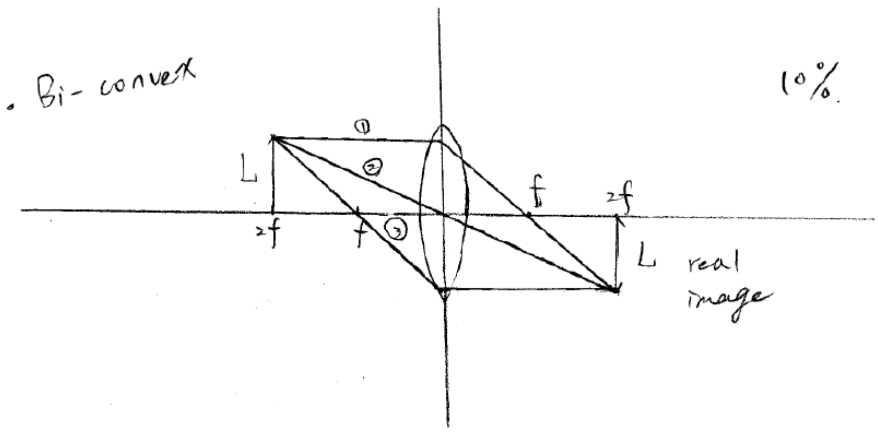
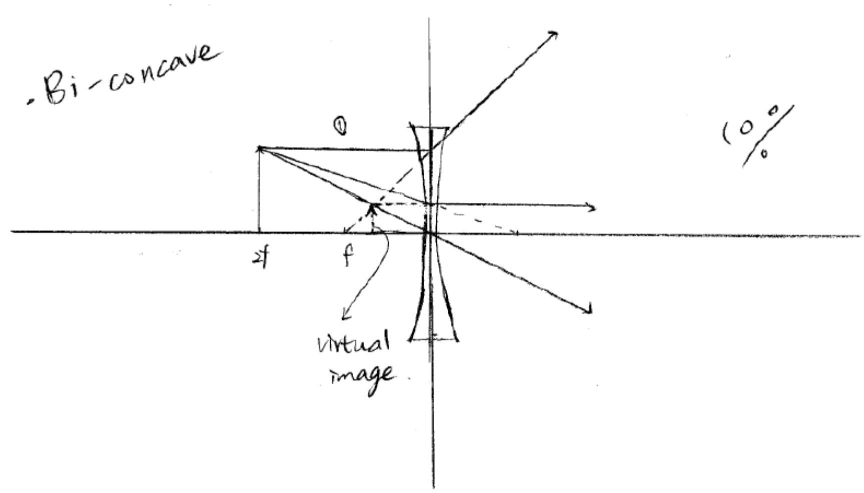
PHY 431 Homework Set #2

Due September 25 at the start of class

1. Ray Tracing (20%)

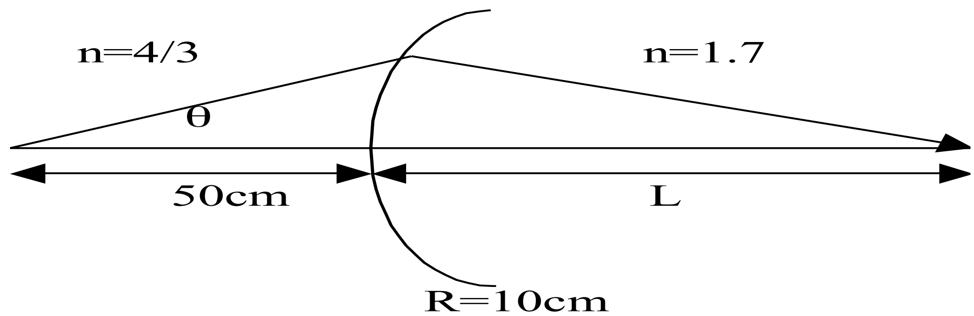
An object of vertical height L is located an axial distance 2f from a thin glass lens. Sketch the rays **to scale** for each of two lenses of equal focal length but opposite sign, i.e., a bi-convex and a bi-concave lens. Draw three rays for each lens that define the image.



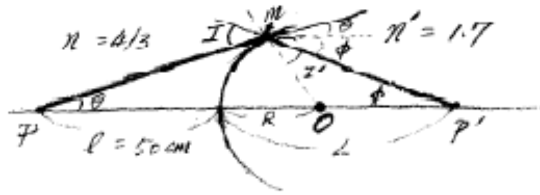


2. ***Paraxial approximation/small angle approximation*** (30%)

You will explore some of the differences between real and paraxial rays in this problem. For each part below, trace the specified ray and determine where it crosses the optical axis. Show all calculations and include a diagram. Report your answers to 4 decimal places.



a) Find L when $\theta = 5°$ with real ray (no paraxial approximation).
b) Find L when $\theta = 0.5°$ with real ray (no paraxial approximation).
c) Repeat part (a) with the paraxial ray (paraxial approximation)
d) Repeat part (b) with the paraxial ray (paraxial approximation)
e) Is there a difference between your answers in (a) and (b)? Is there any difference between your answers in (c) and (d)?
f) Now compare your answers in (b) and either (c) or (d). Why are they so similar?



$R = 10$ cm

- $I = \theta + \phi + I'$ ...... ①
- Snell's law

  $n \cdot \sin I = n' \sin I'$ ...... ②
- from $\triangle PmO$

$$\frac{\overline{PO}}{\sin I} = \frac{\overline{mO}}{\sin\theta}$$

$$\Rightarrow \frac{R+\ell}{\sin I} = \frac{R}{\sin\theta} \quad \cdots\cdots ③$$

- from $\triangle mOP'$

$$\frac{\overline{mO}}{\sin\phi} = \frac{\overline{OP'}}{\sin I'}$$

$$\Rightarrow \frac{R}{\sin\phi} = \frac{L-R}{\sin I'} \quad \cdots ④$$

a) $\theta = 5°$ (I) use ③ $\sin I = \frac{R+\ell}{R}\sin\theta = \frac{10+50}{10}\sin 5° = 0.52$

(II) use ② $\sin I' = \frac{n}{n'}\sin I = \frac{4/3}{1.7} \times 0.5229 = 0.410$

$\therefore I = 31.5270° \quad I' = 24.2138°$

(III) use ① $\phi = I - \theta - I' = 2.3154°$

$\Rightarrow \sin\phi = 0.0404$

(IV) use ④ $L = R\left(\frac{\sin I'}{\sin\phi} + 1\right) = 111.5842$ cm

(b) $\theta = 0.5^\circ$, follow same procedure as (a)

$$\sin I = 0.0529 \qquad I = 3.0013^\circ$$
$$\sin I' = 0.0411 \qquad I' = 2.3526^\circ$$
$$\phi = 0.1477$$
$$L = 168.0769 \text{ cm}$$

(c) Paraxial approximation → small angles

$$\sin\theta \approx \theta,\ \sin I \approx I,\ \sin\phi \sim \phi,\ \sin I' \sim I'$$

③ $$I = \frac{R+\ell}{R}\,\theta$$
② $$I' = \frac{n}{n'}\,I$$
① $$\phi = I - \theta - I' = \frac{R+\ell}{R}\theta - \theta - \frac{n}{n'}\left(\frac{R+\ell}{R}\right)\theta = \left[\cancel{1} + \frac{\ell}{R} - \cancel{1} - \frac{n}{n'}\left(\frac{R+\ell}{R}\right)\right]\theta$$
④ $$\frac{R}{\phi} = \frac{L-R}{I'}$$

$$\frac{R}{\left[\frac{\ell}{R} - \frac{n}{n'}\left(\frac{R+\ell}{R}\right)\right]\cancel{\theta}} = \frac{L-R}{\frac{n}{n'}\left(\frac{R+\ell}{R}\right)\cancel{\theta}}$$

⇒ solve this, we'll get

$$\frac{n'}{L} + \frac{n}{\ell} = \frac{n'-n}{R}$$

$\ell = 50$ cm ⇒ $L = 170.000$ cm

(d) no angle dependence, same as (c)

(e) (a) and (b) are different

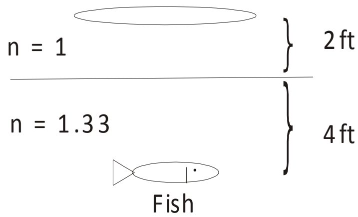
(c) and (d) are same

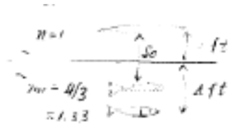
(f) (b), (c) & (d) are similar.

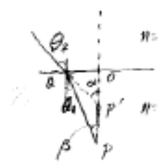
The small angle approximation is the idea of paraxial approximation. The smaller the angle, the less the error induced by paraxial approximation.

3. Imaging - Thin lens (20%)

A small fish, four feet below the surface of Lake Lansing is viewed through a simple thin converging lens with focal length 30 feet. If the lens is 2 feet above the water surface, where is the image of the fish seen by the observer? Assume the fish lies on the optical axis of the lens and that $n_{air}=1$, $n_{water}=1.33$.







(Fig. 1-a)

An object at P in water appears to be at P' as seen by an observer in air, as Fig. 1-a shows. The paraxial light emitted by P is refracted at the water surface, for which

$$n_w \sin\theta_1 = \sin\theta_2$$

As θ_1, θ_2 are small (paraxial approximation → small angle approximation)

$\sin\theta \approx \theta \Rightarrow \quad n_w \theta_1 = \theta_2$

Also,

$$\frac{\overline{OQ}}{\overline{OP'}} = \tan\alpha \approx \alpha = \theta_2$$

$$\frac{\overline{OQ}}{\overline{OP}} = \tan\beta \approx \beta = \theta_1$$

Hence, we have

$$\overline{OP'} = \frac{\overline{OP}}{n_w} = \frac{4}{4/3} = 3 \text{ ft}.$$

Let the distance between the apparent location of the fish and the center of the lens be d, then

$$S_o = 2 + \overline{OP'} = 5 \text{ ft}.$$

From $\frac{1}{f} = \frac{1}{S_o} + \frac{1}{S_i}$, we have

$$\frac{1}{30} = \frac{1}{5} + \frac{1}{S_i} \Rightarrow S_i = -6 \text{ ft}.$$

$$M_T = -\frac{S_i}{S_o} = -\frac{-6}{5} = 1.2$$

Therefore, the image of the fish is still where the fish is, four feet below the water surface, and is magnified by 1.2 (erect virtual image).

4. Compound lens (30%)

a. A compound lens is composed of two thin lenses separated by 10 cm. The first of these has a focal length of +20cm, and the second a focal length of –20cm. Determine the focal length of the combination and locate the corresponding principal points. Draw a diagram of the system.
b. Three identical positive lenses, of focal length *f*, are aligned and separated by a distance *f* from each other. An object is located *f/2* in front of the leftmost lens. Find the position and the magnification power of the resultant image.

4 a

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{+20} + \frac{1}{-20} - \frac{10}{20(-20)}$$

$\therefore f = +40$ cm.

The principal planes are found from (6.9) and (6.10).

$$\overline{O_1H_1} = \frac{fd}{f_2} = \frac{(+40)(10)}{-20} = -20 \text{ cm}$$

$$\overline{O_2H_2} = -\frac{fd}{f_1} = -\frac{(+40)(10)}{20} = -20 \text{ cm}$$

Note: (6.10) on the Hecht has a typo. Should be $H_{02}H_2 = -\frac{fd}{f_2}$

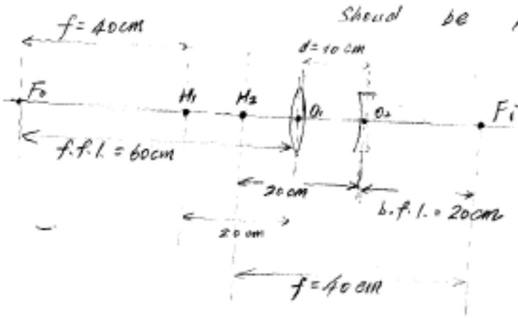


Diagram of the system
Required
⇒ label H_1, H_2
specify F_o, F_i and $f = 40$ cm
Optional
f.f.l. (front focal length)
& b.f.l. (back focal length)

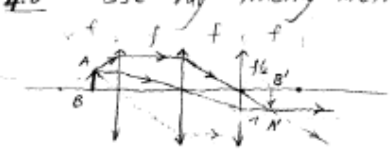
$$\text{f.f.l.} = \frac{f_1(d - f_2)}{d - (f_1 + f_2)} \quad (5.35)$$

$$= \frac{+20(10 - (-20))}{10 - (20 - 20)} = 60 \text{ cm}$$

$$\text{b.f.l.} = \frac{f_2(d - f_1)}{d - (f_1 + f_2)} \quad (5.36)$$

$$= \frac{-20(10 - 20)}{10 - (20 - 20)} = 20 \text{ cm}$$

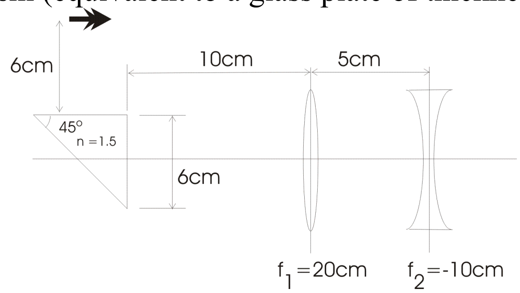
4.b "Use ray tracing method." We construct the image using rays passing through the top of the object.



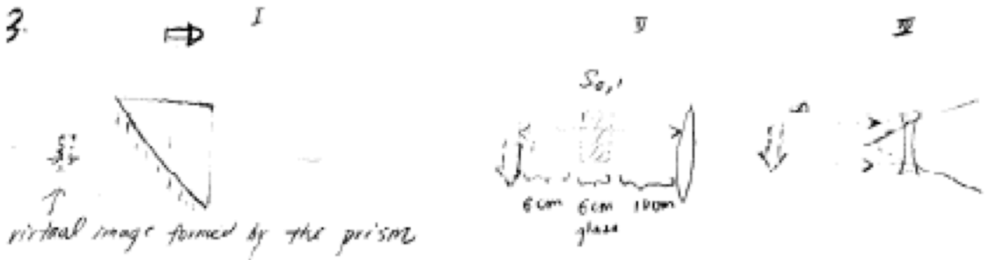
The resultant image is found at a distance f/2 behind the rightmost lens. The magnification is −1 (inverted) (or +1 with Nete's convention)

Imaging - Prism/Thin lens (Extra Credit: 0.25 points)

For the combination of one prism and 2 lenses shown below, find the location and size of the final image when the object, length 1 cm, is located as shown in the figure. [Hint: Treat the prism as a mirror, but you have to take into account the image shift caused by the prism (equivalent to a glass plate of thickness 6 cm)]



3.



① For the right-angle prism, $n = 1.5$, the critical angle for total internal reflection

$\theta_c = \sin^{-1}(\frac{1}{n}) = 42^\circ$, which is smaller than the angle of incidence, 45°, at the hypotenuse of the prism. Therefore total internal reflection occurs, which forms a virtual image.

② The prism, equivalent to a glass plate of thickness 6cm would cause a image shift, the effective distance is

$$\frac{6}{n} = \frac{6}{1.5} = 4 \text{ cm}$$

(small angle approximation, think about previous "fish" problem

③ The effective object distance for the first lens is

$$S_{o,1} = 6 + 4 + 10 = 20 \text{ cm}$$

$$\frac{1}{S_{o,1}} + \frac{1}{S_{i,1}} = \frac{1}{f}$$

As $S_{o,1} = f = 20$ cm, we have $S_{i,1} = \infty$.