PHY 431 Homework Set #5

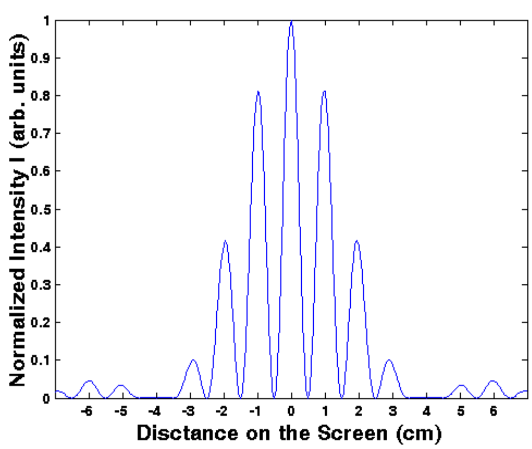
Due Nov. 20 at the start of class

1) ***Newton's rings (10%)***
The radius of curvature of the convex surface of a plano-convex lens is 30 cm. The lens is placed with its convex side down on a plane glass plate, and illuminated from above with red light of wavelength 650 nm.
   A. Find the diameter of the third bright ring in the interference pattern.
   B. Show that for large R this diameter is approximately proportional to $R^{1/2}$.

2) ***Anti-reflection coating (10%)***
A lens is to be coated with a thin film with an index of refraction of 1.2 in order to reduce the reflection from its surface at λ=5000 A°. The glass of the lens has a refractive index of 1.4.
   A. What is the minimum thickness of the coating that will minimize the intensity of the reflected light?
   B. In the above case the intensity of the reflected light is small but not zero. Explain. What need to be changed, and by how much, to make the intensity of the reflected light zero?

3) ***Michelson Interferometer (20%)***
   a. [Hecht 9.36] One of the mirrors of a Michelson Interferometer is moved, and 1000 fringe-pairs shift past the hairline in a viewing telescope during the process. If the device is illuminated with 500-nm light, how far was the mirror moved?
   b. [Hecht 9.37] Suppose we place a chamber 10.0 cm long with flat parallel windows in one arm of a Michelson Interferometer that is being illuminated by 600-nm light. If the refractive index of air is 1.00029 and all the air is pumped out of the cell, how many fringe-pairs will shift by in the process?

4) ***The double slit (20%)***
A plane wave of light from a laser has a wavelength of 6000 A°. The light is incident on a double slit. After passing through the double slit the light falls on a screen 100 cm beyond the double slit. The intensity distribution of the interference pattern on the screen is shown. What is the width of each slit and what is the separation of the two slits?



5) ***Fraunhofer Diffraction (20%)***
   a. Consider the Fraunhofer diffraction pattern due to two unequal slits. Let *a* and *b* be the unequal slit widths and *c* the distance between their centers. Derive and

expression for the intensity of the pattern for any diffraction angle θ, assuming the arrangement to be illuminated by perpendicular light of wavelength λ.

b. Use your formula from (A) to obtain expression for the pattern in the following special cases and make a sketch of those patterns:
   i. $a=b$,
   ii. $a=0$.

6) ***Gratings [20%]***
Green light of λ=530nm falls on a diffraction grating of length l=12 mm and period d=1.5 μm. Calculate the angular dimension of the primary maximum and the resolving power of the diffraction grating.

Example Final Exam Problems:

7) ***Spatial and Spectral Resolution (20%)***

Two stars have an angular separation of 1×10^{-6} radian. They both emit light of wavelengths 5770 and 5790 A°.

A. How large a diameter of the lens in the telescope is needed to separate the image of the two stars?

B. How large a diffraction grating is needed to separate the two wavelengths present?

8) ***Fourier Optics [30%]***

(a) Let a plane monochromatic wave fall on an aperture which is placed on the first (front) focal plane of a positive thin spherical lens. Represent the angle of diffraction in terms of spatial frequencies over the second (back) focal plane of the lens.

(b) Let a plane monochromatic wave fall on a transmission diffraction grating, which has an amplitude transparency function $t(x) = 1 + a\cos(\Omega_0 x)$, with $a < 1$. Find the intensity distribution on a plane parallel to the grating at a distance Δz.

9) **Diffraction: Single Slit, Double Silts & Rectangular Apertures (40%)**

In a diffraction experiment, a collimated beam of laser from a green diode laser (λ=532nm) is to be used.

(A) The laser output beam is approximately circular with a diameter of 2 mm. You need a beam expander to produce a collimated beam of 10 mm for the diffraction experiments. You are given six lenses with the following focal lengths (negative lenses: f=-1 cm, -2 cm, and -5cm; positive lenses: f= +2cm, +10 cm, and +15 cm). **Choose** two lenses to set up such a beam expander (Galilean telescope type). **Specify** the distance between these two lenses and **Draw** a ray diagram to illustrate the beam expansion.

(B) In the 1st experiment, a hole of R=0.5 mm radius is used as an aperture.
Explain Fraunhofer and Fresnel diffraction regimes based on the Huygens-Fresnel principle. **Determine** whether Fresnel or Fraunhofer approximation applies when the screen-to-aperture distance z is (i) 10 cm (ii) 2 m.
[Hint: consider z vs. $\frac{\pi R^2}{\lambda}$ & z^3 vs. $\frac{\pi R^4}{4\lambda}$]

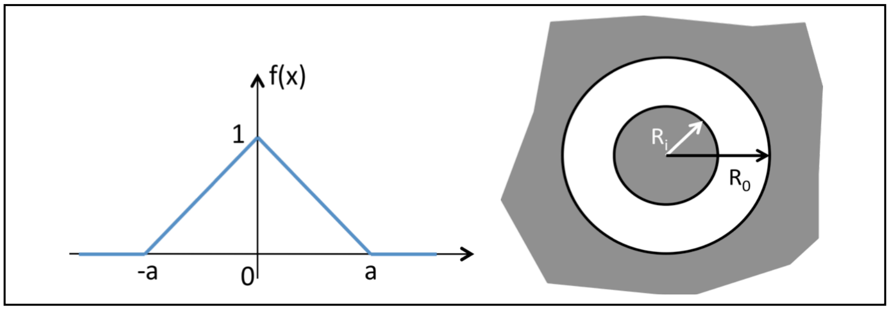
(C) In the 2nd experiment, the collimated beam falls normally on a slit 50 μm wide. A lens of 50 cm focal length placed just behind the slit focuses the diffraction light on a screen located at the focal distance.
Calculate the distance from the center of the diffraction pattern (central maximum) to the first minimum.

(D) In the 3rd experiment, a double slit is used as an aperture. It is found that the fourth secondary maximum is missing.
Determine the ratio of slit width ***a*** to slit separation ***b***. [consider Fraunhofer diffraction]

(E) A square aperture (width=a) is used for the final experiment. **Sketch** the diffraction pattern **and label the first zeros and maxima. Determine** the value of intensity ratio I/I_0 for the first diagonal maximum of the resultant Fraunhofer diffraction pattern. I_0 is the intensity at the central maximum. (The maxima occur at
$\frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right) = 0$, *i.e.* $\alpha = \tan\alpha$. The first diagonal maxima occur at
$\alpha=\beta=1/2k\times a\times\sin(\theta)=\pm1.4303\pi$).

10) ***Fourier Optics / Fraunhofer Diffraction*** (30%)

a. Determine the Fourier transform of the triangular pulse shown below. Make a sketch of your answer. Label the **first and second minimums** in units of $k_x=2\pi/\lambda\times\sin\theta$ on the curves. [The Fourier transform of f(x), $F(k_x)=\int_{-\infty}^{+\infty} f(x)e^{ik_x x}dx$.]



b. A transparent ring on an otherwise opaque mask as shown above is used as an aperture for a diffraction experiment. Assuming uniform normally incident plane-wave illumination. The aperture is circular and has a central obscuration disk (**$R_0/R_i=2$**). A screen is position at a distance z away from the aperture. The irradiance/intensity at the center of the diffraction pattern, P, on the screen is **I_0**.

(i) Find an expression in the Fraunhofer diffraction pattern of the aperture, and

(ii) Determine the irradiance at P when the central obscuration is removed.

[Hint: The electric field amplitude of the Fraunhofer diffraction pattern of a circular aperture (radius R) under uniform illumination U_0 is $U(x,y,z)=U_0\times A\times\frac{e^{-i(kz)}}{-i\lambda z}\times\left[\frac{2J_1(\rho)}{\rho}\right]$, where A=$\pi R^2$ is the area of the aperture, R the radius of the aperture, $\rho=kR\sin\theta$, k the wavenumber, θ the direction of the diffracted ray. The irradiance/intensity $I\propto|U|^2$].

11) ***Irradiance/Human Eyes*** (30%)

a. (5%) Estimate the average number of photons per second per square centimeter reaching Earth's surface from the sun. At the earth's surface, the suns irradiance is approximately 300 W/m^2.
b. (5%) A green laser pointer puts out 1 mW at a wavelength of 532 nm. How many photons are emitted per second?
c. (5%) Supposed you are seated 10 m from a screen onto which a speaker shines his laser pointer. Assume that the screen scatters 20% of the radiation incident upon it, and that you eye's pupil has a diameter of 5 mm. What is the rate at which photons from the laser pointer enter your eye?
d. (15%) Compare the **irradiances** at the retina that result when looking:
   (a) Directly into the sun. The sun subtends an angle of 0.5 degrees. Assume that the pupil of the bright-adapted eye is 2 mm in diameter with focal length of 22.5mm.
   (b) Into a 1 mW green laser pointer (wavelength=532 nm). Assume the beam waist of the laser is 1 mm, and the laser is located 1m from the eye.
   (c) Which one will damage your eye? Eye-damaging intensities are in the range of 10mW/cm^2.

12) Visual Acuity (20%)

A. The separation between cone cells in the fovea corresponds to about 1 min (0.3 mrad).
   a. At close viewing distance of 25 cm, what is the resolution?
   b. What is the diffraction-limited object size (at 25 cm) imposed by the numerical aperture of the eye (if the eye is a diffraction-limited optic)? Use 4mm for the iris diameter and 550 nm for the wavelength.
   c. What magnification is necessary for a telescope to enable a person with "normal" visual acuity to read letters 1 mm high, at a distance 400 ft? Note: the "normal" visual acuity is considered to be 1.0.

B. Looking through a small hole is a well-known method to improve sight. If your eyes are near-sighted and can focus an object 20 cm away without using glasses, estimate the required diameter of the hole through which you would have good sight for object far away. (Assume the eye is 20 mm long.)