

## Aberrations

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In this lab we will explore how a lens deviates from ideality, the subject of *aberrations*. You will observe how the focal length depends on the angular distribution of light (spherical aberrations) and on wavelength (chromatic aberrations). Because these effects may be small, you will need to make several measurements, plot the results to reveal systematic deviations, and perform least-squares fits to compare with theory.

- A. Select a plano-convex lens, about 3.5 inches in diameter, and find its focal length. Use the method of image formation with the thin lens formula; also calculate  $f$  from the lens curvatures, measured with a spherometer, and  $n$  given in the Appendix. Calculate uncertainties in  $f$  and compare results.
- B. Place the lens about  $1.25f$  from the object with the planar surface facing the lamp. Place the smallest aperture (the cardboard with a 15 mm hole) immediately in front of the lens. Be careful to position it on the optical axis. Focus the image and measure the image distance  $s_i$ . With this aperture, assume that the measurement is aberration-free.
- C. Replace the aperture with different ring stops. Measure  $s_i$  and the radii,  $h$ , of the rings.
- D. Make a plot of  $h$  vs  $\Delta(1/s_i)$  and fit to a quadratic function as suggested by the equations in the Appendix. Also, use a linear fit by plotting  $h^2$  vs  $\Delta(1/s_i)$ . **Q1.** What is the goodness of fit? What measure should you use?
- E. Rotate the lens (planar side away from the source) and repeat steps B-D.
- F. Using the equations in the Appendix, calculate the parameter  $L_s$  from the measurements. **Q2.** How does your calculation compare with the least-squares fit parameters? **Q3.** If some of the points deviate systematically from the fits, explain why this happens.
- G. Place one of the colored filters in front of the lens and measure the focal length using the thin lens equation. Repeat with the other filters. Leave  $s_o$  unchanged to improve accuracy.
- H. Plot  $1/f$  vs.  $n$  using the values of the refractive index given in the Appendix and fit to a linear function. **Q4.** Compare your fit results to the radii measured with the spherometer. Make an assessment of the accuracy of the two different types of measurements.

APPENDIX

See Pedrotti Ch. 3-2 for a brief look at aberrations and Chapter 20 for more complete theory (alternative text *Hecht Ch. 6.3*). You may also consult the technical notes on “[Performing Factors](#)” from CVIMellesGriot.com for an intuitive understanding of aberrations.

**Spherical Aberration [Pedrotti 20-3] [Hecht 6.3.1]**

The derivation of an equation for spherical aberration is too lengthy to be given here. For a thin lens we have the reasonably simple formula from third-order theory:

$$L_s = \frac{h^2}{8f^3} \frac{1}{n(n-1)} (A \times q^2 + B \times pq + C \times p^2 + D); A = \frac{n+2}{n-1}, B = 4(n+1), C = (3n+2)(n-1), D = \frac{n^3}{n-1}$$

where  $L_s = \frac{1}{s'_h} - \frac{1}{s'_p}$ .  $s'_h$  is the image distance for an oblique ray traversing the lens at a distance  $h$  from the axis,  $s'_p$  is the image distance for paraxial rays, and  $f$  the paraxial focal length.

The constant  $p = \frac{s_i - s_o}{s_i + s_o} = \frac{2f}{s_o} - 1 = 1 - \frac{2f}{s_i}$  is called the *position facto*. Here  $s_o$  and  $s_i$  are the object and image distances under the paraxial approximation (i.e. applying the first-order/thin-lens imaging equation  $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$ ).

$q = \frac{R_1 + R_2}{R_1 - R_2}$  is the *shape factor* (for  $R_1 \rightarrow \infty, q \rightarrow 1$ ).

The difference between the two image distances,

$$L \bullet SA \equiv \Delta s_i \equiv s'_p - s'_h = s'_p \times s'_h \times L_s$$

is called the *longitudinal spherical aberration*.

The intercept of the oblique ray with the paraxial focal plane,

$$T \bullet SA = s'_p \times h \times L_s$$

is the *transverse spherical aberration* (or *lateral spherical aberration*).

The image distance  $s'_h$  for any ray through any zone is given by  $s'_h = \frac{s'_p}{1 + s'_p L_s}$

\*Assume  $n = 1.52$  when using these equations.

**Chromatic Aberration [Pedrotti 20-7] [Hecht 6.3.2]**

For chromatic aberration, you can use the Lensmaker’s Equation and values of  $n$  for the glass at the filter transmission maxima given in the table below.

Color	$\lambda$ (nm)	n
Violet	420	1.5318
Blue	460	1.5265
Cyan	485	1.5240
Green	540	1.5196
Yellow	580	1.5172
Red	640	1.5145