x} + s$, and $\overline{x} - s$ on your histogram.

2.8 Comparing Data

It is often necessary to compare two different pieces of data or results of two different calculations and determine if they are compatible (or consistent). In just about every experiment in this course you will be asked if two quantities are compatible or consistent. The following describes how to determine if two pieces of data are consistent (or compatible). Use this procedure to answer the question at the end and use it as a reference whenever you are asked if two pieces of data are compatible or consistent. Let us denote the pieces of data by d_1 and d_2. We'll arbitrarily set d_2 as our expected value, 'e' and d_1 as the data, 'd'. Then we'll apply our usual formulas. If $d = e$ or $d - e = 0$, clearly they are compatible. We often use D to denote the difference between two quantities:

$$D = d - e \tag{2.1}$$

This comparison must take into account the uncertainties in the observation of both measurements. The data values are $d \pm \delta d$ and $e \pm \delta e$. To perform the comparison, we need to find δD. δD is the uncertainty for the difference between d and e as shown in formula 2.1. The addition/subtraction rule for uncertainties is:

$$\delta D = \delta d + \delta e \tag{2.2}$$

Our comparison becomes, "is zero within the uncertainty of the difference D?" This is the same thing as asking if:

$$|D| \leq \delta D \tag{2.3}$$

In Fig. 2.4 we demonstrate three possible cases (A, B, and C) involving consistency checks. As we can see for all three cases, the value for $d \pm \delta d$ is 12 ± 3 ($d = 12$ and $\delta d = 3$). But, as we can see for each case, the value for e changes ($e = 8, 6, 5$ respectively), while the value for δe remains the same ($\delta e = 3$). Case A is consistent as the error bars overlap, case B is consistent as the error bars touch, and case C is inconsistent because the error bars do not overlap or touch.

Equation 2.2 and 2.3 express in algebra the statement "d and e are compatible if their error bars touch or overlap" (see Fig. 2.4). The combined length of the error bars is given by Eq. 2.2. $|D|$ is the separation of d and e. The error bars will overlap if d and e are separated by less than the combined

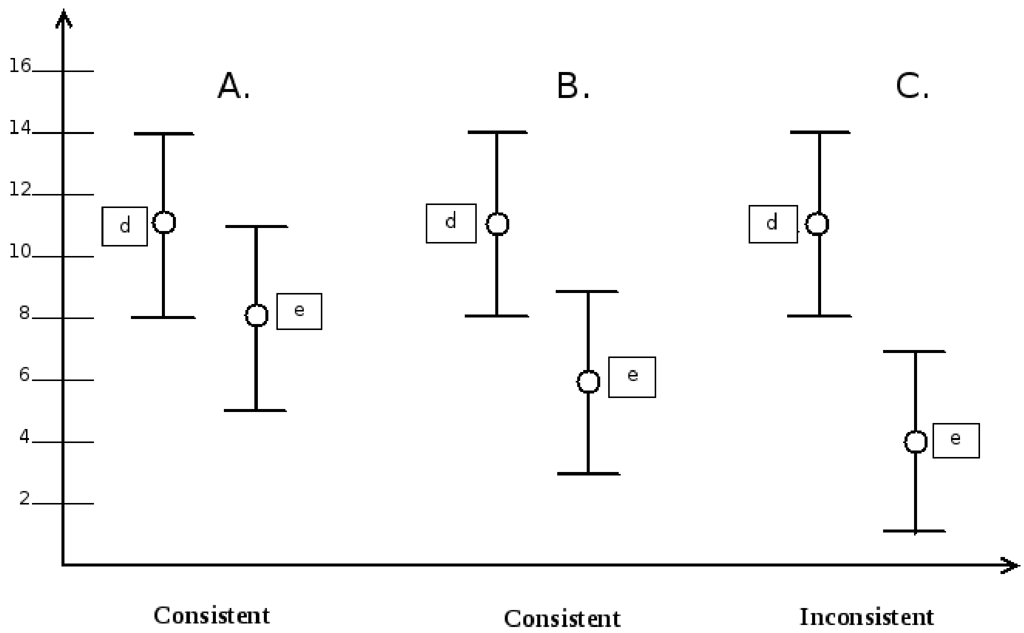


Figure 2.4: Visual representation of a consistency check.

length of their error bars, which is what Eq. 2.3 says. Using Fig. 2.4 and the given equations, we can see that $D = 4, 6, 7$ respectively and $\delta D = 6$ for all cases. We can then see that $|D| \leq \delta D$ for cases A and B, so they are consistent. However, for case C, we can see that $|D| \nleq \delta D$, so the values are not consistent. Sometimes rather than a second measured value you are comparing your data to an expected value. If this is the case, replace $d \pm \delta d$ with $e \pm \delta e$, where $e \pm \delta e$ is the expected value including its uncertainty. For more information on using uncertainties to compare data, see section 4 of Appendix A

2.9 Checklist

1. The filled spreadsheet and formula view.
2. The histogram.
3. Hand calculations.

4. Answered question sheet.

2.10 Questions

1. For all of your 25 measurements indicate on your spreadsheet whether or not each measurement lies between $\overline{x} \pm s$. How many trials would you have expected to be within that range for a pure Gaussian distribution? How many of your trials were in that range for your distribution? If you made many sets of 25 trials of your reaction time, would there always be the same number of trials in that range?

2. Suppose your lab partner was talking to the students at an adjacent lab table when you started the timer. As a result, the time registered on the timer when it was stopped was 10 seconds. How many standard deviations (s) from your mean value does this represent? Should you include this data point with the rest of your data? Why or why not?

3. Compare the mean and standard deviation of $N = 10$ with those for $N = 5$ and $N = 25$. Are the values the same? Why or why not? Explain.

4. If you have already taken 25 measurements, how many more measurements of reaction time would you have to take to reduce s_m by a factor of two, assuming s does not change? Justify your response.

5. Two red blood cell counts are $(4.52 \pm 0.14) \times 10^6 \frac{\text{cells}}{\text{cm}^3}$ and $(4.84 \pm 0.18) \times 10^6 \frac{\text{cells}}{\text{cm}^3}$. Would you conclude that these measurements are consistent with being from the same human? Evaluate the difference and comment. (Use the formulas outlined in the Comparing Data section). Would your answer change if the second blood cell count is $(4.87 \pm 0.18) \times 10^6 \frac{\text{cells}}{\text{cm}^3}$?

Experiment 3

Analysis of a Freefalling Body

3.1 Objectives

- Verify how the distance of a freely-falling body varies with time.
- Investigate whether the velocity of a freely-falling body increases linearly with time.
- Calculate a value for g, the acceleration due to gravity.

3.2 Introduction

Everyday, you experience gravity. This happens because the Earth is so massive, it pulls us down and keeps us on the ground. But happens when we drop something? We notice that as this thing falls to the Earth, it moves faster and faster until it hits the ground. From this we can tell that gravity is **accelerating** the object the entire time the object is in freefall. Today, we will measure how much gravity actually accelerates this object by using the Behr Freefall apparatus and your mathematical skills.

3.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: gravity, velocity, and acceleration

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

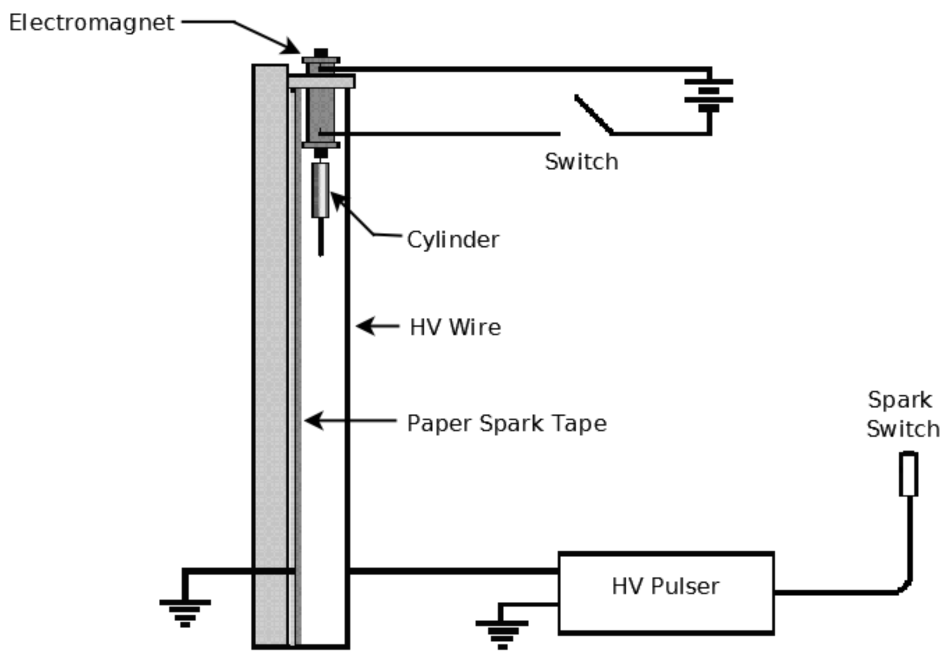


Figure 3.1: Schematic of the Behr Freefall Apparatus.

3.4 Apparatus

A Behr Free-Fall Apparatus and Spark Timing System will be used in this experiment. A schematic representation of the apparatus is shown above in Fig. 3.1 and a digital photograph of the apparatus is shown in Fig. 3.2 on the following page.

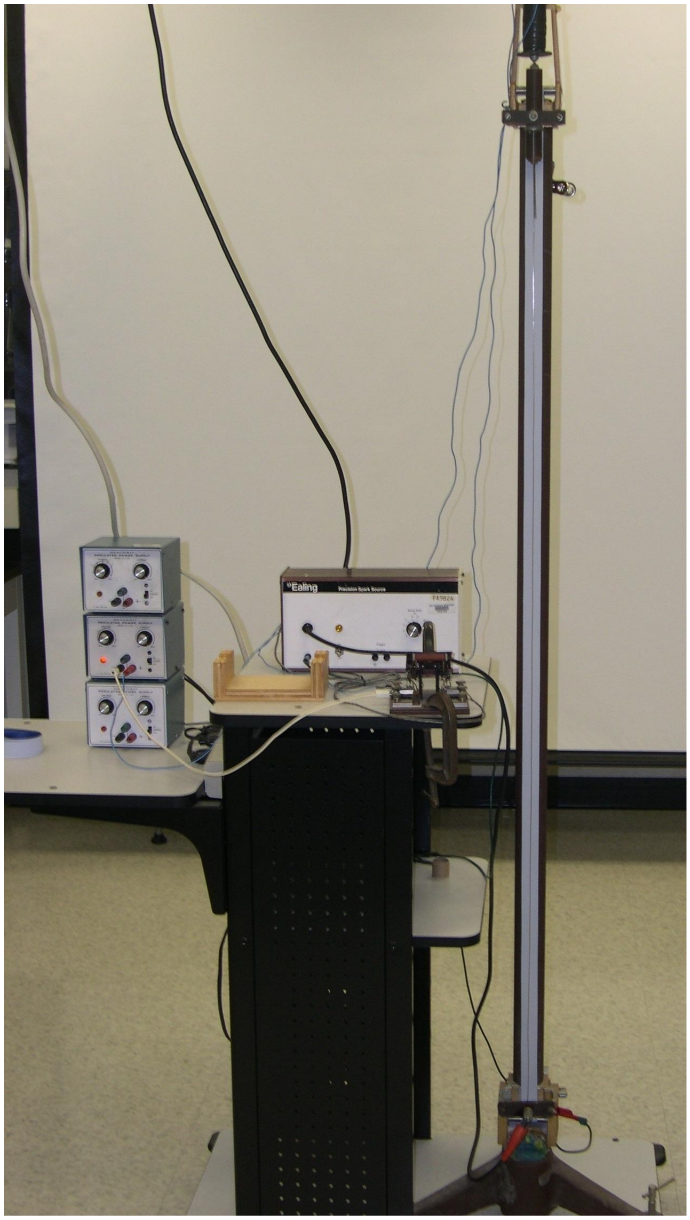


Figure 3.2: Schematic of the Behr Freefall Apparatus.

3.5 Theory

In this experiment a cylinder is dropped and a record of its free fall is made. Before the measurement, the cylinder is suspended at the top of the stand with the help of an electromagnet. When the electromagnet is turned off, the cylinder is released and starts to fall. Simultaneously, the spark timer starts to send high-voltage pulses through two wires which are stretched along the cylinder's path. At the time of each pulse a spark goes through the wires and the cylinder, leaving a mark on the special paper tape that lies between the cylinder and one of the wires. The **time interval** between two adjacent sparks is constant and is denoted by the Greek letter tau "τ". $\tau = 1/60$ of a second. Measuring the distances between any two marks, Δy, and knowing the time interval between the corresponding sparks, Δt, it is possible to calculate the **average** velocity during this interval using the formula

$$v = \frac{\Delta y}{\Delta t} \tag{3.1}$$

If Δt is small enough, we can assume that the velocity at any instant within this interval is approximately equal to this average velocity. In the case where acceleration is constant, the **instantaneous velocity** at the middle of the time interval Δt is exactly equally to the average velocity of the object during the time interval Δt.

In general, for the motion of a body with a constant acceleration a, the velocity v is given by the equation

$$v = at + v_0 \tag{3.2}$$

where v_0 is the velocity of the cylinder at $t = 0$. Since in our case the body is falling freely,

$$a = -g \tag{3.3}$$

where $g = 9.81 \text{m/s}^2$ is the magnitude of the acceleration due to gravity. The negative sign in front of g is to indicate that the direction of the acceleration is in the negative direction (i.e. downward). Therefore it follows from Eq. 3.2 that for a freely-falling body

$$v = v_0 - gt \tag{3.4}$$

Thus g can be determined from a plot of v vs. t since the slope of any velocity versus time graph is just the acceleration. The obtained value of g can then be compared with the known value of the acceleration due to gravity. The position of the cylinder, y, as a function of time is given by the standard equation for an object that is undergoing constant acceleration. If at time $t = 0$ the object has height y_0 and velocity in the vertical direction v_0, then this equation looks like

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{3.5}$$

3.6 In today's lab

At the start of today's lab, your instructor will demonstrate the operation of the Behr Freefall apparatus. In the interest of saving time, you will be supplied with a shock tape from the Behr freefall apparatus. On this tape, you will then measure the distance between the "dots," and using the given τ, you will calculate the value of acceleration for g.

3.7 Equipment

- Behr Freefall Apparatus
- Shock tape
- Meter stick

3.8 Procedure

1. Secure both ends of the shock tape to the desk using masking tape, making sure that the shock tape is as flat as possible.
2. The "bottom" of the tape is defined as the end of the tape with the largest, bold black dot, also where the dots become farther apart. Starting from the **third** dot from the bottom, label each successive point from #25 to #1 in **descending** order. It is ok if you have unused marks left over. Point #25 will now be defined at $y = 0$ and point #1 will be defined at $t = 0$.

3. Using your meter stick, measure each point's distance (in cm) from $y = 0$ (point #25) and write it down on the shock tape.

4. Input your measured distances into excel paying special attention to which point number you are putting the distance into. Also input a reasonable uncertainty for your measurement in excel.

5. Input the correct times for each point using the given value of τ.

6. Calculate the instantaneous velocity v_i for each point y_i. Here we have that $v = \frac{\Delta y}{\Delta t}$. Δy for each point i is defined as $\Delta y = y_{i+1} - y_{i-1}$ and Δt is likewise defined as $\Delta t = t_{i+1} - t_{i-1}$. For example, we can see that
$$v_2 = \frac{y_3 - y_1}{t_3 - t_1}. \tag{3.6}$$
Note that Δt will be the same value for each point, which happens to just be 2τ. Please include at least 1 hand calculation for these values.

7. Now calculate the uncertainty for your velocity at each point using the equation $\delta v_i = \frac{\delta y}{\tau}$.

8. Transfer your data columns for "Time", "y", and "v" into Kaleida-Graph.

9. Make a graph for y vs. t. You do not need a best fit line or error bars for this plot.

10. Make a graph for v vs. t. Be sure to include a best fit line and error bars for v. Be sure to write in plot comments for both of your plots. The error calculation is given in section 3.9.

3.9 Error Calculation

For each measured y_i you assign an error based on how accurately you can measure that point. This error is called δy. This error determines all other errors in this lab. For this lab and for the following formulae it is assumed that the error in τ and m are zero.

There error in Δy at each point i is the same and is given by
$$\delta(\Delta y) = 2\delta y \tag{3.7}$$

The error in the speed at each point i is

$$\delta(v_y) = v_y \frac{\delta(\Delta y)}{\Delta y} = v_y \frac{2\delta y}{\Delta y} = \frac{(\Delta y) 2\delta y}{2\tau(\Delta y)} = \frac{\delta y}{\tau} \tag{3.8}$$

3.10 Checklist

1. Your spreadsheet and formula view.
2. Sample calculations.
3. Plot of the height vs. time Graph I.
4. Plot of velocity vs. time Graph II.
5. Interpretation of the two plots.
6. Answered questions.
7. One member of each group should turn in your spark tape record of the free-fall.

3.11 Questions

1. What is the y-intercept determined from your Graph II (or from the equation of its best-fit line)? What does it mean?

2. Combining equation 3.4 and the equation of the best fit line from Graph II, calculate the time at which $v = 0\ cm/s$. Explain this result.

3. What is your value of the gravitational acceleration in cm/s^2 determined from the slope from graph II? Is this value compatible with the accepted value of the gravitational acceleration? If not suggest a possible source of error (**NEVER** just suggest "human error" or a "mistake").

4. If the initial position and velocity were exactly zero, what would you plot to make graph I linear: y^2 vs. t, y^2 vs. t^2, or y vs. t^2? As always, justify your response.

Experiment 4

Newton's Second Law

4.1 Objectives

- Test the validity of Newton's Second Law.
- Measure the frictional force on a body on a "low-friction" air track.

4.2 Introduction

While dealing with physics, you have most likely heard of the famous Newton's Laws. While each one is important in its own way, Newton's Second Law is probably the most **practically** important. Newton's Second Law states that when a force acts on an object, it will accelerate. The more mass that object has, the more force it will take for that object to accelerate faster.

4.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: Newton's laws

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

4.4 Theory

Newton's Second Law states that the acceleration of a body is proportional to the net force acting on the body ($a \propto F_{NET}$) and inversely proportional to the mass of the body ($a \propto \frac{1}{m}$). Combining these two, we can replace the proportionality with equality. That is,

$$a = \frac{F_{NET}}{m}$$

or

$$F_{NET} = ma$$

F_{NET} is the **sum of all of the forces** acting on the body. In many textbooks this is denoted by $\sum F$. So, **Newton's Second Law** is:

$$\sum_{i=1}^{n} F = ma \tag{4.1}$$

Note: in the cgs system of units, the dyne is the unit of force:

$$(1\ dyne = 1\frac{g\ cm}{s^2})$$

In this experiment a low friction air track will be used to test the validity of Newton's Second Law. A hanging mass will be attached to a glider placed on the air track by means of a light (negligible mass) string. By varying the mass of the hanging mass we will vary the net force acting on this two body system. We will however, keep the total mass of the system constant. This is accomplished by moving mass from the glider to the hanger. With the air track turned on, the hanging mass will be released and the glider will pass through two photogate timers. The photogate timers will be used to measure two velocities. Recall that $v = \frac{\Delta x}{\Delta t}$. In our case Δx will be the length of a fin place on top of the glider. If you know the separation between the two photogate timers, you can use an equation from kinematics to determine the acceleration of the glider:

$$v_2^2 = v_1^2 + 2aS$$

Where v_2 is the velocity measured with the **second photogate**, v_1 is the velocity measured with the **first photogate**, a is the **acceleration** and

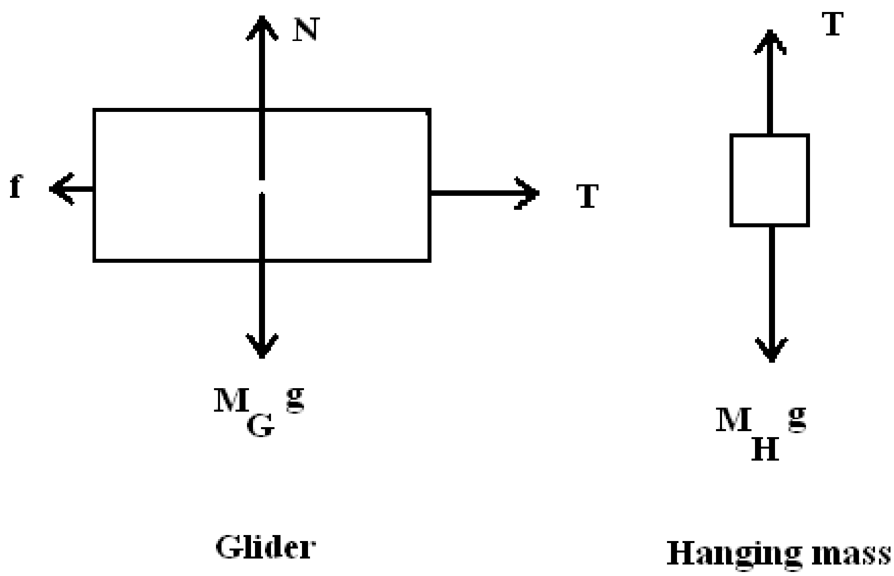


Figure 4.1: Freebody diagrams

S is the distance **between the two photogate timers**. Solving for the acceleration yields:

$$a = \frac{v_2^2 - v_1^2}{2S} \tag{4.2}$$

Separate free body diagrams of the glider and the hanging mass are shown in Fig. 4.1. In the figure, f is the net frictional force acting on the body (assume this includes the frictional forces between the airtrack and the glider and the frictional losses in the pulley; N is the upward force the air track exerts on the glider; T is the **tension** in the string; $M_G g$ is the weight of the **glider**; and $M_H g$ is the weight of the **hanging mass**. Since the air track is horizontal and the glider does not accelerate in the vertical direction, $N = M_G g$. Applying Newton's Second Law to the glider in the horizontal direction and using right as the positive direction yields:

$$T - f = M_G a \tag{4.3}$$

If we now apply Newton's Second Law to the hanging mass, and this time define *downward* as the positive direction we find:

$$M_H g - T = M_H a \tag{4.4}$$

We have no way to directly measure the tension in the string (T), therefore we will combine equations 4.3 and 4.4 to eliminate the tension from the resulting equation:

$$F_H - f = (M_H + M_G)a \tag{4.5}$$

There are only two unbalanced forces acting on our two-mass system (i.e. the weight of the hanging mass and friction). Notice what equation 4.5 states: the left hand side is the net force and the right hand side is the product of the system's mass and its acceleration. This is Newton's Second Law applied to our two body system. If we rearrange equation 4.5 we obtain:

$$F_H = (M_H + M_G)a + f \tag{4.6}$$

Equation 4.6 has the same form as the equation for a straight line $y = mx + b$, where the weight of the hanging mass (F_H) plays the role of y and the acceleration (a) plays the role of x!

4.5 In today's lab

Today we will use the frictionless air track to measure how force affects acceleration. We will measure the velocity of the glider while keeping the total mass of the system the same (total mass = mass of hanger + mass of glider). We'll just redistribute the mass by moving it from the glider to the hanger. Using the velocities at both photogates, we will then be able to find the acceleration of the cart.

4.6 Equipment

Do not move the glider on the track while the air is turned off!

- Air track

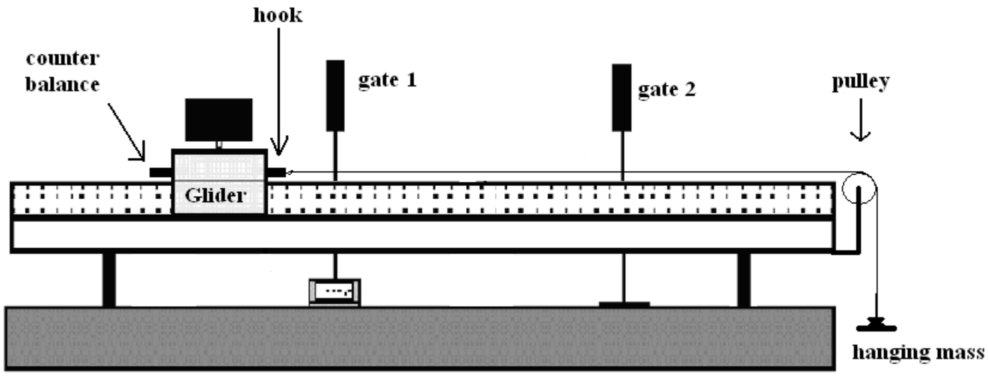


Figure 4.2: Diagram of the apparatus

- Glider
- Hanger
- 2 Photogates
- 5g, 10g, 20g masses

4.7 Procedure

1. Set up the air track as shown in Figure 4.2. With the hanging mass disconnected from the glider and the air supply on, level the air track by carefully adjusting the air track leveling feet. The glider should sit on the track without accelerating in either direction. There may be some small movement due to unequal air flow beneath the glider, but it should not accelerate steadily in either direction.

2. Measure the length (L) of the fin on top of the glider and record it along with its uncertainty in your spreadsheet. See Figure 4.3.

3. Make sure the hook and counter balance are both inserted in the lower hole on the glider.

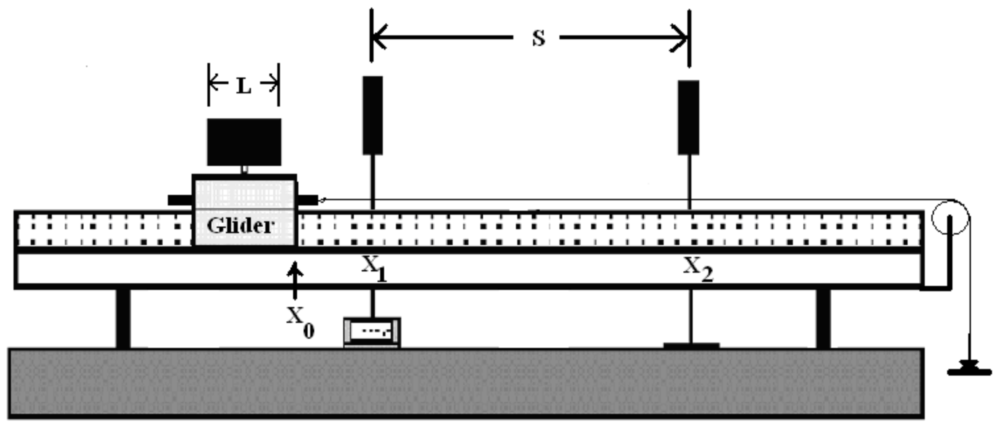


Figure 4.3: Various lengths used throughout this experiment

4. Measure the mass of the empty glider and empty hanger (M_{G_0} and M_{H_0}) and record these masses in your spreadsheet.

5. Using the 5, 10 and/or 20 gram masses, place 40 grams of mass on the glider. Distribute the masses **symmetrically** so that the glider is balanced. Determine the total mass of the glider (M_{G_0} + the mass you just added) and record this in your spreadsheet.

6. Place 10 grams of mass on the mass hanger. Record this in your spreadsheet and have Excel calculate the total mass of the hanger ($M_H = M_{H_0}$ + the mass you just added to the hanger).

7. Determine the total mass of your system ($M_G + M_H$). Note: this should be the sum of the masses you entered in steps 5 and 6. Record the system mass in your spreadsheet. This mass should remain constant thoughout the experiment.

8. Choose a starting position (X_0) for the glider near the end of the track. X_0 must be at least 25 cm away from the first photogate (more on this later). Using the ruler permanently affixed to the air track, measure and record this location in your spreadsheet.

9. Measure and record the locations of the two photogate timers (X_1 and X_2) and assign a reasonable uncertainty to these positions (δx). It is very important that your glider always starts from the same location (X_0) and that the two photogate timers are not moved. If they are accidentally bumped or moved, return them to their original location.

10. Calculate the magnitude of the displacement between the two timers using $S = |X_2 - X_1|$ and record this in your spreadsheet. S must be at least 20 cm. See Figure 4.3. Also, make sure that the hanging mass does **not** touch the ground before the cart moves past x_2.

11. Calculate the uncertainty in this displacement $\delta S = 2\delta x$.

12. Set your photogate timer to `GATE` mode and make sure the memory switch is set to on. The `GATE` mode will only record time when the glider is passing through one of the two photogates. In this mode, the timer will only display the time the glider took to pass through the first photogate (t_1). The time the glider took to pass through the second photogate will be added to the memory. Flipping the toggle switch to read will display the total time the glider took to pass through both photogates (t_{mem}). To obtain the time the glider took to pass through the second photogate, simply subtract t_1 from the time stored in the photogate's memory $t_2 = t_{mem} - t_1$. The uncertainty in a measurement of time is $\delta t = 0.5$ms. Using the rules for addition and subtraction, $\delta t_2 = 2\delta t = 1.0$ms.

13. With the air supply on, hold the leading edge of the glider stationary at X_0; press the reset button on the photogate timer, then release the glider. Make sure the glider does not bounce off the far end of the air track and pass through the second photogate a second time. The time displayed on the photogate's screen will be the time the glider took to pass through the first photogate (t_1). Record t_1 in your spreadsheet. Flip the memory toggle switch to read the displayed time (t_{mem}) in your spreadsheet; use the displayed time (t_{mem}) and t_1 to calculate t_2.

14. Return the glider to X_0. Make sure all of the masses are still on the hanging mass hanger and the string is still over the pulley.

15. Move 10 grams from the glider to the hanger. Calculate the total glider mass and the new total hanger mass. (NOTE: the total system mass has not changed.) Repeat steps 13–14.

16. Repeat step 15 until no mass remains on the glider. You should have data for 10g, 20g, 30g, 40g and 50g added to your hanger.

17. Have Excel calculate v_1, v_2 and their respective uncertainties.

$$v = \frac{L}{t}$$

and

$$\delta v = v\left(\frac{\delta L}{L} + \frac{\delta t}{t}\right)$$

18. Have Excel use equation 4.2 to calculate the acceleration.

19. Excel will calculate the uncertainty in the acceleration δa. **This formula is already programmed in for you.**

$$\delta a = a\left[\frac{\delta S}{S} + \frac{2(v_2\delta v_2 + v_1\delta v_1)}{(v_2^2 - v_1^2)}\right]$$

The acceleration formula (4.2) is derived assuming assuming the point at which the **instantaneous velocity** of the glider at gate 2 (v_2) is a distance S away from the point where the instantaneous velocity of the glider at gate 1 (v_1) is measured. This is not quite true and introduces a small systematic error into our calculations. By keeping $S > 20$cm and $|X_1 - X_0| > 25$cm, we keep this systematic error less than 0.5%. The contribution of this systematic uncertainty is also included in the Excel calculation of δa as described above.

20. Have Excel calculate the weight ($F_H = M_H * g$, where $g = 981\ \frac{\text{cm}}{\text{s}^2}$) of each of the hanging masses. Transfer your data into KaleidaGraph and make a graph of F_H vs. a. Make sure you include horizontal error bars associated with the acceleration (a) and fit your graph with a best-fit line. Your graph should include error bars, a best fit line along with its equation and the uncertainty of the slope. Note that your error bars here will not have a constant value. You will need to import the

uncertainty in the acceleration (δa) data column into KaleidaGraph. Also, when adding error bars rather than choosing `Fixed Value` from the `Error Bars Settings` window, select `Data Columns` and choose the data column containing your values for δa.

4.8 Checklist

1. Excel sheets (data view and formula view)
2. Plot with **horizontal** error bars and best fit line
3. Question sheet

4.9 Questions

1. From the equation of your best fit line and equation 4.6, what is the net frictional force acting on the glider? Is this value significantly different from zero?

2. Discuss the consistency of the slope of your best fit line with the theoretical value (see equation 4.6). If it is not consistent, suggest possible reasons for the discrepancy.

3. Are your results consistent with Newton's Second Law ($F = ma$)? Why or why not?

Experiment 5

Inelastic Collisions

5.1 Objectives

- Measure the momentum and kinetic energy of two objects before and after a perfectly inelastic one-dimensional collision.
- Observe that the concept of **conservation of momentum** is independent of **conservation of kinetic energy**, that is, the total momentum remains constant in an inelastic collisions while the kinetic energy changes.
- Calculate the percentage of KE which will be lost (converted to other forms of energy) in a perfectly inelastic collision between an initially stationary mass and an initially moving mass.

5.2 Introduction

One of the most important concepts in the world of physics is the concept of conservation. We are able to predict the behavior of a system through the **conservation of energy** (energy is neither created nor destroyed). An interesting fact is that while total energy is **always** conserved, kinetic energy is not. However, momentum is always conserved in both elastic and inelastic collisions. In this experiment and the following experiment, we will see how momentum always remains a conserved quantity while kinetic energy does not.

5.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: elastic collision, and inelastic collision.

5.4 Theory

The following two experiments deal with two different types of one-dimensional collisions. Below is a discussion of the principles and equations that will be used in analyzing these collisions. For a single particle, **momentum** is defined as the product of the mass and the velocity of the particle:

$$p = mv \tag{5.1}$$

Momentum is a **vector** quantity, making its direction a necessary part of the data. For the one-dimensional case, the momentum would have a direction in either the $+x$ direction or the $-x$ direction. For a system of more than one particle, the **total momentum** is the vector sum of the individual momenta:

$$p = p_1 + p_1 + ... = mv_1 + mv_2 + ... \tag{5.2}$$

So you just add the momentum of each particle together. One of the most fundamental laws of physics is that the **total momentum** of any system of particles is **conserved**, or constant, as long as the net external force on the system is zero. Assume we have two particles with masses m_1 and m_2 and velocities v_1 and v_2 which collide with each other without any external force acting. Suppose the resulting velocities are v_{1f} and v_{2f} after the collision. **Conservation of momentum** then states that the total momentum before the collision ($p_{initial} = p_i$) is equal to the *total* momentum after the collision ($p_{final} = p_f$):

$$p_i = m_1 v_{1_i} + m_2 v_{2_i} \qquad p_f = m_1 v_{1_f} + m_2 v_{2_f} \qquad p_i = p_f \tag{5.3}$$

In a given system, the **total energy** is generally the sum of several different forms of energy. **Kinetic energy** is the form associated with motion, and for a single particle

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

$$KE = \frac{mv^2}{2} \tag{5.4}$$

In contrast to momentum, kinetic energy is *not* a vector; for a system of more than one particle the total kinetic energy is simply the sum of the individual kinetic energies of each particle:

$$KE = KE_1 + KE_2 + ... \tag{5.5}$$

Another fundamental law of physics is that the **total energy** of a system is **always conserved**. However within a given system one form of energy may be converted to another, such as in the freely-falling body lab where potential energy was converted to kinetic energy. *Therefore, kinetic energy* alone *is often not conserved.*

There are two basic kinds of collisions, elastic and inelastic:

In an **elastic collision**, two or more bodies come together, collide, and then move apart again with *no loss in kinetic energy.* An example would be two identical "superballs," colliding and then rebounding off each other with the same speeds they had before the collision. Given the above example conservation of kinetic energy then implies

$$\frac{m_1 v_{1_i}^2}{2} + \frac{m_2 v_{2_i}^2}{2} = \frac{m_1 v_{1_f}^2}{2} + \frac{m_1 v_{2_f}^2}{2} \qquad KE_{initial} = KE_{final} \tag{5.6}$$

In an **inelastic collision**, the bodies collide and come apart again, but *some kinetic energy is lost.* That is, some kinetic energy is converted to some other form of energy. An example would be the collision between a baseball and a bat.

If the bodies collide and stick together, the collision is called **perfectly inelastic**. In this case, *much of the kinetic energy is lost* in the collision. That is, much of the kinetic energy is converted to other forms of energy.

In the following two experiments you will be dealing with a perfectly inelastic collision in which much of the kinetic energy of the objects is lost, and with a nearly elastic collision in which kinetic energy is conserved. Remember, in both of these collisions total momentum should always be conserved.

Since we are considering inelastic collisions today, let's consider what the kinetic energy should be in the initial and final states. If we look at Eq. 5.4, we can see that the initial kinetic energy is

$$KE_i = \frac{m_1 v_{1_i}^2}{2} + \frac{m_2 v_{2_i}^2}{2} = \frac{m_1 v_{1_i}^2}{2} \tag{5.7}$$

because Cart 2 is initially at rest ($v_{2_i} = 0$).
The final kinetic energy is defined as

$$KE_f = \frac{(m_1 + m_2) v_f^2}{2} \tag{5.8}$$

because the two carts have stuck together after the collision ($v_f = v_{1f} = v_{2f}$ is the common velocity of the two carts).
Using the conservation of momentum, we can calculate the final momentum as

$$m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_i} = (m_1 + m_2) v_f \tag{5.9}$$

Using Eqs. 5.7, 5.8, and 5.9, we arrive at the equation for KE_f in terms of KE_i.

$$\boxed{KE_f = \left(\frac{m_1}{(m_1 + m_2)}\right) KE_i} \tag{5.10}$$

This is the prediction for the final kinetic energy of a perfectly inelastic collision.

5.5 In today's lab

Today you will get to see how inelastic collisions work while you vary the masses on two colliding carts. You will then see how there is a significant energy loss in these types of collisions and will try to figure out where this energy goes.

5.6 Equipment

- Air Track
- Air Supply

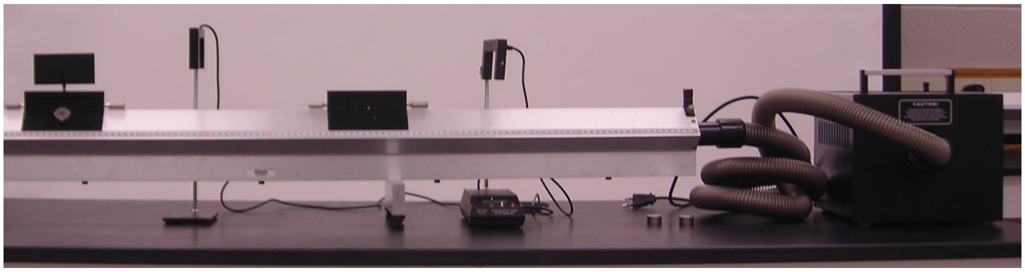


Figure 5.1: Equipment used in lab fully set up.

- Two carts one with needle and one with clay (carts are sometimes called gliders)
- Photogate Circuit
- 4 - 50g masses

5.7 Procedure

Do not move the carts on the air track when the air is not turned on. It will scratch the track and ruin the "frictionless" environment we need to get accurate data.

1. Start by making sure that the air track is level. Your instructor will demonstrate how at the beginning of class.
2. Set up the photogates such that there is sufficient room for the collision to happen in the middle and enough room on the remainder of the track for the carts to move freely.
3. Set the photogates to `GATE` mode.
4. We will define Cart 1 as the cart with the fin and Cart 2 as the cart without. We will always push Cart 1 for each trial and will always start with Cart 2 stationary ($v_{2_i} = 0$cm/s) in the middle. Before placing the carts on the track, measure the mass of them *without* the extra masses. Record the empty cart masses data on the given results sheet.

5. Measure the length of the fin on Cart 1 and record this on your results sheet and in excel. Be sure to put a reasonable uncertainty for the fin length in excel as well.

6. Input the uncertainty for the times measured by the photogate into excel (0.0005 s).

7. Put all four 50g masses on Cart 2 such that it is evenly distributed (2 masses on each side).

8. Place Cart 2 in between the photogates and have one partner hold it steady up until the collision takes place.

9. Place Cart 1 "outside" of the photogates.

10. Making sure that your photogates are reset, give a brief but firm shove to Cart 1 such that it collides and sticks together with Cart 2. Allow the two carts to leave the middle completely before stopping them. **Do not allow the carts to pass through the photogates again until you finish recording their times.**

11. Record the time for Cart 1 to pass through the first photogate (t_i) in excel, then press the `READ` switch and record (t_{mem}) as well.

12. Calculate the time for the combined cart system to pass through the second photogate using the formula $t_f = t_{\text{mem}} - t$ and input that into your notebook and excel file.

13. Note that the initial velocity of Cart 1 (v_{1_i}) is calculated using the formula L/t_i and that the final velocity of the combined cart system is calculated using the formula L/t_f.

14. Make sure that the absolute value of the percent difference between initial and final momentum is **less than** 5% (the spreadsheet does this calculation for you). If it is not, rerun the trial until it is. Compress the putty in the counterbalance in between trials. Also make sure that the fin on top of Cart 1 is completely through the photogate before the collision occurs. If the trial is acceptable, record the times on your worksheet. Always try to keep your best trial written down on your worksheet, even if it does not fit our desired percent difference.

15. Repeat this trial one more time and record the results.
16. Repeat steps 6–15 for the cases when you have:
    - 2 mass disks on Cart 1 and 2 mass disks on Cart 2
    - 2 mass disks on Cart 1 and no mass disks on Cart 2
17. Be sure to include hand calculations for the light blue boxes in excel.

5.8 Uncertainties

In today's experiment we have already input all of the equations into excel for you out of the interest of brevity, but it is important to understand the uncertainties for the values you used in this experiment. The uncertainty for velocity is:

$$\delta v = v\left(\frac{\delta L}{L} + \frac{\delta t}{t}\right)$$

The uncertainty for momentum is:

$$\delta P = P\frac{\delta v}{v}$$

And the uncertainty for kinetic energy is:

$$\delta KE = 2KE\frac{\delta v}{v}$$

The uncertainties for the differences for the momenta and kinetic energies are then:

$$\delta P_{\text{diff}} = \delta P_f + \delta P_i \text{ and } \delta KE_{\text{diff}} = \delta KE_f + \delta KE_i$$

5.9 Checklist

1. Excel sheets
2. Questions
3. Hand Calculations

5.10 Questions

1. For which of your trials was momentum conserved? The scientific way to address this question is to ask, for which of your trials is P_{diff} compatible with zero? Also, if momentum is not conserved for any of your trials, suggest a possible source of error.

2. Was kinetic energy conserved for any of your trials? If not, how much kinetic energy was lost?

3. Compare one of your measured KE_f trials with the $KE_{f\ calc}$ prediction of equation 5.10 for a perfectly inelastic collision. Use your measured masses and KE_i value. Are they compatible? The uncertainty of $KE_{f\ calc}$ is:

$$\left(\delta KE_{f\ calc} = KE_{f\ calc}\frac{\delta KE_i}{KE_i}\right)$$

4. Combine equations 5.7, 5.8 and 5.9 to obtain the expression in equation 5.10. Hint: solve equation 5.9 for v_f, then substitute this into equation 5.8.

Experiment 6

Elastic Collisions

6.1 Objectives

- Measure the momentum and kinetic energy of two objects before and after a one-dimensional collision.
- Try to account for any change in KE in the nearly elastic collision.
- Compare and contrast the results obtained from the inelastic collision experiment with the results obtained from this experiment.

6.2 Introduction

Now that we are acquainted with inelastic collisions, it is time to investigate elastic collisions. This time around, we will observe the conservation of both momentum *and* kinetic energy. Energy can tell us a great deal about how a system works, and if it is conserved, we can understand the process much better than if we cannot. Though we are not in the ideal and impossible conditions of a perfectly frictionless system and in vacuum, we will be able to see these conservation laws with very good precision.

6.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: elastic collision.

6.4 Theory

Please refer to the inelastic collision lab to refresh your knowledge of the theory for collisions.

6.5 In today's lab

In today's lab, we will observe the effects on changing the mass in elastic collisions between two carts. After collecting our data, we will then compare the results of elastic collisions with those of inelastic collisions and try to understand where energy loss can occur.

6.6 Equipment

- Air Track
- Air Supply
- Two carts (one with bumper and one with blade, see Fig. 6.1)
- Photogate Circuit
- 4 50g masses

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

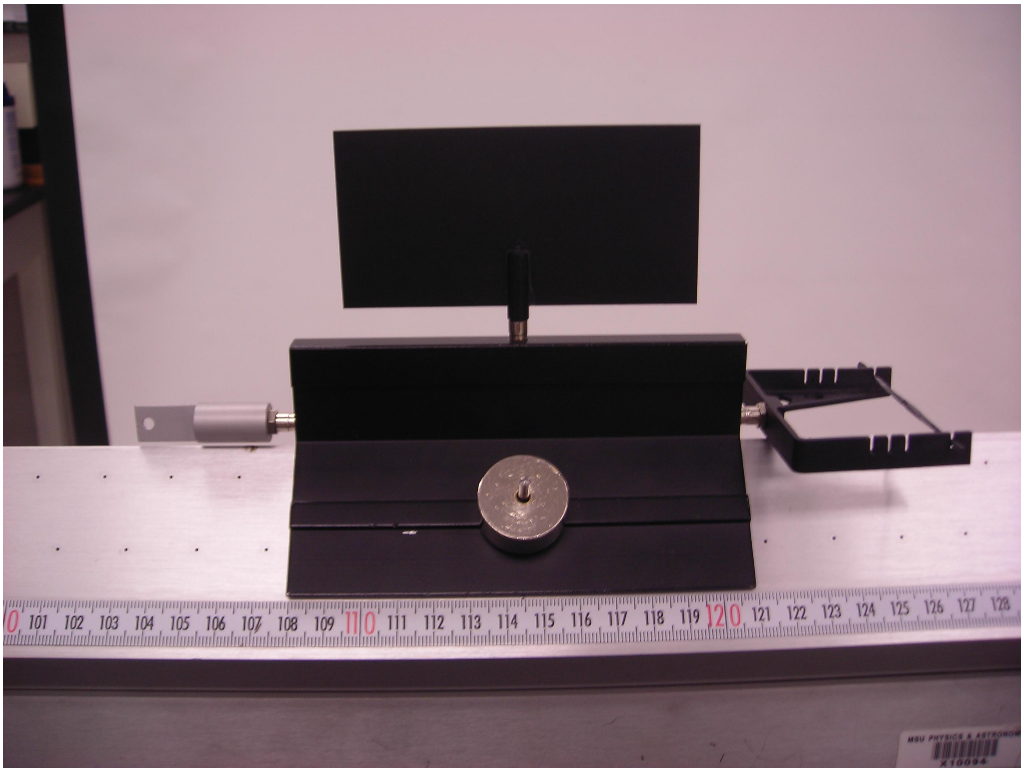


Figure 6.1: Cart with bumper.

6.7 Procedure

1. Start by making sure that the air track is level. Your instructor will demonstrate how at the beginning of class.

2. Set up the photogates such that there is sufficient room to reset the timer before the collision, for the collision to happen in between the photogates, and enough room on the remainder of the track for the carts to move freely.

3. Set the photogates to `GATE` mode.

4. We will define Cart 1 as the cart with the *bumper* and Cart 2 as the cart with the *bumper blade*. We will always push Cart 1 for each trial and will always start with Cart 2 stationary ($v_{2_i} = 0$cm/s) in the

middle. Before placing the carts on the track, measure their mass *without* the extra masses. Record the empty cart masses data on the given results sheet.

5. Measure the lengths of the fins on both carts and record them on your worksheet and in excel. Be sure to put a reasonable uncertainty for the fin length in excel as well.

6. Input the uncertainty for the times measured by the photogate into excel (0.0005 s).

7. Put 2 masses on Cart 2 so they are evenly distributed (1 on each side) and no masses on Cart 1.

8. Place Cart 2 in between the photogates and have one partner hold the cart steady until the collision occurs.

9. Place Cart 1 "outside" of the photogates.

10. Making sure that your photogates are reset, give a brief but firm shove to Cart 1 such that it collides with Cart 2. Allow the two carts to leave the middle completely before stopping them. **Do not let the carts drift back through the photogates until you finish recording the times.** Be sure to have one partner memorize the first time that appears on the photogate (t_i) and reset the photogate **before either cart passes through the photogates following the collision**. It may take multiple tries to get this method correct, so feel free to practice a few times.

11. Record t_i and in Excel. Also record the other times off of the photogate in Excel. **If you reset the timer at the correct moment, the time you read on the photogate prior to flipping the `READ` switch will be either t_{1_f} or t_{2_f} depending on which cart left the middle first. The *other* time will be calculated the same way as before using t_{mem}.**

12. Once again, note that the velocities for each cart are calculated in the same way as before (i.e. L/t).

13. Make sure that the absolute value of the percent difference between initial and final momentum is less than 5% *and* the absolute value of

the percent difference between initial and final kinetic energy is **less than** 10% (the spreadsheet does these calculations for you). If they are not, rerun the trial until they are. Make sure that the fin on top of Cart 1 is completely through the photogate before the collision occurs and that the carts remain outside of the photogates until you have your times recorded. If the trial is acceptable, record the times on your worksheet. Always keep your best trials recorded on the worksheet just in case you run out of time and need to use those.

14. Repeat this trial one more time and record the results.

15. Repeat steps 8–14 for the cases when you have:

    - 2 mass disks on Cart 1 and 2 mass disks on Cart 2
    - 2 mass disks on Cart 1 and no mass disks on Cart 2

    For trials 3 and 4, you can use 1000s for t_{1_f} as in an ideal case, Cart 1 will transfer all of its momentum to Cart 2 and will stop moving after the collision.

16. Be sure to include hand calculations for the light blue boxes in excel.

6.8 Checklist

1. Excel Sheets
2. Questions
3. Hand Calculations

6.9 Questions

1. For which of your trials was momentum conserved? Justify your response. Also, if momentum is not conserved for any of your trials, suggest a possible source of error.

2. For which of your runs was kinetic energy conserved? Justify your response. Also, if energy is not conserved for any of your trials, suggest a possible source of error.

3. Momentum was supposed to be approximately conserved in both the inelastic and elastic collision experiments. Compare the percent change in kinetic energy for the inelastic collisions experiment with the elastic collisions experiment.

4. Where did the kinetic energy go that was lost in the inelastic collisions you performed in the previous experiment?

Experiment 7

Rotational Motion: Moment of Inertia

7.1 Objectives

- Familiarize yourself with the concept of moment of inertia, I, which plays the same role in the description of the rotation of a rigid body as mass plays in the description of linear motion.
- Investigate how changing the moment of inertia of a body affects its rotational motion.

7.2 Introduction

In physics, we encounter various types of motion, primarily linear or rotational. We have already learned how linear motion works and the relevant quantities we need to look at in order to understand it. Today we will investigate rotational motion and measure one of the most important quantities pertaining to that: the moment of inertia. The way mass is distributed greatly affects how easily an object can rotate. For example, if you are sitting in an office chair and start spinning around, you can notice that if you extend your arms away from your body, you will begin to rotate slower than when you started. If you then pull your arms back in as close as possible, you will start to rotate much faster than you just were with your arms extended. This gives us evidence of the reliance that the moment of inertia has on mass and how it is distributed.

7.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: moment of inertia, torque, angular acceleration

7.4 Theory

If we apply a single unbalanced force, F, to an object, the object will undergo a linear acceleration, a, which is determined by the unbalanced force acting on the object and the mass of the object. The mass is a measure of an object's inertia, or its resistance to being accelerated. Newton's Second Law expresses this relationship:

$$F = ma$$

If we consider rotational motion, we find that a single unbalanced torque

$$\tau = \text{(Force)(lever arm)}$$

produces an *angular* acceleration, α, which depends not only on the mass of the object but on how that mass is *distributed*[2]. The equation which is analogous to $F = ma$ for an object that is rotationally accelerating is

$$\tau = I\alpha \tag{7.1}$$

where the Greek letter tau (τ) represents the **torque** in Newton-meters, α is the **angular acceleration** in radians/sec^2, and I is the **moment of inertia** in kg-m^2. The moment of inertia is a measure of the way the mass is distributed on the object and determines its resistance to angular acceleration.

Every rigid object has a definite moment of inertia about any particular axis of rotation. Here are a couple of examples of the expression for I for two special objects:

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

[2] In this lab the lever arm will be the radius at which the force is applied (the radius of the axle). This is due to the fact that the forces will be applied tangentially, i.e., perpendicular to the radius. The general form of this relationship is $\tau = \text{(force)(lever arm)}(\sin(\theta))$ where θ is the angle between the force and the lever arm. However, in this experiment θ is 90° and $\sin(90^\circ) = 1$.)

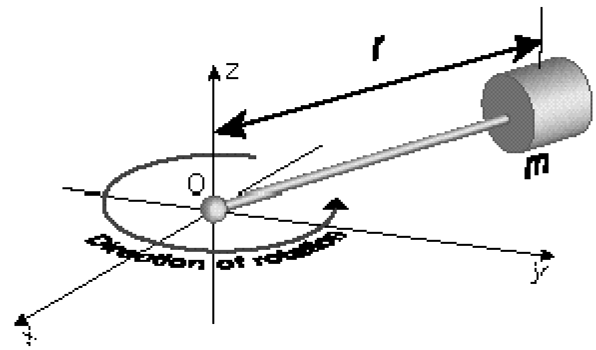


Figure 7.1: One point mass m on a weightless rod of radius r ($I = mr^2$).

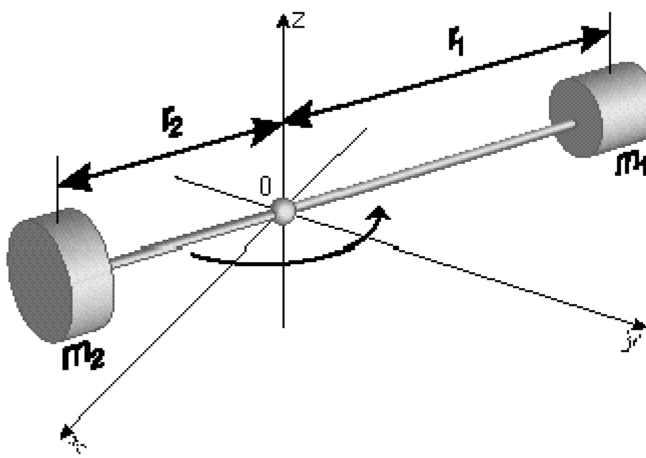


Figure 7.2: Two point masses on a weightless rod ($I = m_1 r_1^2 + m_2 r_2^2$).

To illustrate, we will calculate the moment of inertia for a mass of 2 kg at the end of a massless rod that is 2 m in length (Fig. 7.1 above):

$$I = mr^2 = (2 \text{ kg})(2 \text{ m})^2 = 8 \text{ kg m}^2$$

If a force of 5 N were applied to the mass *perpendicular* to the rod (to make the lever arm equal to r) the torque is given by:

$$\tau = Fr = (5 \text{ N})(2 \text{ m}) = 10 \text{ N m}$$

By equation 7.1 we can now calculate the angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{10 \text{ N m}}{8 \text{ kg m}^2} = 1.25 \frac{\text{rad}}{\text{sec}^2}$$

Note: The moment of inertia of a complicated object is found by adding up the moments of each individual piece (Figure 7.2 above is the sum of two Figure 7.1 components).

7.5 In today's lab

Today we will measure the moment of inertia for multiple mass distributions. We will plot our data and determine the relationship of the moment of inertia and the radii that our masses were placed at.

7.6 Equipment

- 2 Cylindrical Masses
- Hanger
- Small Masses
- Main Axle
- String

In our case, the rigid body consists of two cylinders, which are placed on a metallic rod at varying radii from the axis of rotation. The cylinders and rod are supported by a rotating platform attached to a central pulley and nearly frictionless air bearings. A side view of the apparatus is shown in Figure 7.3 and a top view of the central pulley is shown in Figure 7.4.

In this experiment, we will change the moment of inertia of the rotating body by changing how the mass is distributed on the rotating body. We will place the two cylindrical masses at four different radii such that $r = r_1 = r_2$ in each of the four cases. We will then use our measurements to calculate the moment of inertia (I) for each of the four radial positions of the cylindrical masses (r). The sum of the two cylindrical masses ($m_1 + m_2$) can then be found from a graph of I versus r^2.

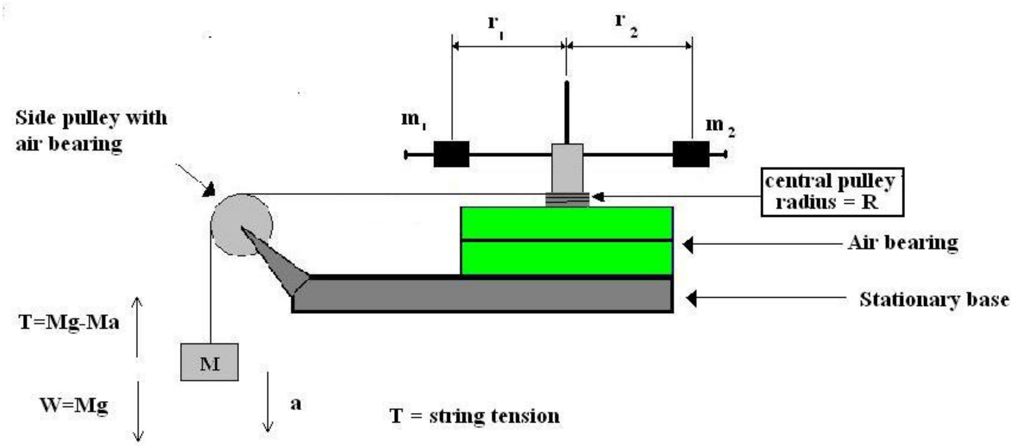


Figure 7.3: Moment of Intertia Apparatus

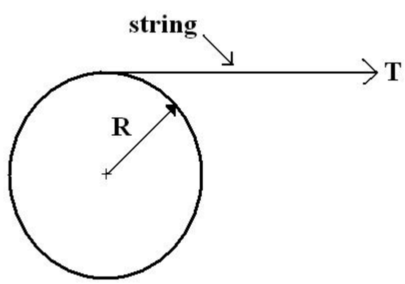


Figure 7.4: Central Pulley (axle)

To set up your rigid body, wrap the string around the central pulley (axle) and run it over the side pulley to a known weight as shown in Figure 7.3.

Consider the following steps:

If we release the weight from rest, the tension in the string will exert a torque on the rigid body causing it to rotate with a constant angular acceleration α. The angular acceleration of the rigid body is related to the linear acceleration of the falling mass by:

$$\alpha = \frac{\text{Linear acceleration}}{\text{Radius of axle}} = \frac{a}{R}$$

or

$$a = R\alpha \tag{7.2}$$

From Figure 7.3 and Newton's Second Law, the tension in the string is:

$$T = Mg - Ma \tag{7.3}$$

The tension in the string causes a net torque on the rigid body. Since Torque = (Lever arm) (Force), the net torque on the rigid body is given by:

$$\tau = R \times T \tag{7.4}$$

The moment of inertia of the rigid body is then found from equation 7.1 ($\tau = I\alpha$).

7.7 Procedure

1. Measure and record the masses of the hanging mass (M) and the two cylinders (m_1 and m_2).

2. Place the cylinders on the horizontal rod such that the axes of the cylinders are along the horizontal rod (as shown in Figure 7.5). Make sure the thumbscrew on each cylinder is tightened. The center of mass of each cylinder must be the same distance (r) from the axis of rotation (i.e. $r_1 = r_2$ in Figure 7.3). Estimate the uncertainty in r (called δr). This should include both the uncertainty in reading your ruler and the uncertainty in locating the cylinder's center of mass.

3. With the air supply on, attach the hanging mass (M) to one end of a string and wind the other end around the central pulley. The string should also pass over the side pulley such that the hanging mass is just below the side pulley. Hold the hanging mass stationary and measure its elevation (y) using the floor as your reference level. Record this elevation in your spreadsheet and assign an appropriate uncertainty to this measurement. Then release the hanging mass and simultaneously start the desktop timer. When the mass hits the floor, stop the timer. For the uncertainty in this time (δt), use the standard deviation of a measurement (denoted by s) from the Reaction Time experiment.

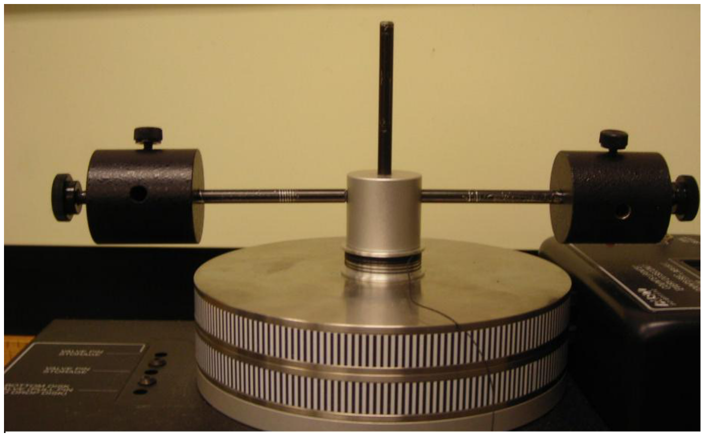


Figure 7.5: View of main axle with 2 masses at same radius r

4. The position, y, of an object released from rest a distance h above the floor is found using: $y = h - \frac{at^2}{2}$. The final position of the mass is $y = 0$, so the acceleration is found using: $a = \frac{2h}{t^2}$.

   Calculate the linear acceleration of the falling mass (M) and use $\delta a = a\left(2\frac{\delta t}{t} + \frac{\delta h}{h}\right)$ to calculate its uncertainty.

5. Use equation 7.3 to calculate the tension in the string (T) and use $\delta T = T\left(\frac{\delta M}{M} + \frac{\delta a}{g-a}\right)$ to calculate its uncertainty.

6. Use $R = 1.27 \pm 0.01$ cm for the radius of the central pulley and equation 7.2 to calculate the angular acceleration of the rotating apparatus. In addition, use $\delta\alpha = \alpha\left(\frac{\delta a}{a} + \frac{\delta R}{R}\right)$ to calculate its uncertainty.

7. Use equation 7.4 to calculate the unbalanced torque on the rotating apparatus and use $\delta\tau = \tau\left(\frac{\delta T}{T} + \frac{\delta R}{R}\right)$ to calculate the uncertainty in this torque. (**Note:** in this equation the Greek letter τ (tau) is the torque and T is the tension in the string.)

8. Use equation 7.1 to calculate the moment of inertia of the rotating apparatus; the uncertainty in moment of inertia is given by: $\delta I = I\left(\frac{\delta\tau}{\tau} + \frac{\delta\alpha}{\alpha}\right)$. Calculate r^2 and its uncertainty, $\delta r^2 = 2r\delta r$.

9. Repeat steps 2–8 for two additional (non-zero) values of r. Make sure that these values differ by at least 2 cm.

10. We would like to place the two cylinders at $r = 0$. To do this, we will use the vertical bar on the support (see Figure 7.6). When you place the cylinders on the vertical bar, make sure they are oriented the same way as in your previous trials, i.e. with the axes of the two cylinders perpendicular to the vertical bar. As before, make sure to tighten the thumbscrews on the cylinders. Follow the procedure in steps 3–8 to calculate the moment of inertia of the body with the two cylinders at $r = 0$. Include this data in your data table.

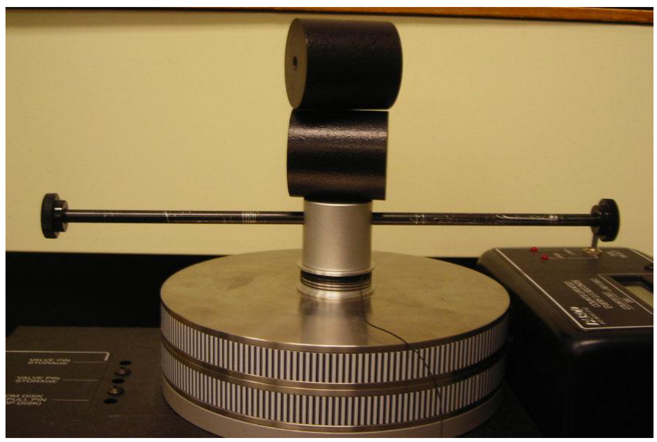


Figure 7.6: View of main axle with 2 masses at radius $r = 0$

11. Transfer your data into KaleidaGraph and make a plot of I vs. r^2. Your data points should have both horizontal and vertical error bars. Also, fit your data with a best fit line, display its equation with the uncertainties in the slope and intercept. When the two cylinders are placed *on the axis of rotation*, the measured moment of inertia I_0 is the moment of inertia of the rotating apparatus alone plus the moment of inertia of each of the two cylinders about an axis through their own centers of mass.

$$I = I_0 \tag{7.5}$$

If the two masses are now each placed a distance r from the axis of rotation then equation 7.5 becomes:

$$I = (m_1 + m_2)r^2 + I_0 \tag{7.6}$$

If you compare equation 7.6 to the form of an equation for a straight line:

$$y = mx + b$$

You can see that a plot of I vs. r^2 should be a straight line. The slope of this line is the sum of the masses $(m_1 + m_2)$ and the intercept is I_0.

7.8 Checklist

1. Excel Sheets
2. Plot of I vs. r^2 with proper error bars and fit line.
3. Questions

7.9 Questions

1. The moment of inertia of a body depends not only on its mass, but also on how the mass is distributed. Does your data support this? Why or why not?

2. Discuss the consistency of the slope of the plot of I vs. r^2 with the value you measured for $(m_1 + m_2)$. If they are not consistent, suggest possible sources of error.

3. In your plot of I vs. r^2, why did you use r^2 and not r in the plot? What are the units of the slope of I vs. r^2?

4. In step 6 of the procedure, you were given that $R = 1.27 \pm 0.01$ cm. Using only the experimental apparatus and a meter stick, how would you verify this radius with an uncertainty of less than or equal to 0.01 cm?
(**Hint:** You cannot get this uncertainty by holding the meter stick next to the axle and measuring the diameter. Also note that the string is a part of the apparatus.)

Experiment 8

The Pendulum

8.1 Objectives

- Investigate the functional dependence of the period (τ) of a pendulum on its length (L), the mass of its bob (m), and the starting angle (θ_0). The Greek letter tau (τ) is typically used to denote a time period or time interval.
- Use a pendulum to measure g, the acceleration due to gravity.

8.2 Introduction

Everyday we experience things moving in a periodic manner. For example, when you go to a park, you can see children playing on a swingset. As they move back and forth, they are undergoing periodic motion, much like that of a pendulum. Pendula are great tools for measuring time intervals accurately, but they also can be used to measure gravity if you know how. Today, we will investigate how a pendulum works, what affects it period, and try to measure gravity (once again) using what we know in physics.

8.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: angular acceleration, pendulum

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

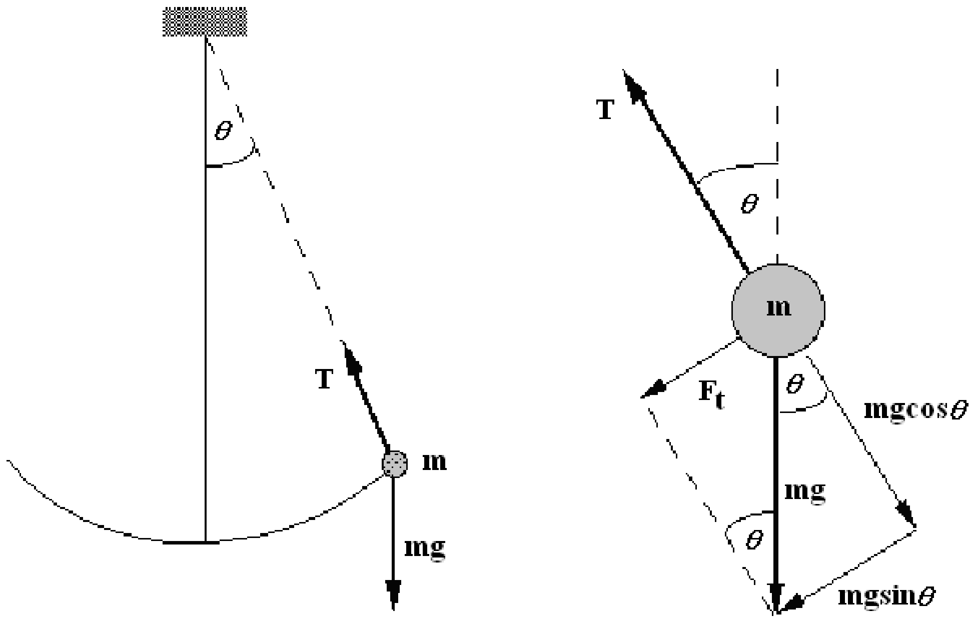


Figure 8.1: Force diagram of a pendulum

8.4 Theory

In the analysis of the motion of a pendulum we should realize that

1. The motion is part of a circle so angular acceleration (α) is a useful variable
2. The angular acceleration will not be a constant throughout the motion

Consider the pendulum shown in Figure 8.1. The weight at the end of the string is called the "bob" of the pendulum. The acceleration, a_t, of the bob **tangent** to the arc "drawn" by the pendulum as it swings is determined by F_t, the force tangent to the arc. Since the tension in the string (T) always acts along the **radius**, it does not contribute to F_t. Decomposing the gravitational force mg into components perpendicular and parallel to the string as shown in Figure 8.1, we find that

$$F_t = mg\text{sin}\theta$$

Therefore, the acceleration tangent to the circle is given by:

$$a_t = \frac{F_t}{m} = g\sin\theta$$

The angular acceleration α is then found by the relationship for circular motion

$$\alpha = -\frac{a_t}{r} = -\frac{g}{L}\sin\theta$$

Thus, as we have suggested, *the angular acceleration α is not a constant* but varies as the sine of the displacement angle of the pendulum.

For small angles (about $\theta < 0.5$ radians), angular accelerations can be shown (with a little calculus which we will skip) to lead to an oscillation of the angle θ by

$$\theta = \theta_0\cos\left(\frac{2\pi t}{\tau}\right)$$

where θ_0 is the angle at time $t = 0$ (when we release the pendulum), and τ is the period of the motion. The period is the time it takes to complete *one full cycle* of the motion.

The period (τ) of a pendulum *depends only on its length (L) and the acceleration due to gravity (g).* The period (τ) is independent of the mass of the bob (m) and the starting angle (θ_0). The period of a simple pendulum is given by:

$$\tau = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad \tau = \frac{2\pi}{\sqrt{g}}\sqrt{L}$$

This equation has the same form as the equation of a straight line, $y = mx + b$, with an intercept of zero (i.e. $b = 0$). Notice in this equation, the period (τ) corresponds to y and $\sqrt{L}$ corresponds to x.

8.5 In today's lab

Today we will change various parameters of the pendulum and see how they affect its period. We will change the

1. Mass
2. Starting angle
3. Length

of the pendulum independently of each other and measure the period. We will then plot our results and see if we can accurately measure gravity using a pendulum.

8.6 Equipment

- Masses
- String
- Photogate
- Compass
- Meter Stick

8.7 Procedure

1. Measure the masses of the point masses provided. Use the six heaviest masses in order to get the most accurate data.
2. Measure the length, L_0 you will use for the trials where you vary the mass m and starting angle θ_0 of the pendulum and record it in the appropriate cells in your excel sheet.
3. Put the photogate on the `PEND` setting.
4. Set a starting angle such that $\theta_0 = 15°$ from vertical and record it in your data sheet.
5. Place a mass on the end of your pendulum and record it in your data sheet.
6. Move the pendulum to that angle and release it, allowing it to complete one full oscillation. Record the time shown on the photogate in your data sheet. This is your period τ.
7. Repeat steps 5 and 6 five more times using different masses each time.

8. Now choose one of the masses you've just used as the mass to use for the starting angle and length trials, hang it on the end of the pendulum, and record the mass in the appropriate cells in your data sheet.

9. Keeping the same string length, record 6 periods for trials where you vary your starting angle θ_0. Here you can choose any starting angle as long as θ_0 remains less than 30° from the vertical. Make sure that each starting angle is at least 4° different from any other starting angle you used. Also, one of your trials should be $\theta_0 = 15°$.

10. Keeping the same mass on the end of the string and starting your pendulum at the same starting angle θ_0 you chose in step 4, measure the period for 6 trials where you change the length of the pendulum and record it in your data sheet. Make sure that there is at least a 5 cm difference between the lengths you choose to use. One of the lengths used should be the one used in steps 7 and 9.

11. Using the correct formula and the "fill down" method in Excel, calculate $\sqrt{L}$ in your data sheet.

12. Create plots for

    - Period (τ) vs. m (for fixed θ_0 and L)
    - Period (τ) vs. θ_0 (for fixed m and L)
    - Period (τ) vs. $\sqrt{L}$ (for fixed m and θ_0)
    - You do not need a formula view for this experiment

    in KaleidaGraph. Use the same scale for the axis displaying period τ. You can determine this scale by selecting the smallest and largest values out of **all** of your trials and using those as your minimum and maximum values respectively for your axis limits. Be sure to fit each plot with a best fit line.

8.8 Checklist

1. Excel Sheets

2. Plot for Period (τ) vs. m

3. Plot for Period (τ) vs. θ_0
4. Plot for Period (τ) vs. $\sqrt{L}$
5. Questions

8.9 Questions

1. Comment on how the period depends on each of the three parameters (m, θ, and $\sqrt{L}$). The period is independent of a parameter if the slope of your best fit line is consistent with zero.

2. By what percent does the period vary with each parameter. Use $\tau(L_0, 15^\circ)$ as your reference?

3. Use the slope of the graph of τ vs. $\sqrt{L}$ to calculate g and its uncertainty.

$$\delta g = 2g\frac{\delta(\text{slope})}{\text{slope}}$$

4. Is your value of g consistent with 981 cm/sec^2? If not, suggest a possible source of error.

Experiment 9

The Spring: Hooke's Law and Oscillations

9.1 Objectives

- Investigate how a spring behaves when it is stretched under the influence of an external force. To verify that this behavior is accurately described by Hooke's Law.
- Measure the spring constant (k) in two independent ways.

9.2 Introduction

Springs appear to be very simple tools we use everyday for multiple purposes. We have springs in our cars to make the ride less bumpy. We have springs in our pens to help keep our pockets/backpacks ink free. It turns out that there is a lot of physics involved in this simple tool. Springs can be used as harmonic oscillators and also as tools for applying a force to something. Today we will learn about the physics involved in a spring, and why the spring is such an interesting creation.

9.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: Hooke's Law, oscillation

9.4 Theory

Hooke's Law

An ideal spring is remarkable in the sense that it is a system where the generated force is **linearly dependent** on how far it is stretched. Hooke's law describes this behavior, and we would like to verify this in lab today. In order to extend a spring by an amount Δx from its previous position, one needs a force F which is determined by $F = k\Delta x$. Hooke's Law states that:

$$F_S = -k\Delta x \tag{9.1}$$

Here k is the **spring constant**, which is a quality particular to each spring, and Δx is the distance the spring is stretched or compressed. The force F_S is a restorative force and its direction is opposite to the direction of the spring's displacement Δx.

To verify Hooke's Law, we must show that the spring force F_S and the distance the spring is stretched Δx are proportional to each other (that just means linearly dependant on each other), and that the constant of proportionality is $-k$.

In our case the external force is provided by attaching a mass (m) to the end of the spring. The mass will of course be acted upon by gravity, so the force exerted downward on the spring will be $F_g = mg$ (see Figure 9.1). Consider the forces exerted on the attached mass. The force of gravity (mg) is pointing downward. The force exerted by the spring ($-k\Delta x$) is pulling upwards. When the mass is attached to the spring, the spring will stretch until it reaches the point *where the two forces are equal but pointing in opposite directions*:

$$F_S - F_g = 0 \text{ or } mg = -k\Delta x \tag{9.2}$$

This point where the forces balance each other out is known as the **equilibrium point**. The spring + mass system can stay at the equilibrium

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

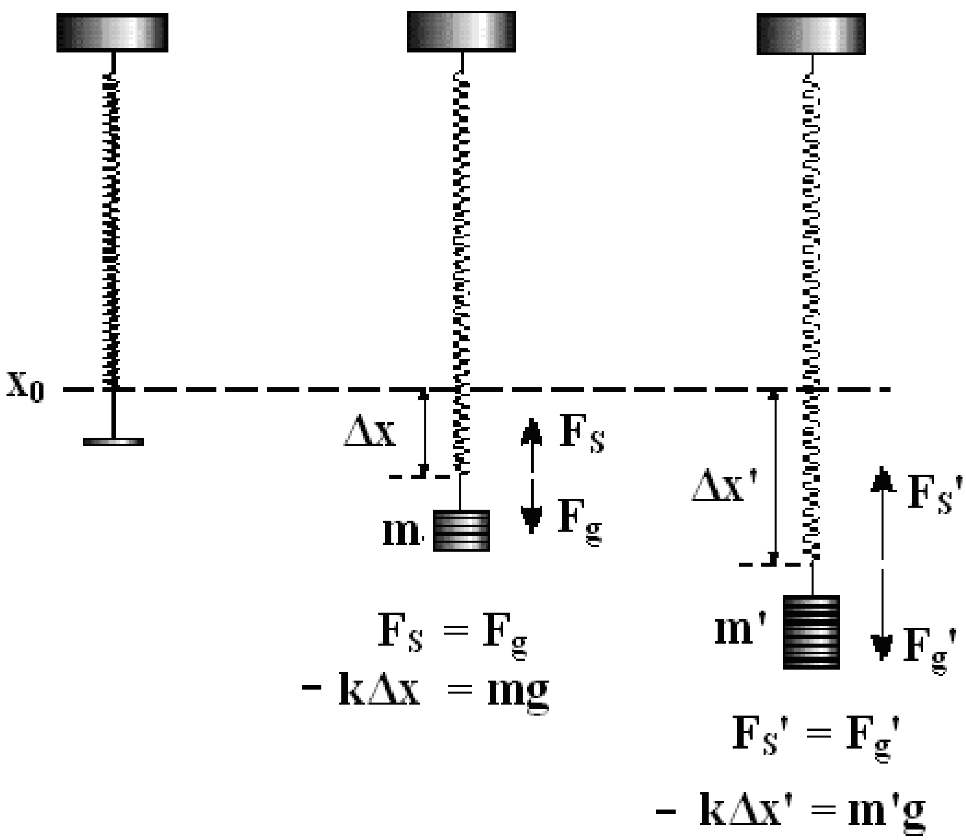


Figure 9.1: Force diagram of a spring in gravity.

point indefinitely as long as no additional external forces come to be exerted on it. The relationship in Eq. 9.2 allows us to determine the spring constant k when m, g, and Δx are known or can be measured. This is one way in which we will determine k today.

Oscillation

The position where the mass is at rest is called the equilibrium position ($x = x_0$). As we now know, the downward force due to gravity $F_g = mg$ and the force due to the spring pulling upward $F_S = -k\Delta x$ cancel each other. This is shown in the first part of Figure 9.2. However, if the string is stretched *beyond its equilibrium point* by pulling it down and then releasing it, the mass will accelerate upward ($a > 0$), because the force due to the spring is *larger* than gravity pulling down. The mass will then pass through the equilibrium point and continue to move upward. Once above the equilibrium position, the motion will slow because the net force acting on the mass is now downward (i.e. the downward force due to gravity is constant while the upwardly directed spring force is getting smaller). The mass and spring will stop and then its downward acceleration will cause it to move back down again. The result of this is that the mass will oscillate around the equilibrium position. These steps and the forces (F), accelerations (a), and velocities (v) are illustrated in Figure 9.2 for the first complete cycle of an oscillation. The oscillation will proceed with a characteristic period, τ, which is determined by the spring constant and the total attached mass. This period is the time it takes for the spring to complete one oscillation, or the time necessary to return to the point where the cycle starts repeating (the points where x, v, and a are the same). One complete cycle is shown in Figure 9.2 and the time of each position is indicated in terms of the period τ, where

$$\tau = 2\pi\sqrt{\frac{m}{k}} \tag{9.3}$$

By measuring the period for given masses the spring constant can be determined. This is the second way we will determine k today. You will use this value of k to verify that the proportionality constant you determined for Hooke's Law in the first part is indeed the correct k for the spring.

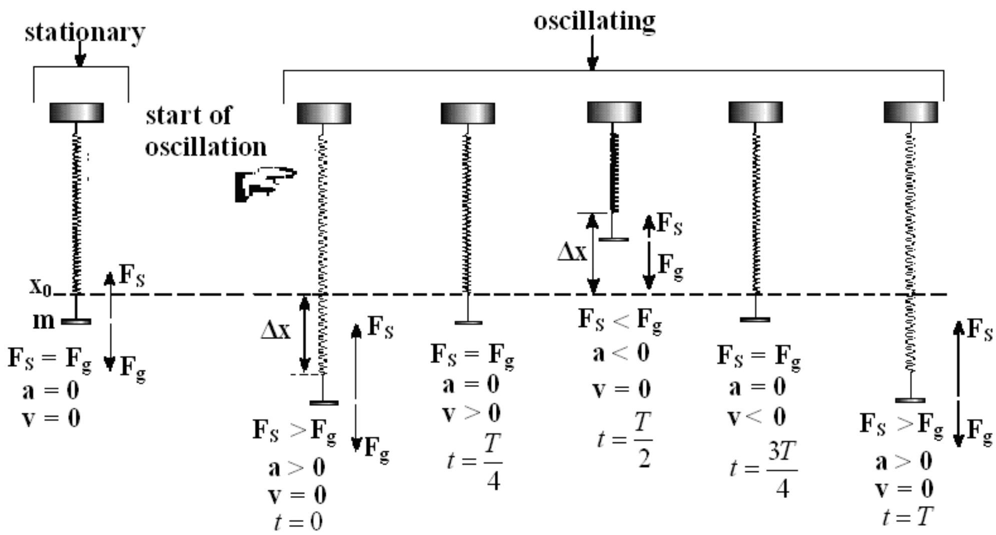


Figure 9.2: Oscillation of a spring.

9.5 In today's lab

Today we will measure the spring constant of a given spring in two ways. First we will add mass gradually to the spring and measure the displacement, then plot the results to find the spring constant. Then, we will find the period of oscillation for the spring after attaching varying mass to the bottom. Once again, we will plot these results to find the spring constant a different way.

9.6 Equipment

- Spring
- Photogate
- Masses
- Hanger

9.7 Procedure

Part I: Hooke's Law

1. Record the mass of the mass hanger, $m_H = 50.0$ g, in your data sheet.
2. Measure the rest length (nothing on the end) of the spring and record it in your data sheet.
3. Attach the **empty** mass hanger to the spring and measure the position X_0 of the end of the spring with the zero end of the meter stick **on the table**. Be sure to include a reasonable uncertainty.
4. Increase the **total mass** on the end of the spring to 120 g (this includes the mass of the hanger). Measure the height of the spring now and record it in your data sheet.
5. Increase the mass by 10 g increments, making sure to measure and record the height at each step, until you reach 220 g.
6. Calculate $\Delta m = m - m_H$, $\Delta X = X - X_0$, $\delta(X - X_0) = 2\delta X_0$, and $F_S = -k\Delta X = \Delta mg$ (we are measuring the distances when the spring is in equilibrium) for each trial on your data sheet.
7. Graph F_S vs. ΔX in KaleidaGraph. Include horizontal error bars and a best fit line. If you have a straight line, this verifies Hooke's Law already. Here, the slope will tell you the spring constant and it's uncertainty.

Part II: Period of Oscillation

1. Set the photogate to the `PEND` setting.
2. Starting at a mass of 120 g on the end of the spring, measure the period of oscillation by causing the masses to oscillate through the photogate. You can adjust the height of the photogate and the height of the spring to allign the equilibrium position with the photogate. When displacing the mass for oscillation, this should be a small displacement; **do not** stretch the spring more than 5 cm. Do this in 20 g intervals up to 220 g.

3. Calculate the mass of the spring using the given spring density and the rest length of the spring. Record this value in your data sheet.

4. Calculate τ^2 in Excel for each trial.

   This gives us an equation in the same form as a straight line $y = mx+b$ with intercept $b = 0$. The value m in Equation 9.4 is the total mass felt by the spring.

5. Calculate the total mass using the formula $m = m_H + m + \frac{\text{spring mass}}{3}$ in Excel. Note that this is a different m than you used in Part 1. Here the total mass experienced by the spring is the mass of the hanger, the masses added to the hanger, and 1/3 of the mass of the spring.

6. Make a plot of τ^2 vs. m in KaleidaGraph. Be sure to include a best fit line on this plot. In the question you will use the slope of your graph to find the spring constant. Note that squaring both sides of Equation 9.3 we get

$$\tau^2 = \frac{4\pi^2}{k} m \tag{9.4}$$

9.8 Checklist

1. Excel Sheets

2. Plot of F_S vs. ΔX

3. Plot of τ^2 vs. m

4. Questions

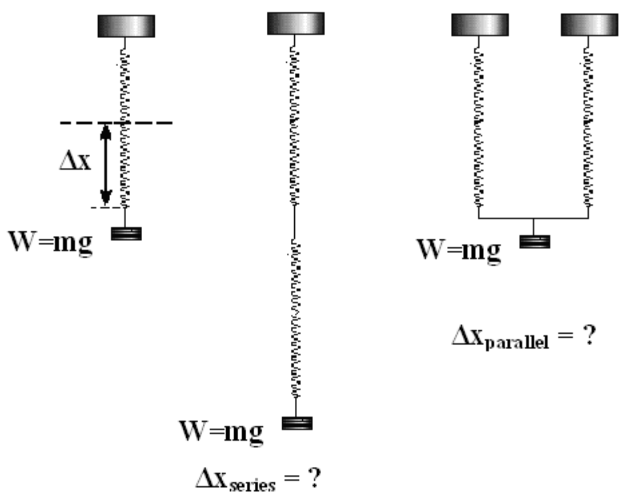
9.9 Questions

1. Is your data consistent with Hooke's Law? Specifically, is the spring force linearly dependent on how much the spring is stretched? Is it a restorative force? Why or why not? From the slope of your graph, what is the spring constant and it's uncertainty?

2. Calculate the spring constant and its uncertainty using the information obtained from your graph of τ^2 vs. m. **Hint:** The fractional uncertainty in the spring constant is equal to the fractional uncertainty in the slope ($\frac{\delta k}{k} = \frac{\delta \text{slope}}{\text{slope}}$).

3. You obtained the spring constant in two independent ways. Discuss the consistency of your two measurements of the spring constant. If they are not consistent, suggest a possible source of error.

4. When a mass m is attached to a spring it exerts a force $W = mg$ on the spring and the length of the spring is changed by Δx. If the single spring is replaced with a) two identical springs in series, what happens to Δx_{series} compared to the case of a single spring? b) If the single spring is replaced by two identical springs in parallel, what happens to $\Delta x_{\text{parallel}}$ compared to the case of a single spring? Assume all springs have the same spring constant and always compare to the single spring case. Answer each question by stating if Δx increases, decreases or remains unchanged. Also, what are Δx_{series} and $\Delta x_{\text{parallel}}$ in terms of Δx for the single spring case? **Hint:** draw a force diagram of the system – the net force on the mass must be zero.



Δx
W=mg
W=mg
Δxseries = ?
W=mg
Δxparallel = ?

Experiment 10

Vibration Modes of a String: Standing Waves

10.1 Objectives

- Observe resonant vibration modes on a string, i.e. conditions for the creation of standing wave patterns.
- Determine how resonant frequencies are related to the number of nodes, tension of the string, length of the string, and string density.
- Determine the velocity of transverse waves in the string.

10.2 Introduction

Everything you can see is due to waves. A wave is defined as an oscillation through space. The reason we can see things is because photons oscillate through space in the form of a wave and enter our eyes. This sends a signal to our brain and thus we can see. Today we will investigate waves on a much large scale. When you apply an oscillation to the end of a tight string, it begins to form waves with given frequencies. The **resonant frequency** is the frequency at which the wave oscillates freely without constructive interference adding to its amplitude.

10.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: standing waves on a string, resonance, transverse waves

10.4 Theory

A wave in a string can be characterized by its **wavelength**, λ, just like a sound wave or a light wave. For a string that is fixed on both ends, a **standing wave** can develop *if an integer number of half wavelengths fit into the length, L, of the string:*

$$n\left(\frac{\lambda_n}{2}\right) = L \tag{10.1}$$

Here n refers to the number of maxima (also called antinodes) in the wave pattern as demonstrated in Figure 10.1. The **resonant frequency**, f_n, for wavelength λ_n with wave speed c is[2]

$$f_n = \frac{c}{\lambda_n} \tag{10.2}$$

Combining equations 10.1 and 10.2 we obtain:

$$f_n = \left(\frac{c}{2L}\right) n \tag{10.3}$$

If a force acts on a string with a resonant frequency, the amplitude of the vibration will grow very large. This is a common behavior in many physical systems. An example of such behavior is pushing a child on a swing. A swing oscillates with a **characteristic frequency**. If someone exerts a push on the child with that frequency, after several cycles the amplitude of the swing becomes large, even if the pushes are gentle. If pushes are given with a different frequency, some of the pushes will be out of phase; meaning that the child will be pushed against his motion and the amplitude will

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

[2] The pitch of musical instruments is determined by the resonant frequency, whether it is a string instrument, a wind instrument or a percussion instrument. Since instruments are not driven at a fixed frequency, the vibrations are composed of a mixture of several harmonic frequencies.

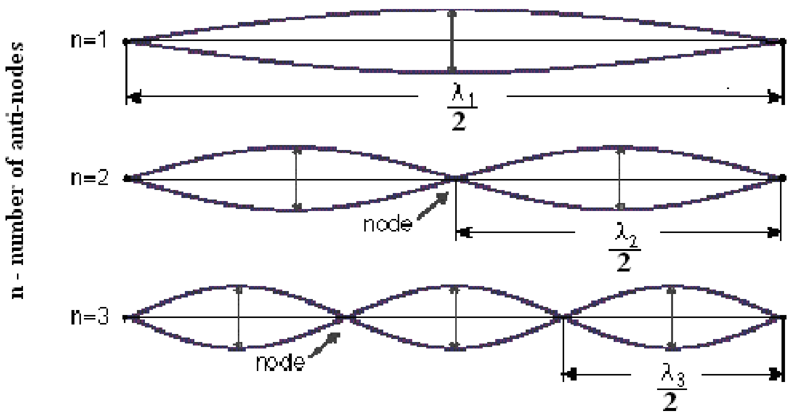


Figure 10.1: Three lowest characteristic frequencies of a string (with $n = 1, 2$ and 3 maxima).

not have a chance to grow. A string has *many* characteristic frequencies and the string's amplitude will grow whenever the driving force has *any* of these characteristic frequencies. If a string is set to vibrate at one of these characteristic frequencies a standing wave is set up on the string. When a standing wave is present, nodes and antinodes will be visible on the string. A node is a location on the string where the string does not move. On the other hand, an antinode is a location that undergoes a vibration with very large amplitude. Figure 10.1 shows the lowest three characteristic frequencies for a given string under constant tension.

The speed, c, of a transverse wave in a string depends on the string's mass per unit length ρ and the tension T (ρ is the Greek letter rho and is frequently used to represent mass density). By setting the tension with the pulley system shown in Figure 10.2 and by measuring the mass density, one can determine the speed of the transverse wave by

$$c = \sqrt{\frac{T}{\rho_s}} \tag{10.4}$$

The fractional uncertainty in the stretched string density is

$$\frac{\delta\rho_s}{\rho_s} = \left(\frac{\delta l_s}{l_s} + \frac{\delta l_0}{l_0}\right) \tag{10.5}$$

Here, l_0 and l_s are the respective lengths of the string when it is unstretched and stretched. Both of these quantities are required to calculate ρ. The uncertainty of the unstretched string δl_0 includes both the uncertainty in reading the meter stick and the uncertainty associated with aligning the unstretched string with the meter stick (the string needs to be in a straight line and at the same time not stretched). Similarly, the uncertainty of the length of the stretched string δl_s includes both the uncertainty associated with reading the meter stick and the uncertainty of aligning the meter stick with the string. In this case, the string changes directions as it passes over the pulley as shown in Figure 10.3. You should make reasonable estimates of both of these uncertainties.

The fractional uncertainty in the speed c found by equation 10.4 is given by:

$$\frac{\delta c}{c} = \frac{1}{2}\frac{\delta \rho_s}{\rho_s} \tag{10.6}$$

Because the fractional uncertainties in the string's mass and the tension in the string are both small compared to the fractional uncertainties of l_0 and l_s, they are not included in the fractional uncertainty equation 10.6.

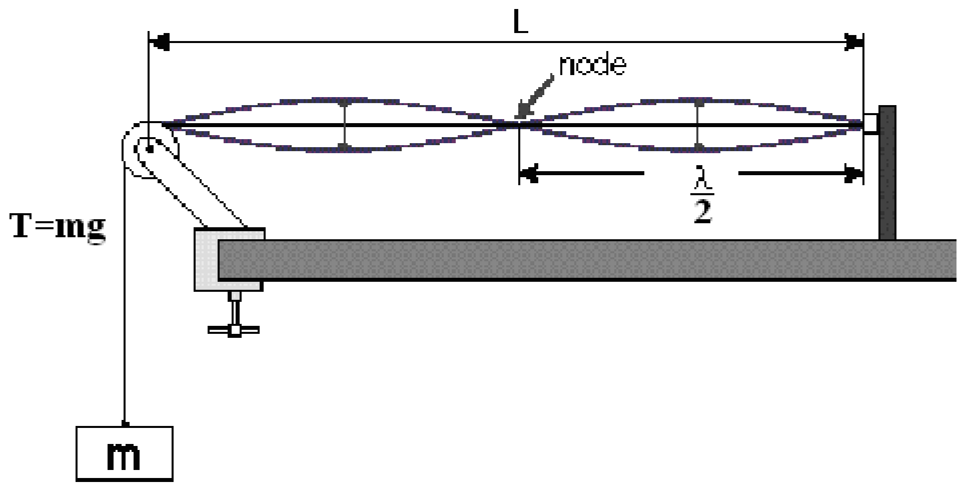


Figure 10.2: Diagram of how the string looks when driven at the second lowest resonant frequency. This configuration has two anti-nodes (points of maximum oscillation).

10.5 In today's lab

Today we will investigate the wave speed traveling through a string under tension in 3 ways. First we will find the resonant frequencies of the string for 1–11 antinodes. We will then calculate the wave speed using Equation 10.3 for each resonant frequency and average the values together. Next, we will calculate the wave speed using Equation 10.4. Finally, we will plot our results and use the graph to measure our wave speed. We will then see whether or not all 3 methods are consistent with one another.

10.6 Equipment

- Variable Frequency Oscillator
- Pulley and Weight System
- String

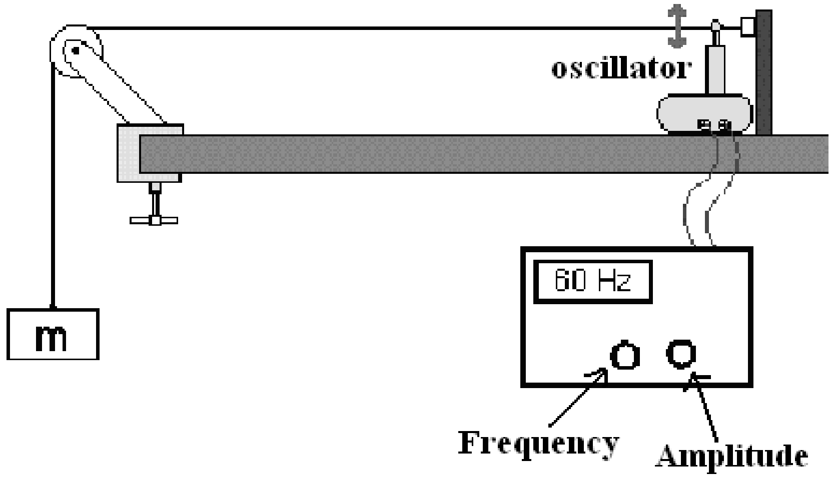


Figure 10.3: Diagram showing the experimental apparatus.

- 2 Meter Stick
- Mass Scale

10.7 Procedure

1. Measure the rest length l_0 of the string and record it in your data sheet.
2. Measure the mass of your hanger and mass system and record it in your data sheet.
3. Attach the mass system to the end of your string and hang it over the pulley. Measure the length L (see Figure 10.2) and record it in your data sheet. L should be approximately 150 cm.
4. Measure the **total** stretched length of the string with your mass system hanging on the end and record it in your data sheet. Using this value, calculate the stretched string density using the density of the unstretched string, $\rho_0 = (m_0/l_0) = 0.0375$[g/cm].

5. Using Equation 10.4, $T = mg$, and ρ_s, calculate the wave speed and record it in your data sheet.
6. Position the oscillator near the fixed end of the string and adjust the oscillator to "`Hz 1-100`".
7. Starting around 100 Hz, find the first frequency mode where you have 11 antinodes in the string. Be sure to finely adjust the frequency until you get the largest possible peaks. Record this frequency in your data sheet.
8. Gradually decrease the frequency until you find the next lowest integer of antinodes ($n = 10$ in this case) and record the frequency in your data sheet.
9. Repeat step 8 for $n = 9$ to $n = 1$ antinodes. Then using equation. 10.1 to find λ_n.
10. Using the formula $c_n = \lambda_n f_n$, calculate the wave speed for each resonant frequency in Excel and calculate the mean value of this speed. You may use the formula "`=AVERAGE(E23:E33)`" for this calculation.
11. In Excel, calculate the standard deviation and standard deviation of the mean for your resonant frequencies as well. For the standard deviation, you may use the formula "`=STDEV(E23:E33)`". The standard deviation of the mean, $s_m = s/\sqrt{N}$ will be your uncertainty for this wave speed calculation (N = Number of Trials).
12. Make a plot of f_n vs. n in KaleidaGraph and include a best fit line. You do not need error bars for this plot.
13. Answer question # 2 and fill in the respective cell in Excel before you print your Excel sheets.

10.8 Checklist

1. Excel Sheets
2. Plot of f_n vs. n
3. Questions

10.9 Questions

1. What is the slope of your graph and its uncertainty?

2. From the slope and uncertainty of the slope, calculate the speed of the wave and its uncertainty. **Hint:** $\delta c = c\left(\frac{\delta\text{slope}}{\text{slope}} + \frac{\delta L}{L}\right)$.

3. Is the speed of the wave measured from your graph consistent with the mean value of your eleven $c = \lambda_n f_n$ calculations?

4. Is the speed of the wave measured from your graph consistent with the value you obtained using Equation 10.4?

Appendix A

Contents of a Lab Report

Write clearly and neatly in full sentences. Avoid wordiness and excessive detail. This is a general list of items and sections which should be included in every lab report.

Data and Spreadsheet

- Write your name and your lab partner's name at the top of your Excel spreadsheet.
- The spreadsheet should have the data columns labeled, **including units**.
- Include any calculations that the lab manual asked you to do.
- Include a print-out of the formula view of your spreadsheet. To go to the formula view use the `Ctrl+~` keys. Make sure none of the formulas are cut-off, you may need to resize some columns.
- Fit the Excel sheets to 1 page: go to `File` ▶ `Page Setup` ▶ `Scaling`: Fit "1" page wide by "1" page tall. If the page is not legible try changing the orientation to landscape.

Graphs

Every graph should have the following (see example in Fig. A.1):

1. Title - should describe the physical situation the graph represents, not just the units, and be in the format of 'vertical axis' vs. 'horizontal axis'.
2. Axes labelled with the quantity being plotted including units
3. Curve fit (if appropriate)
4. Legend (if needed)
5. Error bars (when appropriate)
6. Observations - hand written on the bottom of each graph should be a $\sim 3-4$ sentence observation which covers the following points:
   - What does this plot represent? Why did you make this particular plot?
   - What did you expect the plot to look like? What is the expected functional form of the equation describing the data?
   - What does the graph actually tell you? What can you conclude?

Answers to questions

- Answer the questions at the end of the lab and turn in those sheets. Space is provided for your answers.
- If your measurement is incompatible with the expected value give an explanation for why that might be. "Human error" does not count as an explanation, be more specific if your results are different from what you expected.

Ordering of Pages

1. Coversheet
2. Data in Excel spreadsheet
3. Formula view of Excel spreadsheet
4. Graphs with observations
5. Answers to the questions

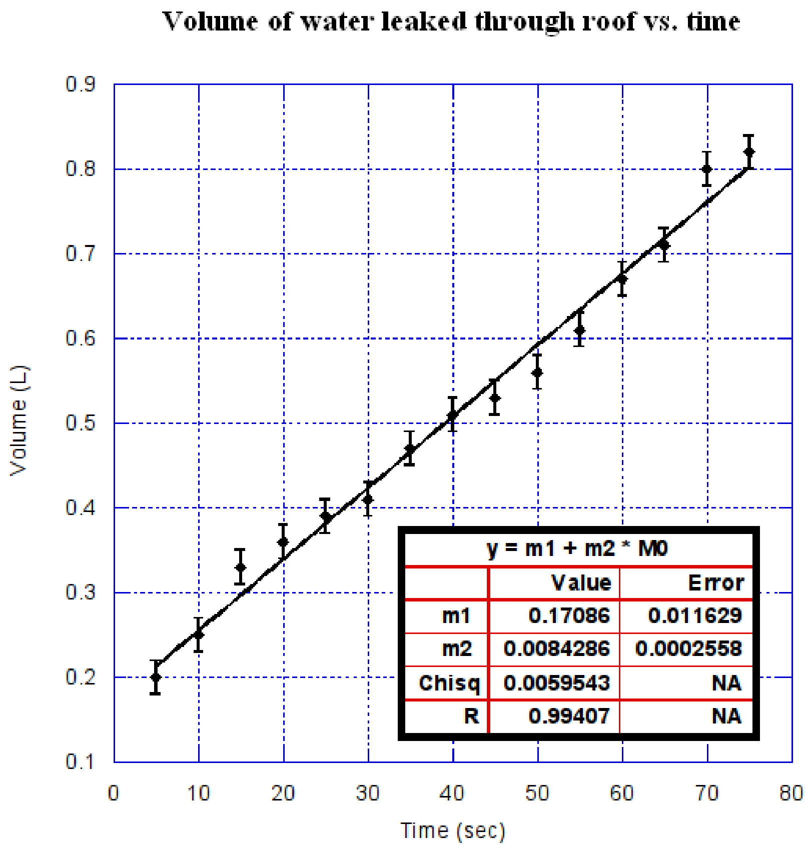


Figure A.1: The graph represents the volume of water leaking through a roof versus time. The water volume is expected to follow the linear equation V = R*t. The leak rate R is given by the slope of the best-fit line and is 0.0084 ± 0.0003 L/s. The data generally follows the expected linear trend, except that the volume at t=0 is not consistent with zero as the formula would predict.

Appendix B

Dealing with uncertainty

B.1 Overview

- An uncertainty is always a positive number $\delta\mathbf{x} > \mathbf{0}$.
- If you measure x with a device that has a precision of u, then δx is at least as large as u.
- **Fractional uncertainty:**
  - If the fractional uncertainty of x is 5%, then $\delta x = 0.05x$.
  - If the uncertainty in x is δx, then the fractional uncertainty in x is $\delta x/x$.
- **Propagation of uncertainty:**
  - If $z = x + y$ or if $z = x - y$, then
$$\delta z = \delta x + \delta y. \tag{B.1}$$
  - If $z = xy$ or if $z = x/y$, then
$$\frac{\delta z}{|z|} = \frac{\delta x}{|x|} + \frac{\delta y}{|y|} \tag{B.2}$$
  - For $f = x^n y^m z^p$, where n, m, and p are exact,
$$\frac{\delta f}{f} = n\frac{\delta x}{x} + m\frac{\delta y}{y} + p\frac{\delta z}{z} \tag{B.3}$$

- For an arbitrary function $f(x)$,

$$\delta f(x) = |f(x+\delta x) - f(x)| \tag{B.4}$$

as long as $\delta x \ll x$.

- **Percent error.** If d is data and e is the expected value, the difference, D, is

$$D = d - e. \tag{B.5}$$

The percent error is given by

$$\%\,\text{error} = \frac{D}{e} \times 100\% \tag{B.6}$$

They are compatible if $|d-e| < \delta d + \delta e$, that is they are compatible when their difference is equal to 0 with the uncertainty of the difference.

B.2 Concise notation of uncertainty

If, for example, $y = 1\,234.567\,89$ U and $\delta y = 0.000\,11$ U, where U is the unit of y, then $y = (1\,234.567\,89 \pm 0.000\,11)$ U. A more concise form of this expression, and one that is in common use, is $y = 1\,234.567\,89(11)$ U, where it is understood that the number in parentheses is the numerical value of the standard uncertainty referring to the corresponding last digits of the quoted result. This explanation is from Ref. (Mohr 2011).

B.3 Significant figures

There is no actual information carried by figures which represent values much smaller than the uncertainty of a measurement. For example, if you do a calculation and your calculator, or Excel, gives you $x = 12.3456789$, but when you calculate the uncertainty, you get $\delta x = 0.01234$, according to the previous section you would naturally write this as

$$12.346(12).$$

This is because there is hardly ever justification for reporting an uncertainty to more than 2 figures (so that 0.01234 should be reported as just 0.012).

Therefore the decimal places in x beyond the reported uncertainty are obviously carrying information corresponding to a tiny fraction of your actual knowledge of x (i.e. much smaller than δx). Thus $12.3456789 \to 12.346$ since the last reported position in the uncertainty is 0.012, 3 figures to the right of the decimal point; and in the reported x value, the trailing 56789 would be rounded up to 6.

B.4 Using uncertainties to compare data and expectations

Simple Measurements: The smallest division estimate

Suppose you use a meter stick ruled in centimeters and millimeters, and you are asked to measure the length of a rod and obtain the result $L_0 = 5.73\,\text{cm}$, seen in Fig. B.1a. A good estimate of the *uncertainty* here is **half the smallest division on the scale**, 0.05 cm. Thus, the length of the rod would be specified as

$$L_0 = 5.73 \pm 0.05\,\text{cm}. \tag{B.7}$$

This says that you are very confident that the length of the rod falls in the range 5.73 cm−0.05 cm to 5.73 cm+0.05 cm — that is, the length falls in the range of 5.68 cm to 5.78 cm, as in Fig. B.1b.

Manufacturer's tolerance

Suppose I purchase a nominally 100 Ω resistor from a manufacturer. It has a gold band on it which signifies a 5% tolerance. What does this mean? The tolerance means $\delta R/R = 0.05 = 5\%$, that is, the fractional uncertainty. Thus, $\delta R = R \times 0.05 = 5\,\Omega$. We write this as

$$R = R_{\text{nominal}} \pm \delta R = 100 \pm 5\,\Omega. \tag{B.8}$$

It says that the company certifies that the true resistance R lies between 95 and 105 Ω. That is, $95 \le R \le 105\,\Omega$. The company tests all of its resistors and if they fall outside of the tolerance limits the resistors are discarded. If your resistor is measured to be outside of the limits, either (a) the manufacturer made a mistake, (b) you made a mistake, or (c) the manufacturer shipped the correct value but something happened to the resistor that caused its value to change.

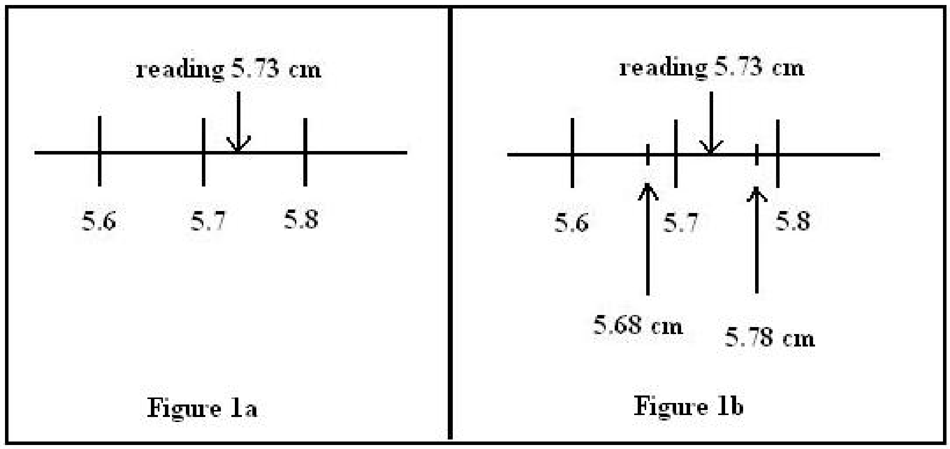


Figure B.1: Measuring a length with a ruler. If the value is read as in (a), then a reasonable uncertainty is show in (b).

Reading a digital meter

Suppose I measure the voltage across a resistor using a digital multimeter. The display says 7.45 V and doesn't change as I watch it. The general rule is that **the uncertainty is half of the value of the least significant digit**. This value is 0.01 V, so the uncertainty is half of it — 0.005 V. Here's why: The meter can only display two digits to the right of the decimal, so it must round off additional digits. So if the true value is between 7.445 V and 7.454 V, the display will get rounded to 7.45,V. Thus the average value and its uncertainty can be written as 7.45 ± 0.005 V.

When you record this in your notebook, be sure to write 7.45 V. Not 7.450 V. Writing 7.450 V means that the uncertainty is 0.0005 V.

Note that in this example we assumed that the meter reading is **steady**. If, instead, the meter reading is fluctuating, then the situation is different. Now you need to estimate the range over which the display is fluctuating, then estimate the average value. If the display is fluctuating between 5.4 and 5.8 V, you would record your reading as 5.6 ± 0.2 V. The uncertainty due to the noisy reading is much larger than your ability to read the last digit on the display, so you record the larger error.

Using uncertainties in calculations

We need to combine uncertainties so that the error bars almost certainly include the true value.

Adding and subtracting

Let's look at the most basic case. We measure x and y and want to find the error in z.

If $z = x + y$, then

$$\delta z = \delta x + \delta y \tag{B.9}$$

If $z = x - y$, then

$$\delta z = \delta x + \delta y \tag{B.10}$$

Note that the uncertainty for subtracting has exactly the same form as for adding.

The most important errors are simply the biggest ones, since they impact the precision of your result the most.

Example:

$$(7 \pm 1\,\mathrm{kg}) - (5 \pm 1\,\mathrm{kg}) = 2 \pm 2\,\mathrm{kg} \tag{B.11}$$

Multiplying and dividing

If $a = bc$, then

$$\boxed{\frac{\delta a}{a} = \frac{\delta b}{b} + \frac{\delta c}{c}} \tag{B.12}$$

For dividing, if $w = x/y$, the rule is the same as for multiplication;

$$\frac{\delta w}{w} = \frac{\delta x}{x} + \frac{\delta y}{y} \tag{B.13}$$

It is simplest to just remember the single boxed rule, Eq. B.12, for multiplication and division.

If the expression contains a constant, the uncertainty of that constant is zero.

The most important errors in multiplication and division are the largest *fractional* errors, not absolute errors. This makes sense if you consider that

b and c need not have the same units — there is no way to compare the absolute sizes of quantities with different units.

Example:

$$\begin{aligned} V &= IR \\ I &= 7 \pm 1\,\text{mA} \\ R &= 20 \pm 2\,\Omega \end{aligned} \tag{B.14}$$

$$V = 140\,(\text{mA} \cdot \Omega) = 140\,\text{mV} = 0.14\,\text{V}$$

The uncertainty is given by

$$\begin{aligned} \frac{\delta V}{V} &= \frac{\delta I}{I} + \frac{\delta R}{R} \\ &= \frac{1\,\text{mA}}{7\,\text{mA}} + \frac{2\,\Omega}{20\,\Omega} \\ &= 0.24 \end{aligned} \tag{B.15}$$

$$\begin{aligned} \delta V &= 0.24 \times (0.14\,\text{V}) \\ &= 34\,\text{mV} \end{aligned}$$

Our formula for multiplication indicates that multiplying by a perfectly known constant has no effect on the *fractional* error of a quantity. For example, the speed of light in vacuum, c, is 299 792 458 m/s with no uncertainty.[1] If we measure the time it takes for light to travel as 12 ± 1 s, then we can find the distance that it traveled.

$$c = 299\,792\,458\,\text{m/s} \qquad t = 12 \pm 1.5\,\text{s}$$

$$\begin{aligned} d &= ct \\ d &= (299\,792\,458\,\text{m/s}) \times (12\,\text{s}) \\ d &= 3\,597\,509\,496\,\text{m} \end{aligned} \tag{B.16}$$

[1] This is because the meter is defined as the distance light travels in 1/299 792 458 seconds in a vacuum.

The uncertainty is given by

$$\begin{aligned}\frac{\delta d}{d} &= \frac{\delta c}{c} + \frac{\delta t}{t} \\ &= \frac{0\,\mathrm{m/s}}{299\,792\,458\,\mathrm{m/s}} + \frac{1.5\,\mathrm{s}}{12\,\mathrm{s}} \\ &= \frac{1.5}{12}\end{aligned} \tag{B.17}$$

and thus $d = 3\,597\,509\,496 \pm 449\,688\,687\,\mathrm{m}$. Note that the value of the speed of light did not matter in the calculation of the fractional uncertainty, since it was multiplied by its zero uncertainty.

The uncertainty $\delta d = d\frac{\delta t}{t} = ct\frac{\delta t}{t} = c\delta t$.

So, δd is just the constant c times δt.

Multiples

If $f = cx + dy + gz$, where c, d, and g are positive or negative constants, then from the multiplication rule, we find that

$$\begin{aligned}\delta(cx) &= |c\delta x| \\ \delta(dy) &= |d\delta y| \\ \delta(gx) &= |g\delta z|\end{aligned} \tag{B.18}$$

From the addition rule,

$$\delta f = |c\delta x| + |d\delta y| + |g\delta z| \tag{B.19}$$

Powers

If $f = x^p y^q z^r$, where p, q, and r are positive or negative constants,

$$\frac{\delta f}{f} = \frac{\delta\left(x^p\right)}{x^p} + \frac{\delta\left(y^q\right)}{y^q} + \frac{\delta\left(z^r\right)}{z^r} \tag{B.20}$$

and thus

$$\frac{\delta f}{f} = \left|p\frac{\delta x}{x}\right| + \left|q\frac{\delta y}{y}\right| + \left|r\frac{\delta z}{z}\right| \tag{B.21}$$

General formula

Suppose we want to calculate $f(x)$, a function of x, which has uncertainty δx. What is the uncertainty in the calculated value f? We simply calculate f at x, and again at $x' = x + \delta x$, then take the absolute value of the difference:

$$\delta f = \left|f(x') - f(x)\right|, \text{ where } x' = x + \delta x. \tag{B.22}$$

For example, if $f(x) = \sin x$, and $x = 30 \pm 1°$, then

$$\begin{aligned} \delta f &= |\sin(31°) - \sin(30°)| \\ &= |0.515 - 0.500| \\ &= 0.015 \end{aligned} \tag{B.23}$$

What happens when there is more than one variable? We do the calculation for each variable separately and combine the resulting uncertainties:

$$\delta f(x,y) = |f(x+\delta x, y) - f(x,y)| + |f(x, y+\delta y) - f(x,y)| \tag{B.24}$$

When are errors negligible?

Errors are only negligible in comparison to something else and in the context of a particular calculation. So it's hard to give general rules, but easier for specific cases. Here's an example of how to think about this question.

You measure a long thin ribbon (that is, something rectangular). Its length is $10 \pm 0.02\,\text{m}$, and its width is $2 \pm 0.1\,\text{cm}$. Which uncertainty is more important? The answer depends on what you want to calculate.

First, imagine that you are feeling festive and want to border the ribbon with glitter. To know how much glitter you'll need, you must find the length of the perimeter P of the rectangle formed by the ribbon. The perimeter is given by

$$P = 2(L + W). \tag{B.25}$$

We apply our addition rule:

$$\delta P = 2(\delta L + \delta W). \tag{B.26}$$

Now, $\delta L = 0.02\,\text{m}$ and $\delta W = 0.1\,\text{cm} = 0.001\,\text{m}$. Since δL is 20 times the size of δW, we can neglect δW — that is, ignore it. Note that we had to put δL and δW into the same units to compare them: 0.02 m is much larger than 0.2 cm.

Now, imagine that instead of just the border, you want to cover the entire area of one side of the ribbon with glitter. For this you need to find the area, A, of the ribbon.

$$A = LW \tag{B.27}$$

The multiplication rule gives

$$\begin{aligned}\frac{\delta A}{A} &= \frac{\delta L}{L} + \frac{\delta W}{W}\\ &= \frac{0.02\,\text{m}}{10\,\text{m}} + \frac{0.1\,\text{cm}}{2.0\,\text{cm}}\\ &= 0.002 + 0.05\end{aligned} \tag{B.28}$$

In this case, the uncertainty due to δL is negligible compared to that from δW, the opposite conclusion as for the perimeter calculation! That's because we are multiplying and now need to compare not δL vs δW, but instead $\dfrac{\delta L}{L}$ vs $\dfrac{\delta W}{W}$.

Using uncertainties to compare data and expectations

One important question is whether your results agree with what is expected. Let's denote the data by d and the expected value by e. The ideal situation would be $d = e$, or $d - e = 0$. We'll use D to denote the difference between two quantities:

$$D = d - e \tag{B.29}$$

The standard form for comparison is always (result) − (expected), so that your difference D will be negative if your value is lower than expected, and positive if it is higher than expected.

This comparison must take into account the uncertainty in the observation, and perhaps, in the expected value as well. The data value is $d \pm \delta d$ and the expected value is $e \pm \delta e$. Using the addition/subtraction rule for uncertainties, the uncertainty in $D = d - e$ is just

$$\delta D = \delta d + \delta e \tag{B.30}$$

Our comparison becomes, "is zero within the uncertainties of the difference D?" Which is the same thing as asking if

$$|D| \leq \delta D \tag{B.31}$$

Eqs. B.30 and B.31 express in algebra the statement "d and e are compatible if their error bars touch or overlap." The combined length of the error bars is given by Eq. B.30. $|D|$ is the magnitude of the separation of d and e. The error bars will overlap (or touch) if d and e are separated by less than (or equal to) the combined length of their error bars, which is what Eq. B.31 says.

Example

Now we have all we need to do comparisons. For example, if we measure a length of 5.9 ± 0.1 cm and expect 6.1 ± 0.1 cm (measured by the TA), the difference is

$$\begin{aligned} D &= d - e \\ &= 5.9\,\text{cm} - 6.1\,\text{cm} \\ &= -0.2\,\text{cm} \end{aligned} \tag{B.32}$$

while the uncertainty of that difference is

$$\begin{aligned} \delta D &= \delta d + \delta e \\ &= 0.1\,\text{cm} + 0.1\,\text{cm} \\ &= 0.2\,\text{cm} \end{aligned} \tag{B.33}$$

We conclude that our measurement is indeed (barely) consistent with expectations. If we had instead measured 6.4 cm, we would not have been consistent.

A good form to display such comparisons is:

d [cm]	δd [cm]	e [cm]	δe [cm]	D [cm]	δD [cm]	compatible?
5.9	0.1	6.1	0.1	−0.2	0.2	YES
6.4	0.1	6.1	0.1	+0.3	0.2	NO
6.2	0.2	6.1	0.1	+0.1	0.3	YES
6.4	0.2	6.1	0.0	+0.3	0.2	NO

If only one comparison is to be made, your lab report might contain a sentence like the following: "The measured value was 6.4 ± 0.2 cm while the expected value was $6.10 \pm 0.0\,\text{cm}$, so the difference is $+0.3 \pm 0.2\,\text{cm}$ which means that our measurement was close to, but not compatible with, what was expected."

Appendix C

Spreadsheet Commands

Table C.1: List of Spreadsheet commands

Operation or Function	Mathematical Description	Command
addition	$11 + 12$	`=11+12`
subtraction	$29 - 21$	`=29-21`
multiplication	30×15	`=30*15`
division	$44/12$	`=44/22`
(combination of above)	$3 + \frac{4}{5 \times 2} - 3 \times 7$	`=3+4/(5*2)-(3*7)`
square root	$\sqrt{5}$ or $\sqrt{7 \times (5/3)}$	`=sqrt(5)` or `=sqrt(7*5/3)`
power	6^3 or $7^{0.5}$	`=6^3` or `7^(0.5)`
the constant "pi"	π	`=pi()`
sum of numbers	$\sum a_i$	`=sum(...)`, where `...` can be a list of cells
(example of sum)	A1 + A2 + A3	`=sum(A1,A2,A3)` or* `=sum(A1:A5)`

Continued on next page

Table C.1 — continued from previous page

operation or function	mathematical description	command
mean value	$(\texttt{A1}+\texttt{A2}+\texttt{A3})/3$	`=average(A1:A3)`
standard deviation	$\sqrt{\frac{\sum(x_i-\bar{x})^2}{N-1}}$	`=stdev`(series of cells)
sine	$\sin x$ or $\sin(2\pi x)$	`=sin(x)` or `=sin(2*pi()*x)`
cosine	$\cos x$	`=cos(x)`
arctangent (inverse tangent)	$\arctan x$ or $\tan^{-1} x$	`=atan(x)`

* This second option can be used when the spreadsheet command references cells in the same column and adjacent rows, or in the same row and adjacent columns. You can also combine methods of defining cells. For example, if you wanted to find the sum of the contents of cells `B3` through `B28`, `B32`, and `B40` through `B100`, the spreadsheet command you would use is `=sum(B3:B28,B32,B40:B100)`

Some other useful hints

- If in doubt, use parentheses to make sure things get calculated in the right order. For example, `=3+5/2` results in 5.5. But, `=(3+5)/2` results in 4. In the first case, it would be better to use `=3+(5/2)` in your spreadsheet program.
- Pushing the `Ctrl+‘` keys will display the formulas for the entire spreadsheet (the backquote (‘) is to the left of the number 1 on the US keyboard). Pressing these two keys again reverts back to the calculated numbers.