**PHY 431 Homework Set #3**

Due October 9 in class

***1) Convex-Plano vs. Plano-Convex lens (25%)***

The rays incident on the outer edge of a lens (outside of the paraxial regime) suffer from spherical aberration. This is because the nonparaxial rays are too strongly bent. Consider the plano-convex lens as seen above. Depending on which surface faces the incident rays, the amount of spherical aberration can be reduced. In this problem, you will decide which lens configuration is better in terms of spherical aberration.

a. Find the focal length of the lens in the paraxial regime.
b. Let's assume we have a ray parallel to the optical axis incident on the lens as shown above. Calculate where the ray crosses the optical axis (L'). Neglect the thickness of the lens. (Hint: You will use Snell's law twice)
c. Now flip the lens around so light is incident on the planar side. Repeat parts (a) and (b). (Hint: You will use Snell's law only once)
d. Which one is better in terms of spherical aberration? (Hint: compare the focal length obtained in (a), (b), and (d))

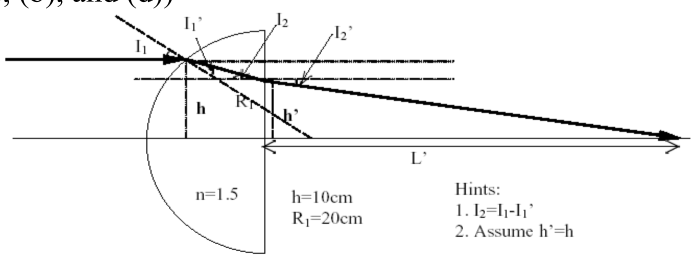
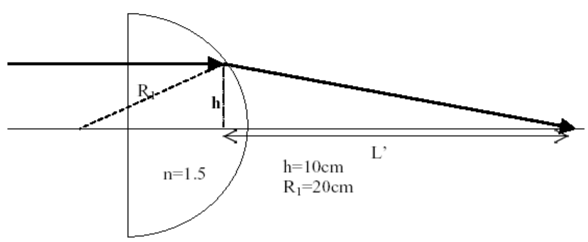



Fig.1-(a)




2) ***Mirror/Magnifying glasses and Imaging*** (25%)

**(a)** An object is 100 cm from a spherical bi-convex lens with radius of curvature 40 cm.
Does a real image form?
Determine the image distance from the vertex of the lens.
Determine the transverse and longitudinal magnifications ($M_T$ and $M_L$).
(Assuming the refractive of the lens material n=1.5)

**(b)** Same as above, but now an object is 20 from the lens.
Does a real image form?
Determine the image distance from the vertex of the lens.
Determine the transverse and longitudinal magnifications ($M_T$ and $M_L$)

**(c)** A person whose face is 25 cm away looks into the bowl of a spherical soupspoon with an approximate radius of curvature R = 3 cm.

Determine the magnification of her image reflected by the spoon.
(use the mirror formula : $\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$ )

$$f = R/2$$

3) ***Wave*** (10%) Show that (a) $\psi(x,t)=f(x \pm vt)$, and (b) $\psi(x,t) = A \sin(kx-wt)$ is a solution of the one-dimensional differential wave equation $\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2}$.

4) ***Plane wave*** (10%)

A plane wave propagating in a given medium is expressed as

$$\mathbf{u}(x,y,z,t)=\mathbf{u_0} \exp[i(2x+3y+4z)\times 10^6 - i\, 10^{15} t]$$

Where the spatial coordinate (x,y,z) and the time are measure in the MKS unit.

a. Find the unit vector for the direction of propagation.
b. The direction of propagation is defined as $e_x=\sin\theta\cos\phi$, $e_y= \sin\theta \sin\phi$, $e_z=\cos\theta$. What are the values of $\theta$ and $\phi$ characterizing the direction of propagation?
c. Find the spatial frequency $f_s$ (in units of line/m) of the plane wave.
d. Find the wavelength of the plane wave in the medium and the refractive index $n$ of the medium.

5) ***Polarization*** (10%)

Describe the polarization of the following transverse, monochromatic waves:

**a.** $\mathbf{E_x} = \sin(wt-kz)$, $\mathbf{E_y} = \sin(wt-kz-\pi)$
**b.** $\mathbf{E_x} = 3\cos(wt - kz)$ , $\mathbf{E_y} = 4\cos(wt - kz)$
**c.** $\mathbf{E_x} = \sin(wt-kz)$, $\mathbf{E_y} = \sin(wt-kz-\pi/2)$
d. $\mathbf{E_x} = 2\sin(wt-kz)$, $\mathbf{E_y} = \sin(wt-kz-\pi/2)$

6) ***Polarizer*** (20%)

a. The irradiance of a beam of natural light is $400 W/m^2$. It impinges on the first of two consecutive ideal linear polarizers whose transmission axes are 40.0° apart. How much light emerges from the two?

**b.** Imagine that you have two identical perfect linear polarizers and a source of natural (unpolarized) light. Place them on behind the other and position their transmission axes at 0 and 50°, respectively. Now insert between them a third linear polarizer with its transmission axis at 25°. If $I_o$ is the incident light intensity, how much will emerge with and without the middle polarizer in place?

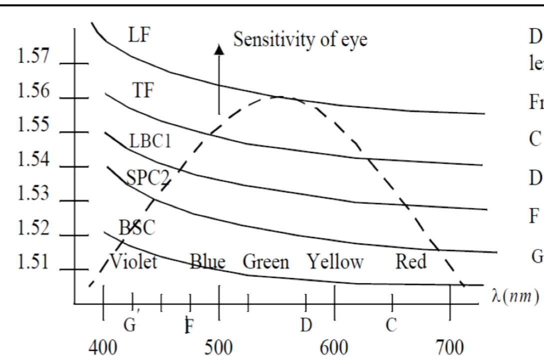
*__Extra Credits: Achromatic Doublet Lens (0.5 point)__*

Table : REFRACTIVE INDICES OF TYPICAL OPTICAL MEDIA FOR FOUR COLORS

| Medium | Desig-nation | ICT type | $\nu$ | $n_C$ | $n_D$ | $n_F$ | $n_{G'}$ |
|---|---|---|---|---|---|---|---|
| Borosilicate crown | BSC | 500/664 | 66.4 | 1.49776 | 1.50000 | 1.50529 | 1.50937 |
| Borosilicate crown | BSC-2 | 517/645 | 64.5 | 1.51462 | 1.51700 | 1.52264 | 1.52708 |
| Spectacle crown | SPC-1 | 523/587 | 58.7 | 1.52042 | 1.52300 | 1.52933 | 1.53435 |
| Light barium crown | LBC-1 | 541/599 | 59.7 | 1.53828 | 1.54100 | 1.54735 | 1.55249 |
| Telescope flint | TF | 530/516 | 51.6 | 1.52762 | 1.53050 | 1.53790 | 1.54379 |
| Dense barium flint | DBF | 670/475 | 47.5 | 1.66650 | 1.67050 | 1.68059 | 1.68882 |
| Light flint | LF | 576/412 | 41.2 | 1.57208 | 1.57600 | 1.58606 | 1.59441 |
| Dense flint | DF-2 | 617/366 | 36.6 | 1.61216 | 1.61700 | 1.62901 | 1.63923 |
| Dense flint | DF-4 | 649/338 | 33.9 | 1.64357 | 1.64900 | 1.66270 | 1.67456 |
| Extra dense flint | EDF-3 | 720/291 | 29.1 | 1.71303 | 1.72000 | 1.73780 | 1.75324 |
| Fused quartz | $SiO_2$ | | 67.9 | | 1.4585 | | |
| Crystal quartz (*O* ray) | $SiO_2$ | | 70.0 | | 1.5443 | | |
| Fluorite | $CaF_2$ | | 95.4 | | 1.4338 | | |

An achromatic lens is to be made of BSC (crown) and DF-2 (flint) glasses and is to have a focal length of 8.00 cm (see Table above). If the flint glass lens is to have its outer face flat and the combination is to be cemented (i.e. set $R_{11}=\infty$ and $R_{12}=R_{21}$. Since the lens is to be cemented, one surface of the negative lens must fit a surface of the positive lens). Find (a) the power of the lens, (b) the *V* values of the two glasses [consider D-line (589.2nm/Yellow) as the center of the spectrum between C-line (656.3nm/Red) and F-line (486.1nm/Blue), then V-number/Abbe number $v=\frac{n_D-1}{n_F-n_C}$], (c) the powers of the two lenses, and (d) the radii of the three curved surfaces. The lens combination is to be corrected for the C and F lines.

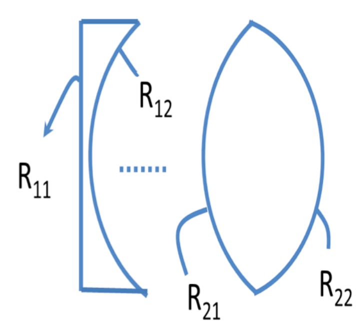
**Recipe:** (1) Start with desired f and P=1/f, (2) Choose the glass materials and calculate V-number, $V_1$ and $V_2$, (3) Find $P_1=1/f_1$ and $P_1=1/f_1$ (First, you need to derive $P_1=P\frac{V_1}{V_1-V_2}$ and $P_2=-P\frac{V_2}{V_1-V_2}$ by using $P=P_1+P_2$ (i.e. $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$) and $f_{1Y}\times V_1+f_{2Y}\times V_2=0$ derived in class), (4) Choose radii (here the radii are obtained by setting $R_{22}=\infty$ and $R_{21}=R_{12}$)




Different glasses for use in lenses.

Fraunhofer designations.

| | | |
|---|---|---|
| C | H | 656.3 nm |
| D | Na | 589.2 |
| F | H | 486.1 |
| G' | H | 434.0 |



***Extra Credit: Liquid crystal half-wave plate (0.25 point)***

A half-wave plate has a phase retardation of $\pi$. Assume that the plane is oriented so that the azimuth angle (i.e., the angle between the x-axis and the slow axis of the plate) $\Psi=0$.

c. Find the polarization state of the transmitted beam, assuming that the incident beam is linearly polarized in the y direction.
d. Show that a half-wave plate will convert right-hand circularly polarized light into left-hand circularly polarized light, and vice-versa.
e. E7 is a nematic liquid crystal with $n_0=1.52$ and $n_e=1.75$ at $\lambda=577$ nm. Find the half-wave-plate thickness at this wavelength.