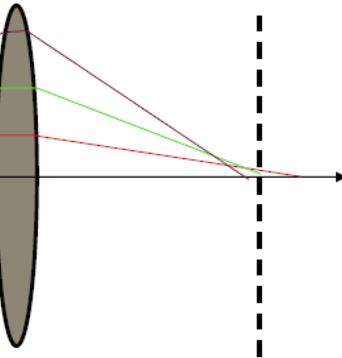
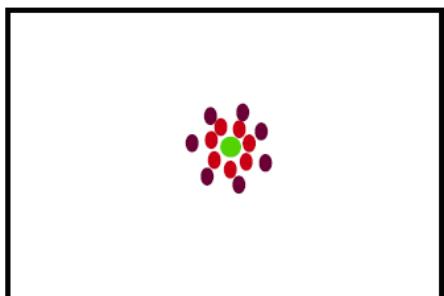


# When Paraxial Approximation Fails: Ray Tracing + Diffraction



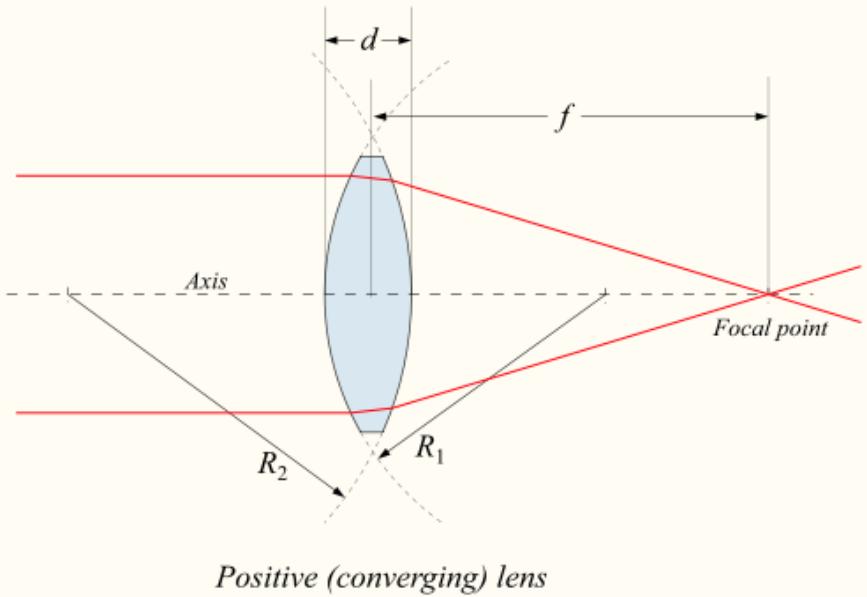
Exact ray-tracing



ray scatter diagram ( $\Leftrightarrow$  defocus)

- Databases of common lenses and elements
- Simulate aberrations and ray scatter diagrams for various points along the field of the system (PSF, point spread function)
- Standard optical designs (e.g. achromatic doublet)
- Permit optimization of design parameters (e.g. curvature of a particular surface or distance between two surfaces) *vs designated functional requirements* (e.g. *field curvature and astigmatism coefficients*)
- Also account for diffraction by calculating the at different points along the field modulation transfer function (MTF) [Fourier Optics]

# Thin Lenses → Thick Lenses



Positive (converging) lens

Lens maker's formula

$$\frac{1}{f} = (n - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} + \frac{(n - 1)d}{nR_1R_2} \right],$$

“Thin” lens → d is negligible

$$\frac{1}{f} \approx (n - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right].$$

Paraxial approximation

$$\sin(\theta) \approx \tan(\theta) \approx \theta$$

$$\cos(\theta) \approx 1$$

Review the following equations in Ch. 2.

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$$

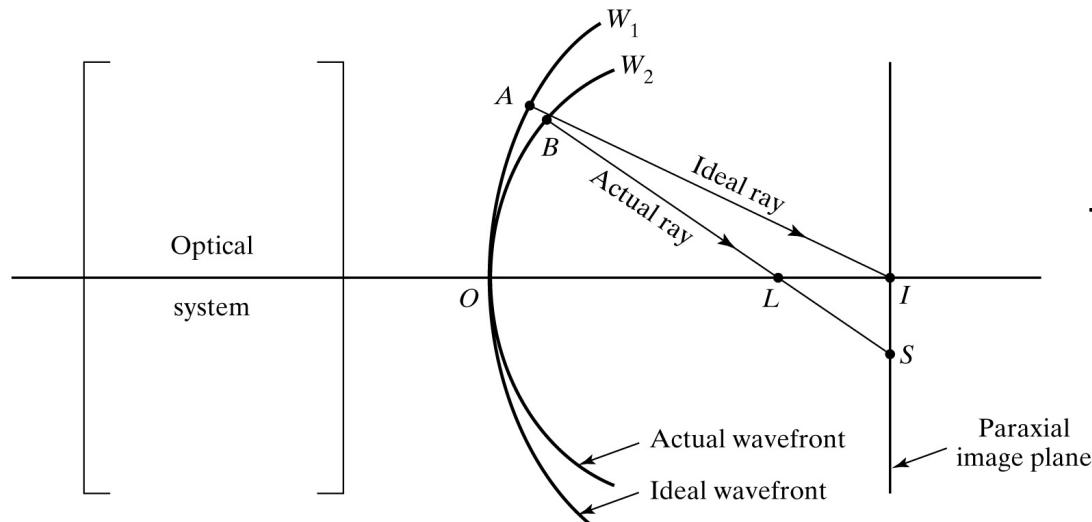
$$x_0 x_i = f^2$$

$$M_T \equiv \frac{y_i}{y_0} = -\frac{s_i}{s_o}$$

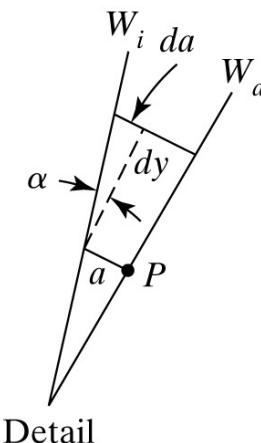
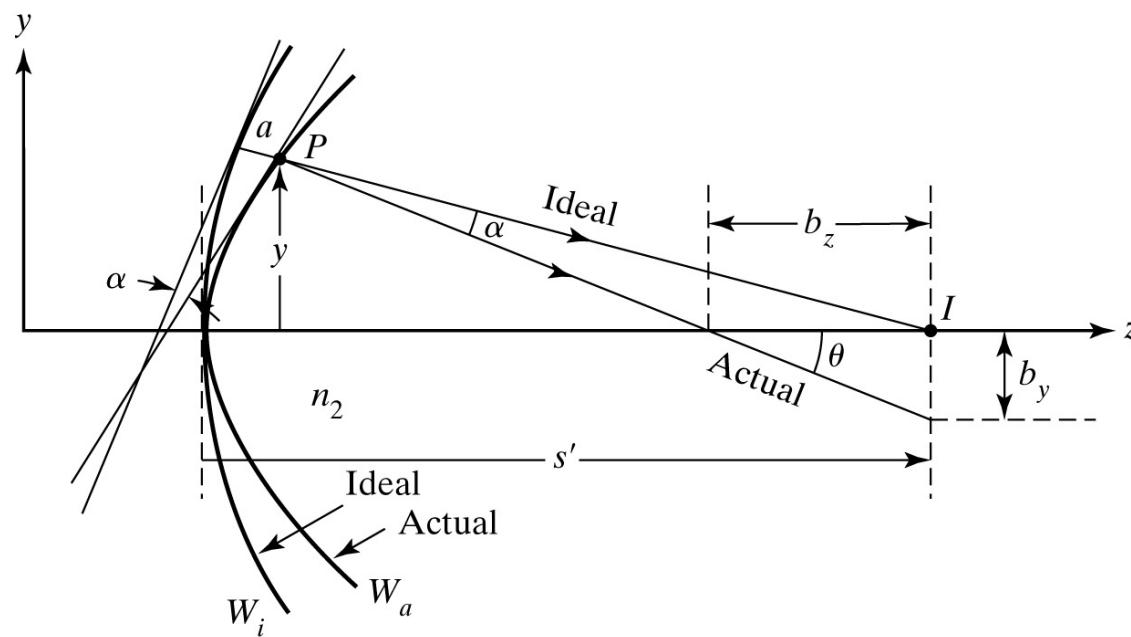
$$M_L \equiv \frac{dx_i}{dx_0} = -\frac{f^2}{x_0^2}$$

**“Sign” convention is of paramount importance!**  
**(See Pedrotti^3, Table 2-1)**

# Ray and Wave Aberrations



Ray Aberrations:  
Longitudinal aberration  
Transverse or lateral aberration



# Aberrations (a brief description)

- Chromatic
  - is due to the fact that the refractive index of lenses, etc. varies with wavelength; therefore, focal lengths, imaging conditions, etc. are wavelength-dependent
- Geometrical (monochromatic)
  - are due to the deviation of non-paraxial rays from the approximations we have used so far to derive focal lengths, imaging conditions, etc.; therefore, rays going through imaging systems typically do not focus perfectly but instead scatter around the “paraxial” (or “Gaussian”) focus

Refractive index  $n$  is dispersive!  
 $n(\omega)$

Third-order or Seidel aberrations  
*Deteriorate the image:*

- Spherical aberration
- Coma
- Astigmatism

*Deform the image:*

- Field curvature
- Distortion

**Departures from the idealized conditions of Gaussian Optics (e.g. paraxial regimes).**

$$\sin \varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \dots$$

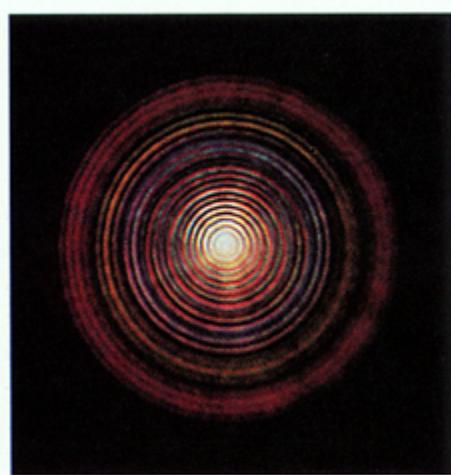
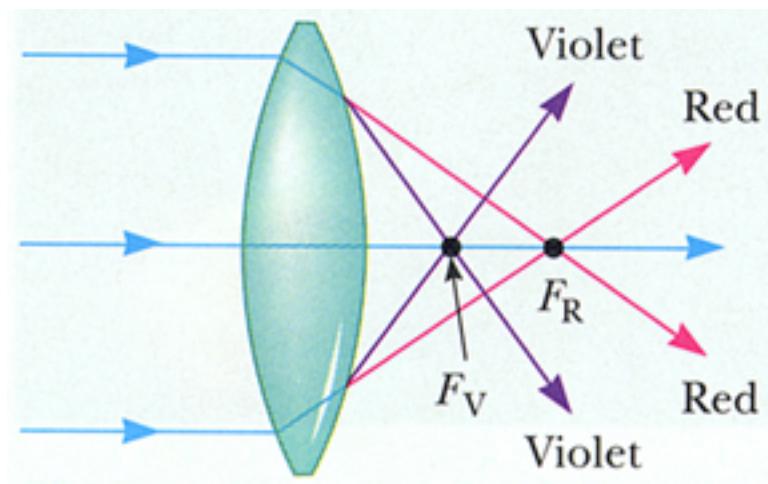
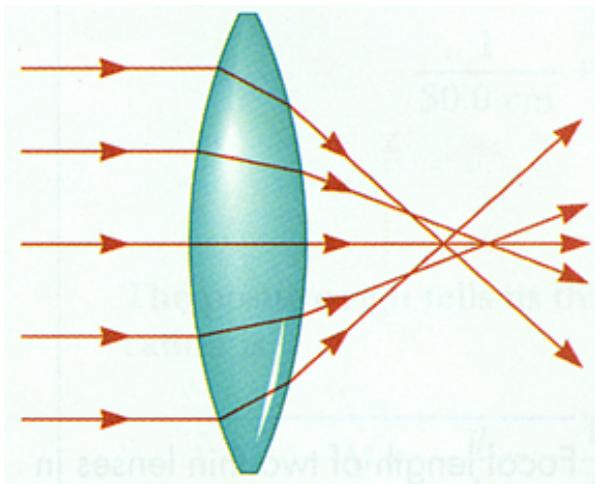
$$\cos \varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \dots$$

Paraxial approximation

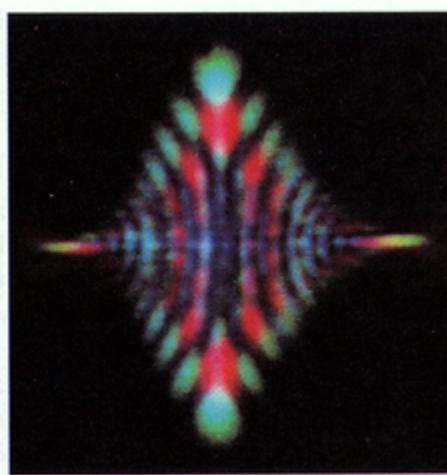
$$\sin(\varphi) \approx \tan(\varphi) \approx \varphi$$

$$\cos(\varphi) \approx 1$$

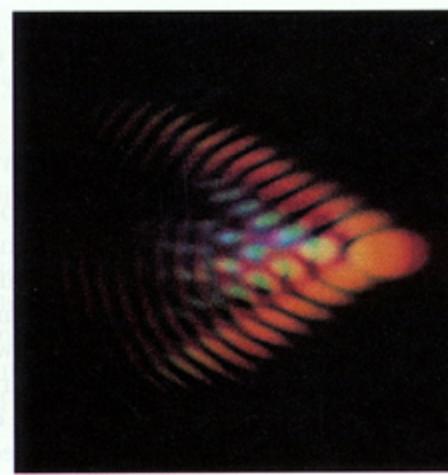
# Lens Aberrations



(a)  
Spherical & chromatic



(b)  
Astigmatism

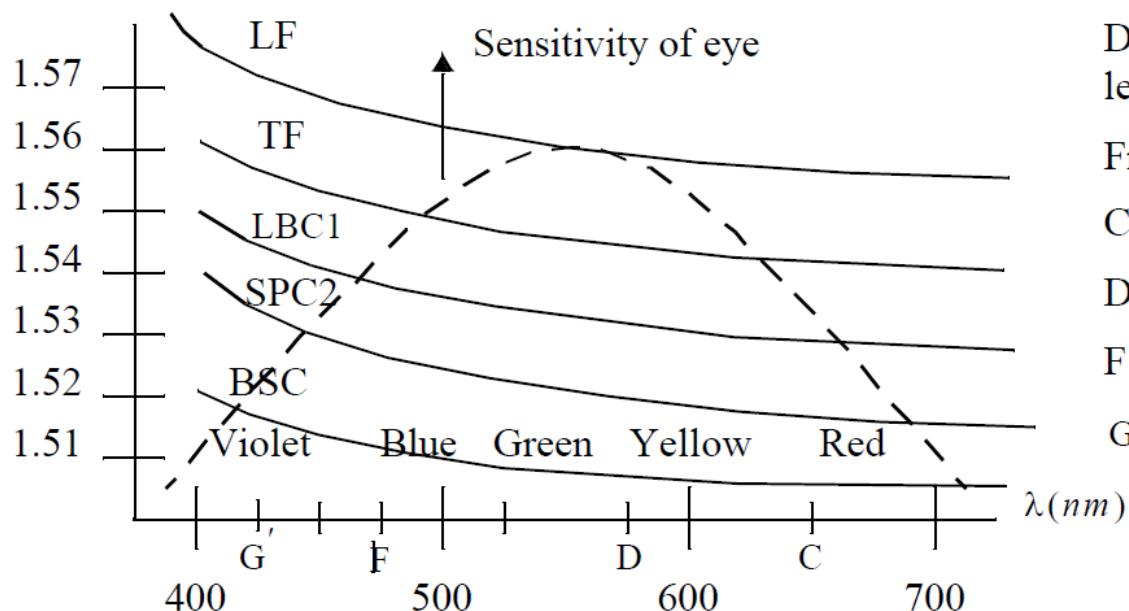
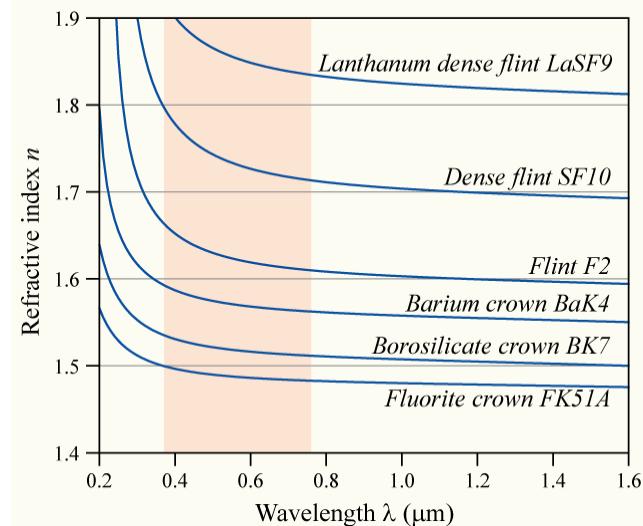


(c)  
Coma

# Chromatic Aberration

Pedrotti^3, Ch. 3-2 & Ch. 20-7

$$\frac{1}{f} \approx (n - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right].$$



Different glasses for use in lenses.

Fraunhofer designations.

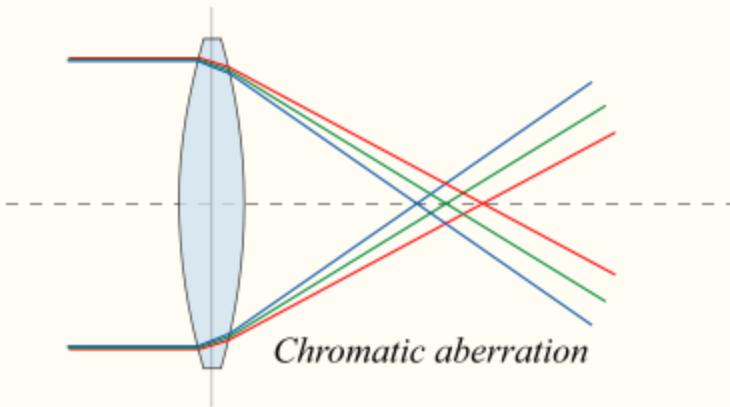
C H      656.3 nm

D Na      589.2

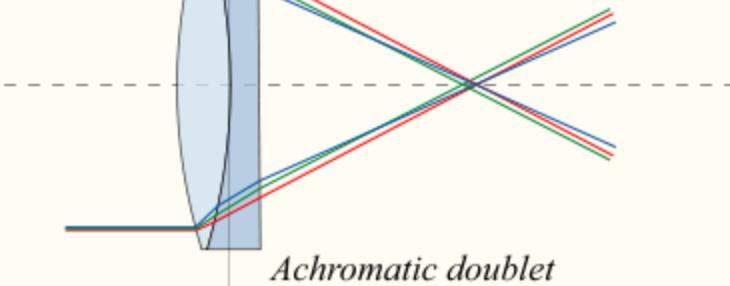
F H      486.1

G' H      434.0

# Chromatic Aberration



Crown      Flint       $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$



Solutions:

1. Combine lenses (achromatic doublets)
2. Use mirrors

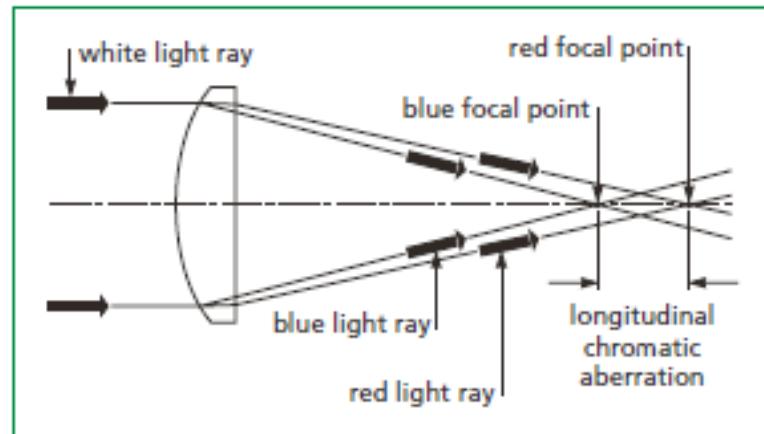


Figure 1.21 Longitudinal chromatic aberration

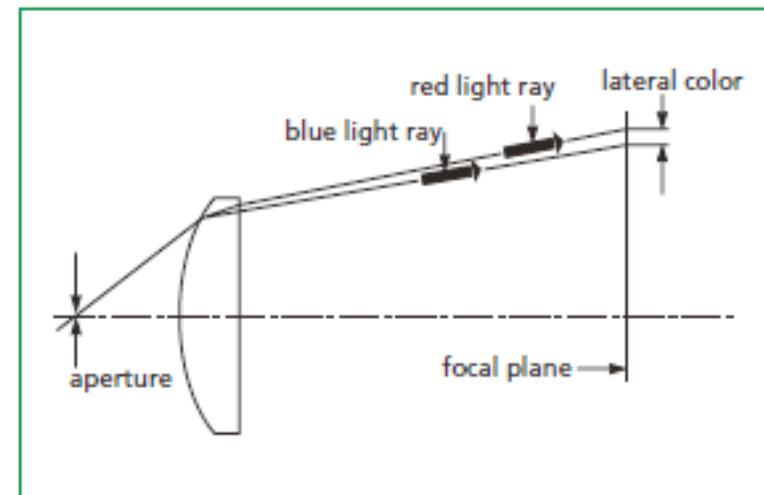
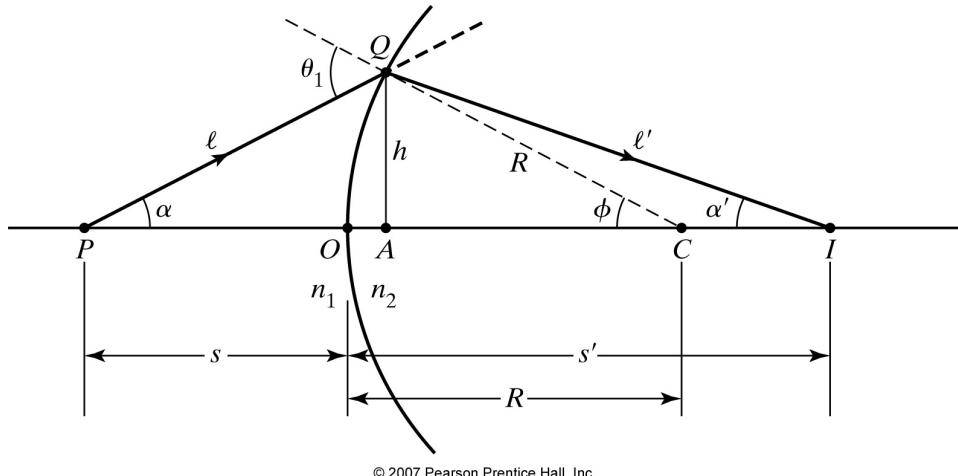


Figure 1.22 Lateral color

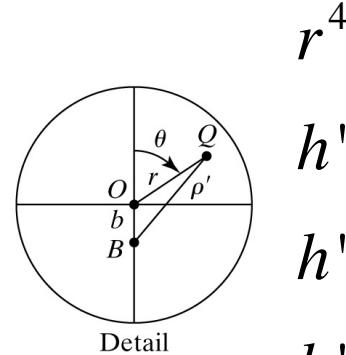
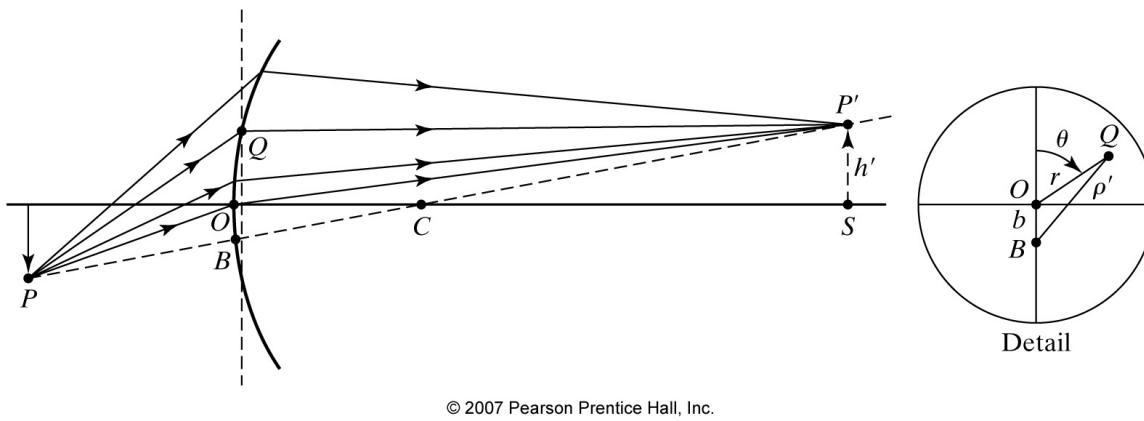
# The Five monochromatic, or Seidel, Aberrations



The aberration at Q

$$a(Q) = (PQI - POI)_{opd}$$

opd: the optical-path difference



$r^4$  Spherical aberration

$h' r^3 \cos \theta$  Coma

$h'^2 r^2 \cos^2 \theta$  Astigmatism

$h'^2 r^2$  Curvature of field

$h'^3 r \cos \theta$  Distortion

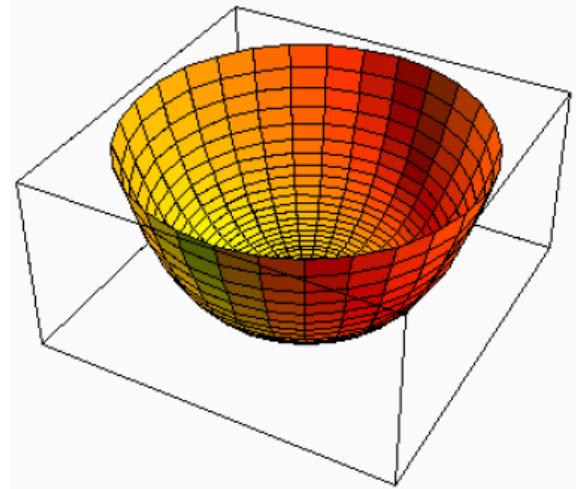
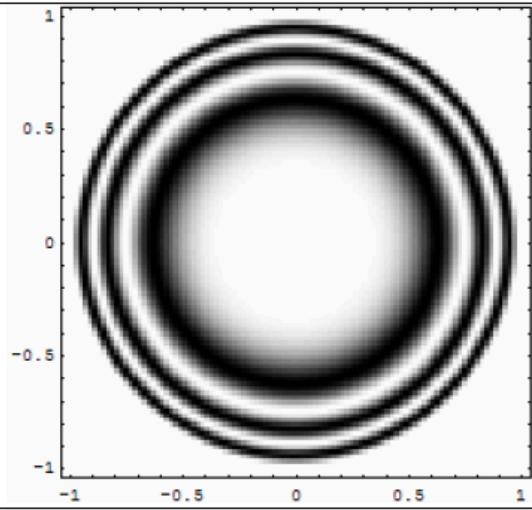
# Optical Path Difference: Spherical Aberration

<http://wyant.optics.arizona.edu/OPDPlotsMargy/OPDplots.htm>

$$\text{SA} = \mathbf{W}_{040} \rho^4$$

$$W_y = W_{040} y_p^4$$

$$W_x = W_{040} x_p^4$$



# Spherical Aberration

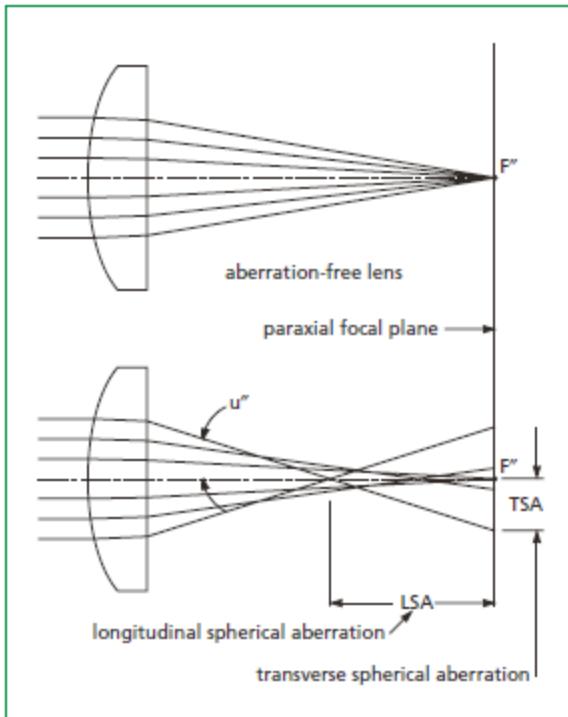
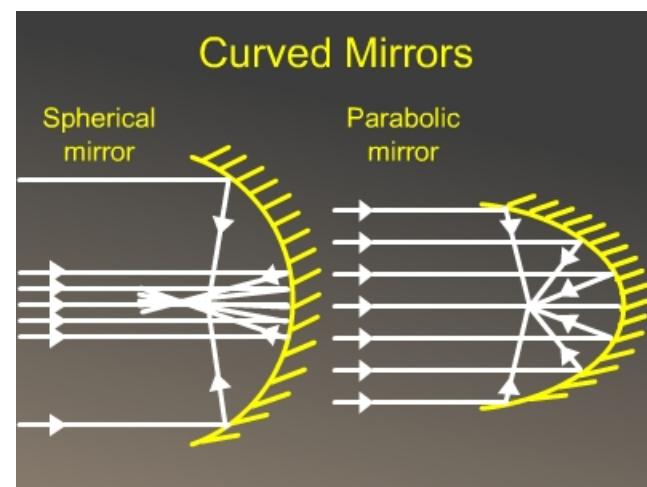
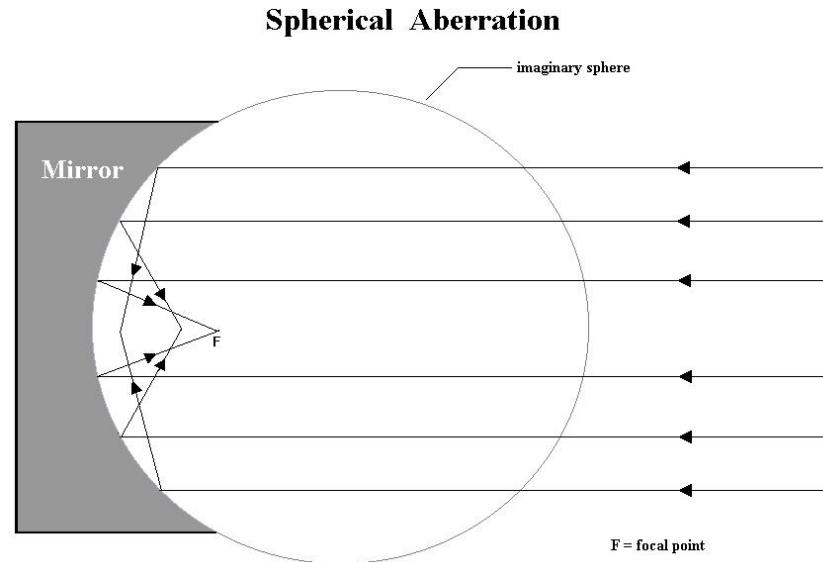
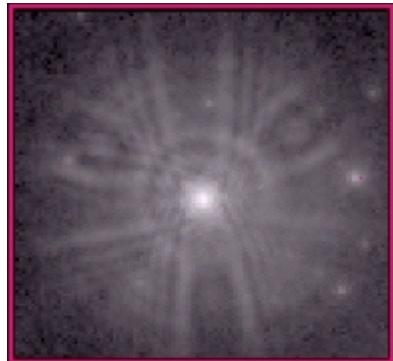


Figure 1.15 Spherical aberration of a plano-convex lens

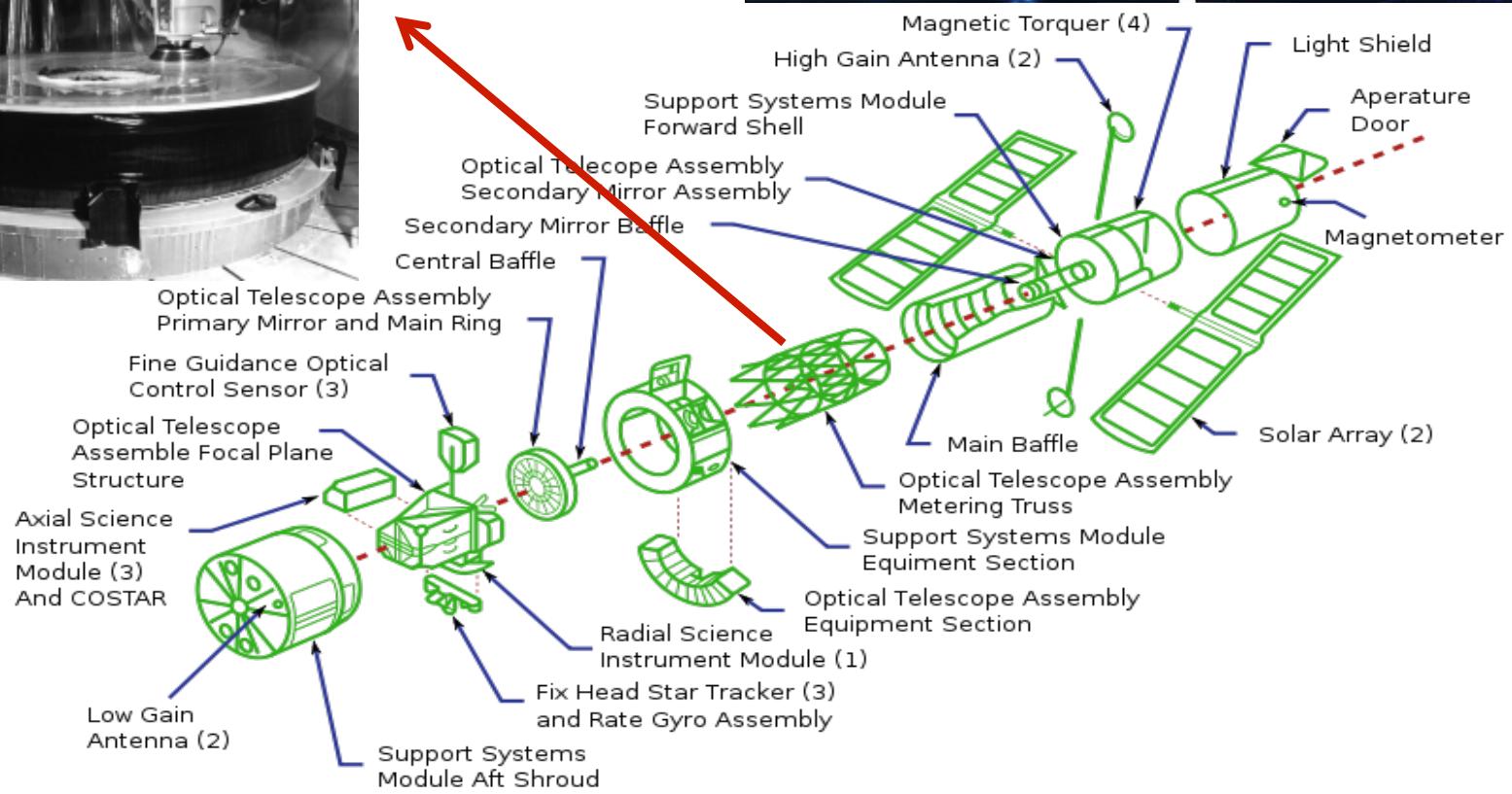
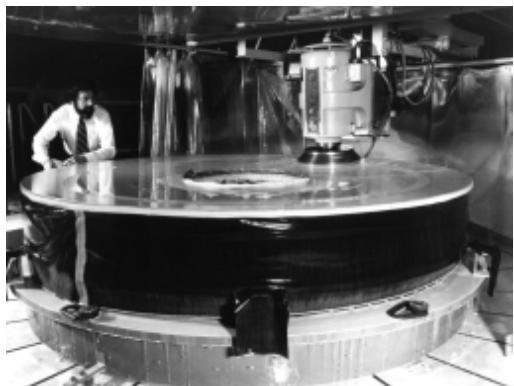


**Solution I: Aspheric Mirrors or Lenses**

# Hubble Telescope

It was probably the most precisely figured mirror ever made, with variations from the prescribed curve of only 10 nanometers, it was too flat at the edges by about 2.2 microns.

Source: wikipedia



# Lens Shape

Solution II: Choose a proper shape of a singlet lens for a given image-object distance.

$$\frac{1}{f} \approx (n - 1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right].$$

For a given desired focal length, there is freedom to choose one of the radii for a singlet. The spherical aberration and coma depend on the particular choice, so these aberrations can be minimized by the designed form.

$$q = \frac{(R_1 + R_2)}{(R_2 - R_1)}$$

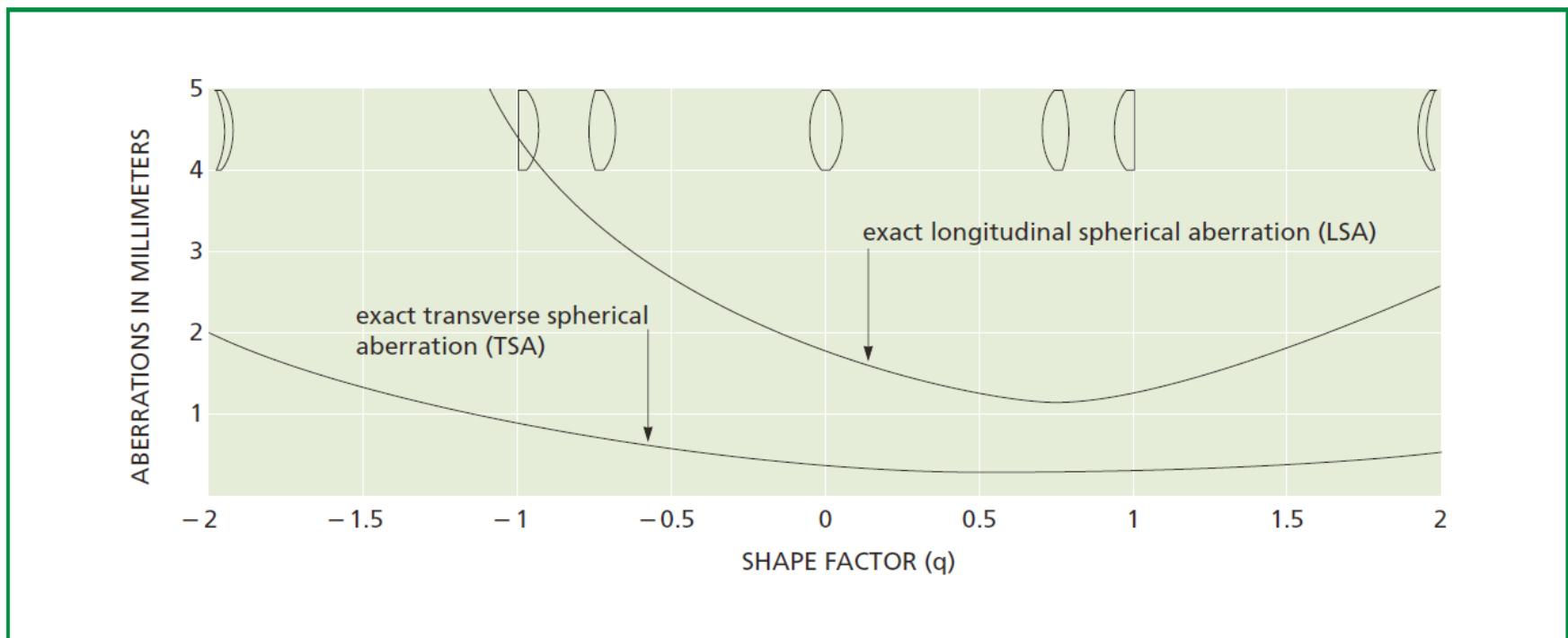
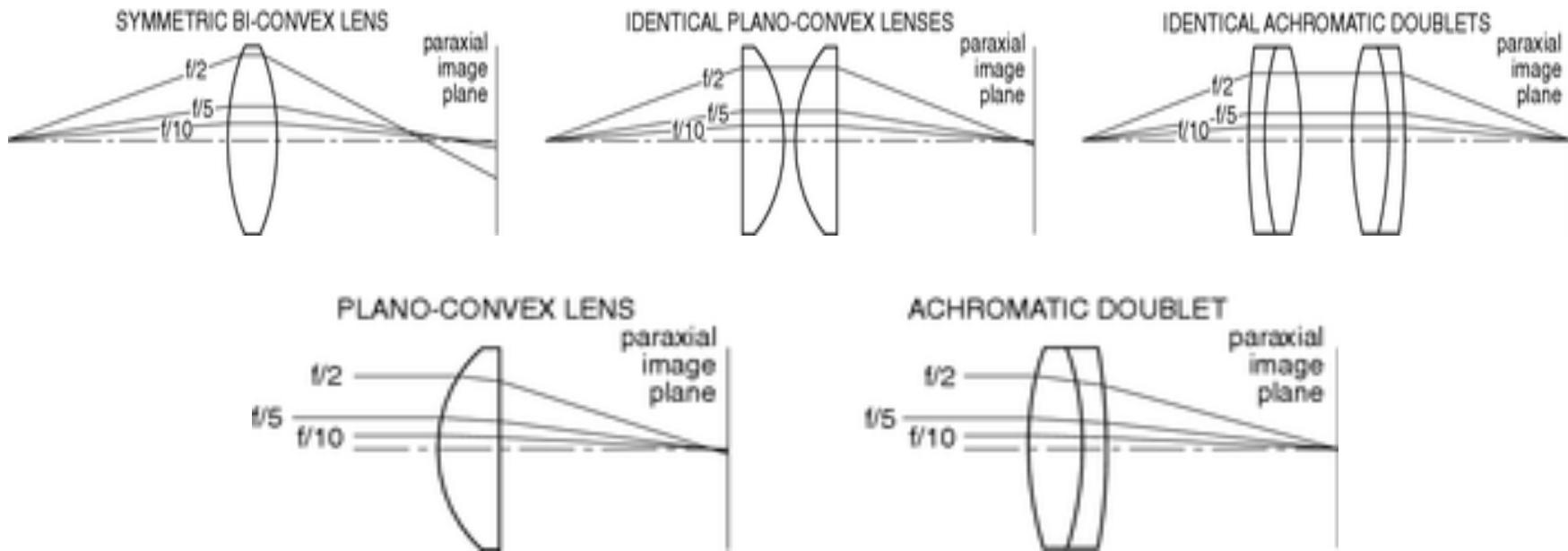


Figure 1.23 Aberrations of positive singlets at infinite conjugate ratio as a function of shape

# Lens Selection Guide



<http://www.newport.com/Lens-Selection-Guide/140908/1033/catalog.aspx#>

# Astigmatism

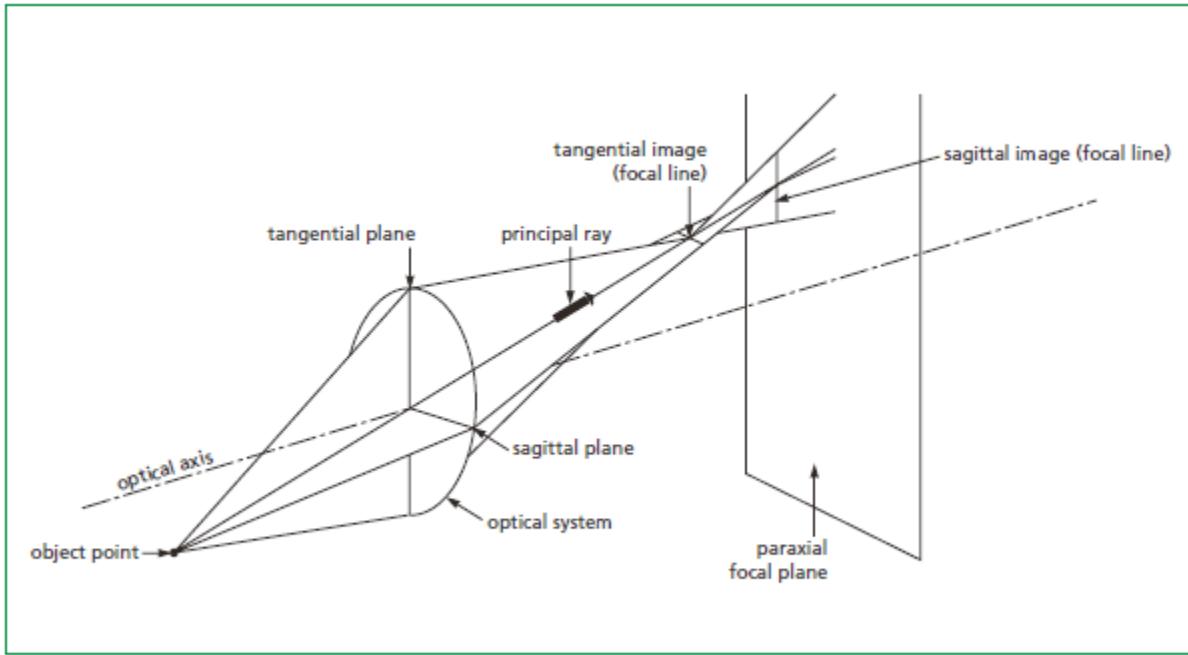


Figure 1.16 Astigmatism represented by sectional views

# Coma and Deformation

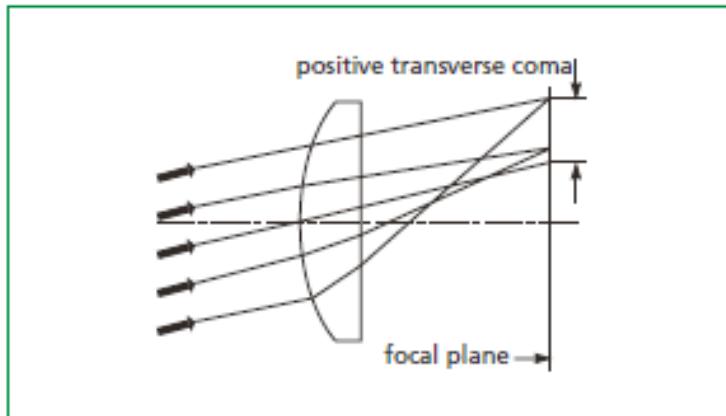


Figure 1.18 Positive transverse coma

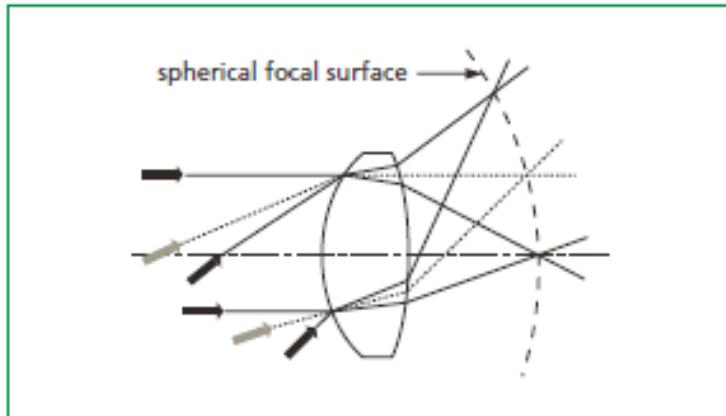


Figure 1.19 Field curvature

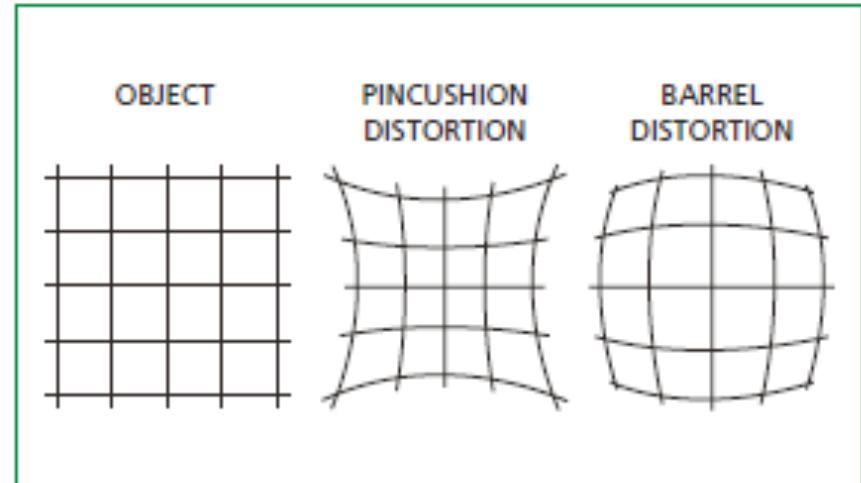
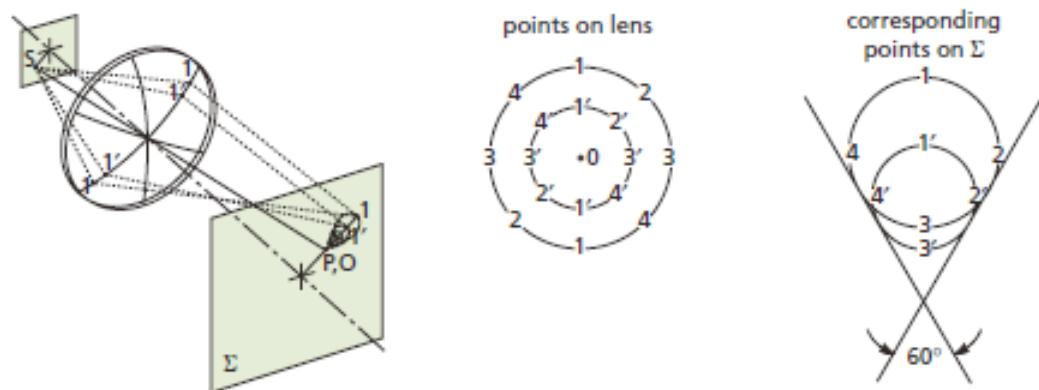


Figure 1.20 Pincushion and barrel distortion