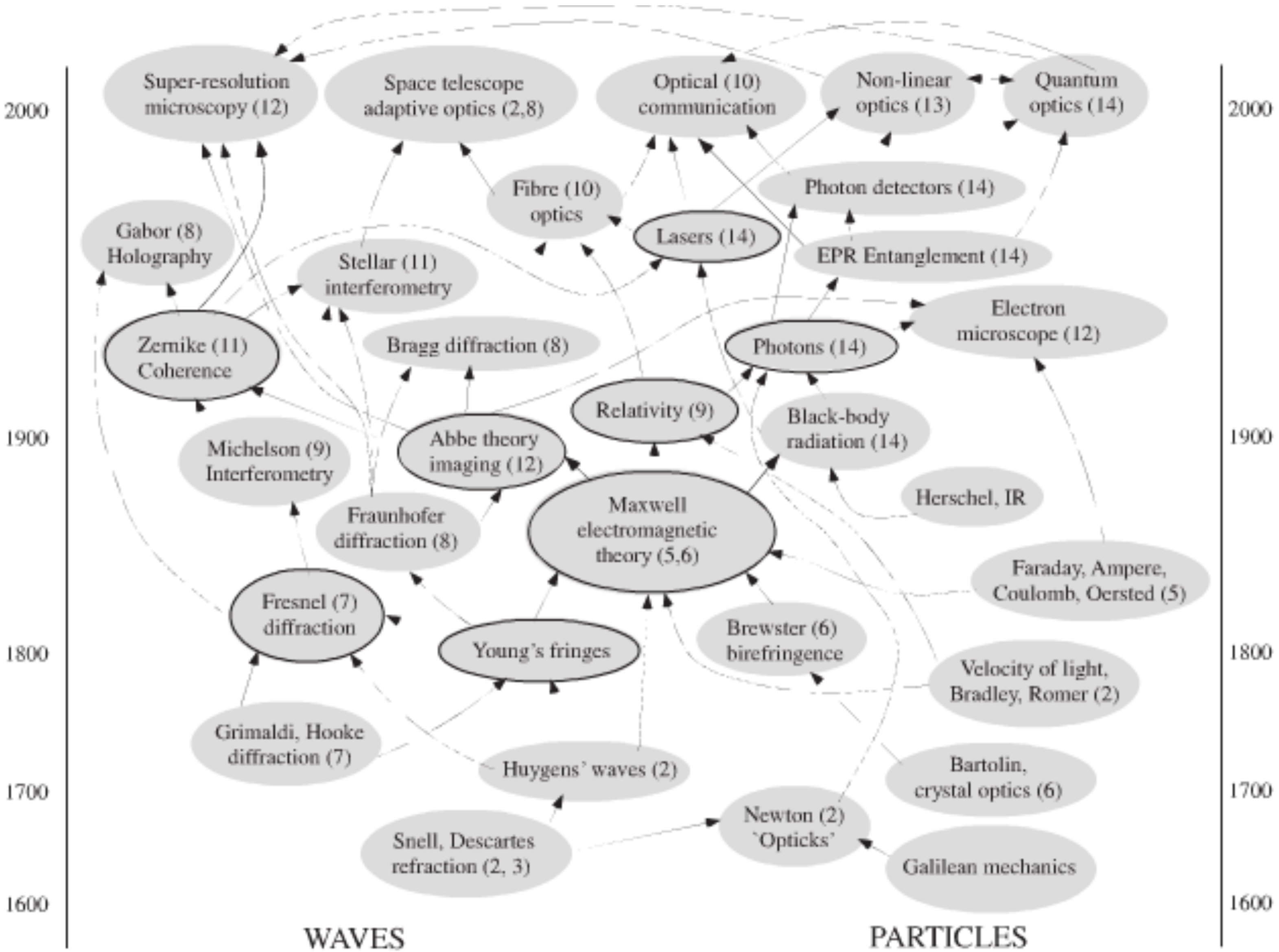


Fig. 1.1. The development of optics, showing many of the interactions. Note that some of the interaction arrows on the left connect with arrows on the right (wrapped around).

Source: "Optical Physics", Lipson et al.

Derive Snell's Law by Translation Symmetry

A *homogeneous* surface can not change the transverse momentum.
The propagation vector is proportional to the photon's momentum.

$$E = pc$$

$$p = \hbar k$$

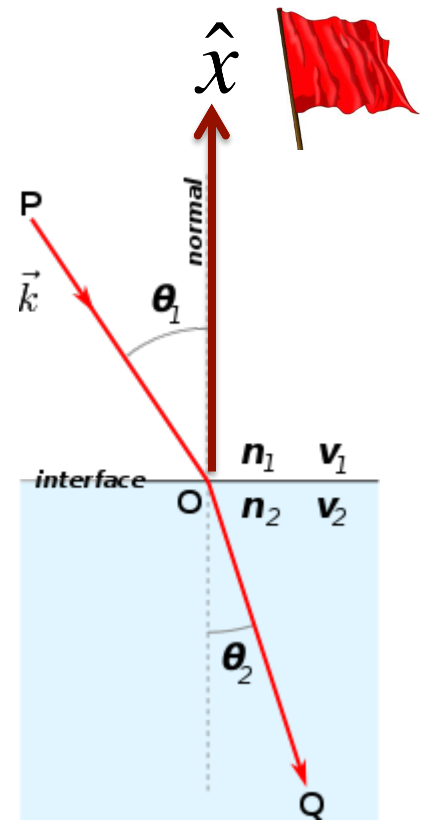
The transverse wave number must remain the same.

$$\vec{k}_1 \bullet \hat{x} = \vec{k}' \bullet \hat{x} = \vec{k}_2 \bullet \hat{x}$$

$$k_1 \sin\theta_1 = k' \sin\theta' = k_2 \sin\theta_2$$

$$k_1 = k' = \frac{2\pi}{\lambda_1} = \frac{2\pi}{\lambda_0 / n_1} = \frac{2\pi}{\lambda_0} n_1 = k_0 n_1$$

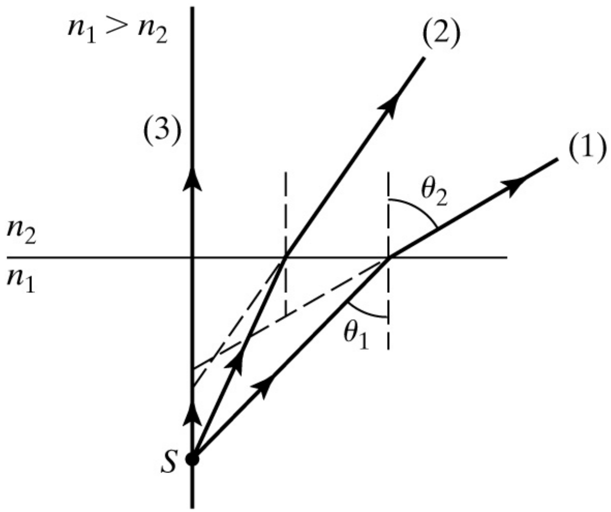
$$k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi}{\lambda_0} n_2$$



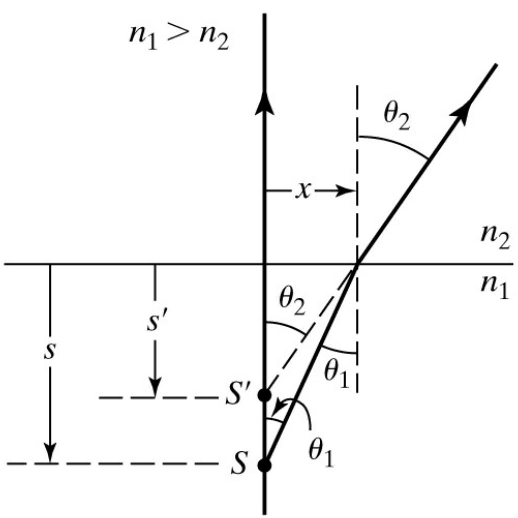
$n_1 k_0 \sin\theta_1 = n_2 k_0 \sin\theta_2$
$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c}$$

Refraction through Plane Surfaces



(a)



(b)

$n_1>n_2$: The refracted rays bend away from the normal.

$n_1<n_2$: The refracted rays bend toward the normal

A source point S below an interface emerge into an upper medium of lower refractive index.

➔ No unique image point is determined. These rays have no common intersection of virtual image point below the surface.

Small angle approximation, i.e. consider only *Paraxial Rays (making small angles with the optical axis)*

$$\sin\theta \cong \tan\theta \cong \theta \quad (\text{in radians})$$

Snell's law can be approximated by

$$n_1 \tan\theta_1 \cong n_2 \tan\theta_2$$

$$n_1\left(\frac{x}{s}\right) = n_2\left(\frac{x}{s'}\right)$$

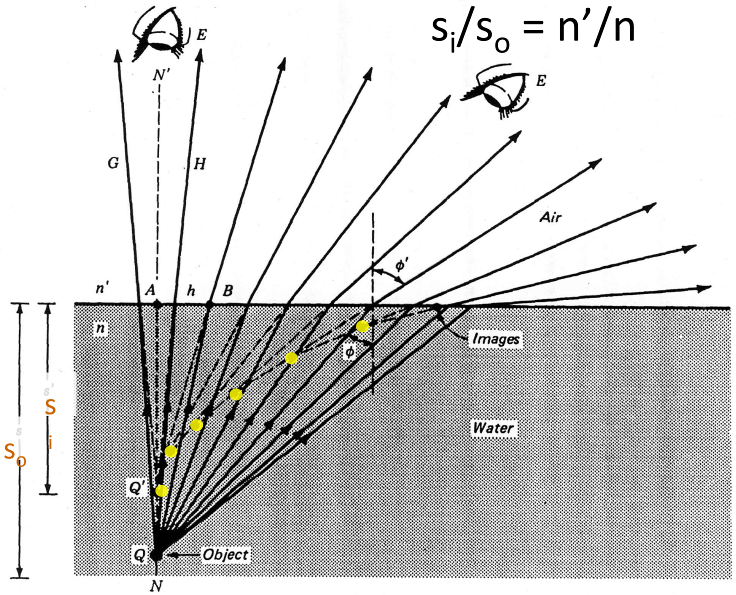
s': the image distance

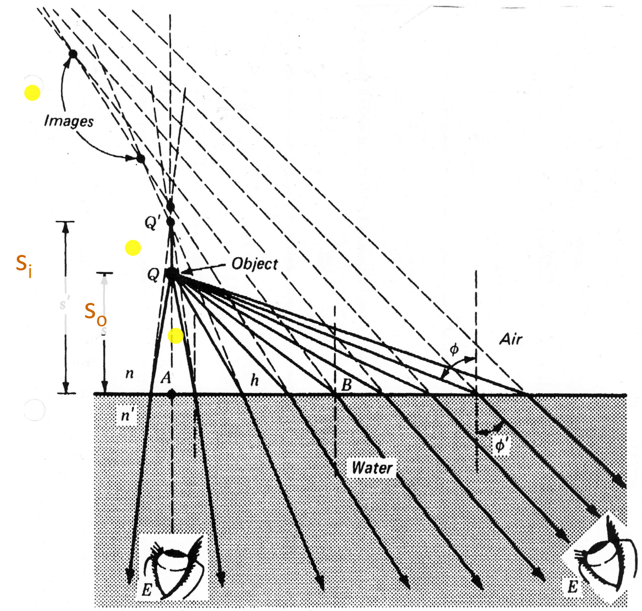
s : the depth of the object

$$s' = \left(\frac{n_2}{n_1}\right)s$$ for a small viewing angle θ_2; vary with the angle of viewing

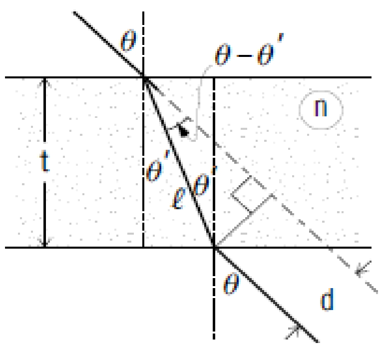
Refraction: the dolphin problem…

$s_i/s_o = n'/n$



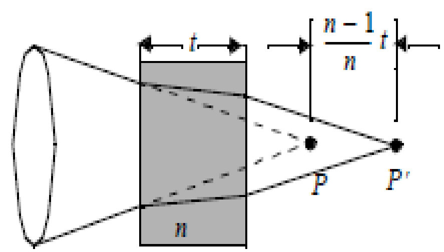


Displacement by a glass plate



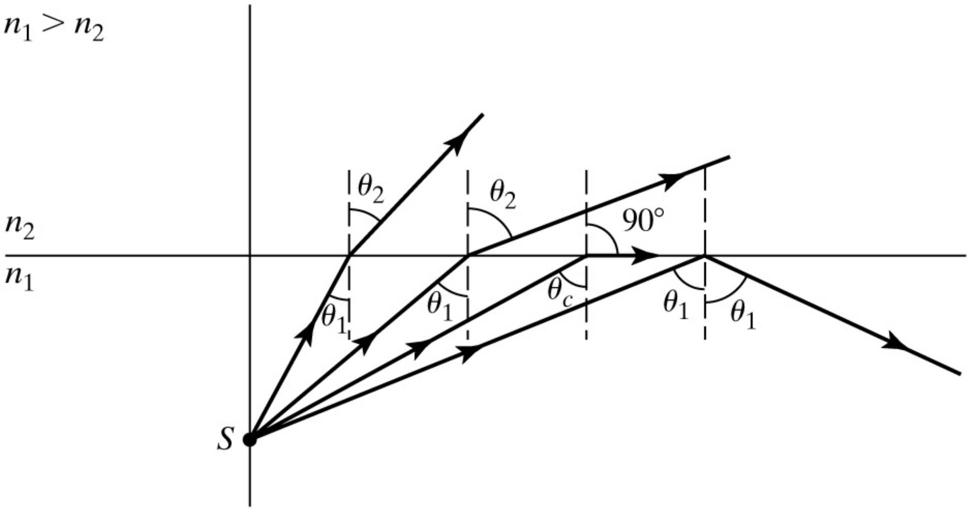
$$d = \frac{t \sin\left(\theta - \theta'\right)}{\cos\left(\theta'\right)}$$

Plane parallel plate placed in between a lens and its focus:



A simple calculation based on the paraxial approximation shows that the focus is displaced by amount $\boxed{\frac{n-1}{n}t}$. However, at steeper incidence angles, the focal shift becomes a function of the incidence angle, which leads to spherical aberration.

Total Internal Reflection



(c)

The critical angle θ_{crit} is the value of θ_1 for which θ_2 equals 90°:

Example : Water n_2 =1.33 (=4/3)
Air n_1=1.00

$$\theta_{\text{crit}} = \arcsin\left(\frac{n_2}{n_1}\sin\theta_2\right) = \arcsin\frac{n_2}{n_1} = 48.6°.$$

Assignments:
Glass n = 1.52
Diamond n = 2.42

The largest possible angle of incidence which still results in a refracted ray is called the **critical angle**; in this case the refracted ray travels along the boundary between the two media.

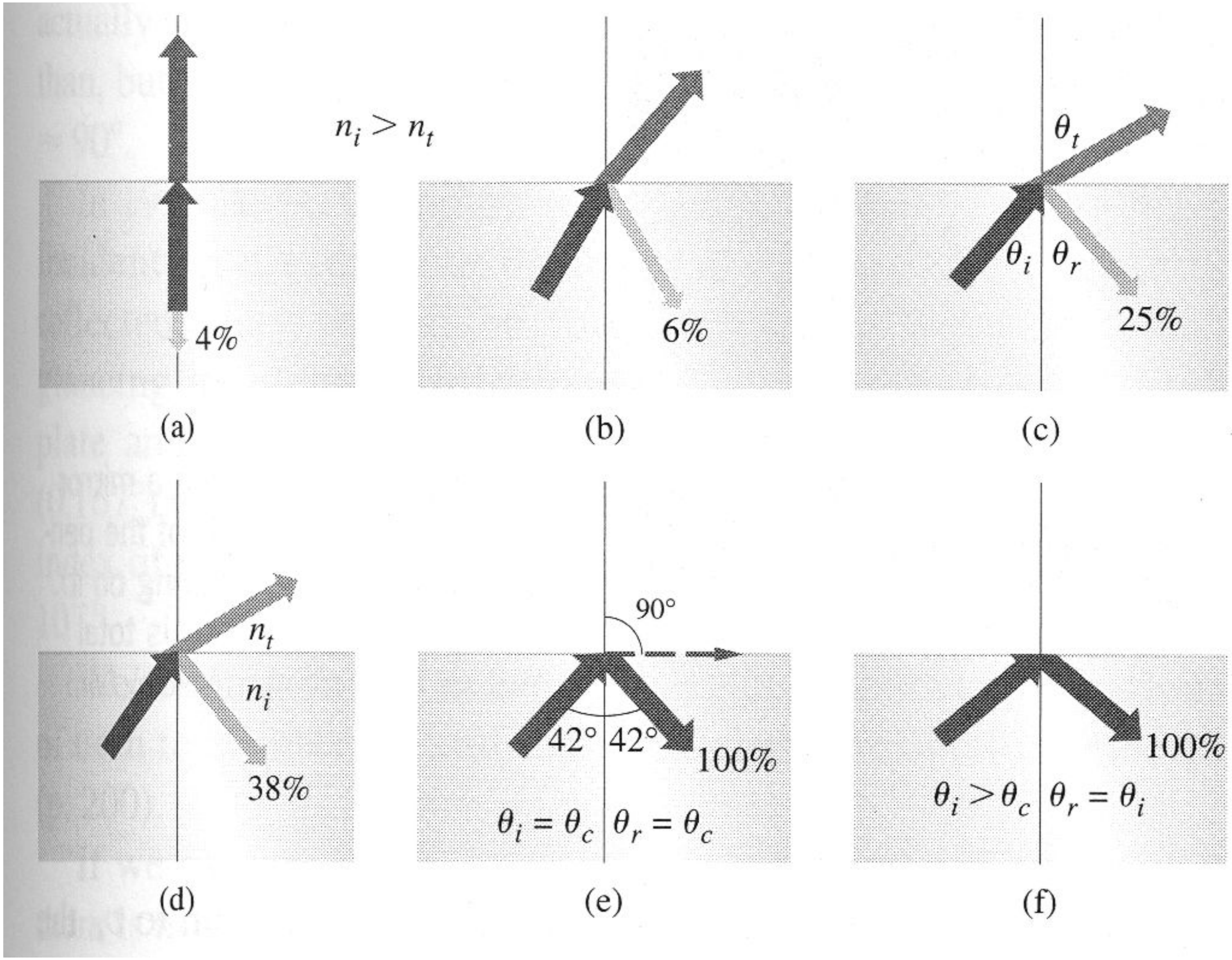
For example, consider a ray of light moving from water to air with an angle of incidence of 50°. The refractive indices of water and air are approximately 1.333 and 1, respectively, so Snell's law gives us the relation

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = 1.333 \cdot 0.766 = 1.021,$$

which is impossible to satisfy.

Quiz: What is the critical angle for diamond?

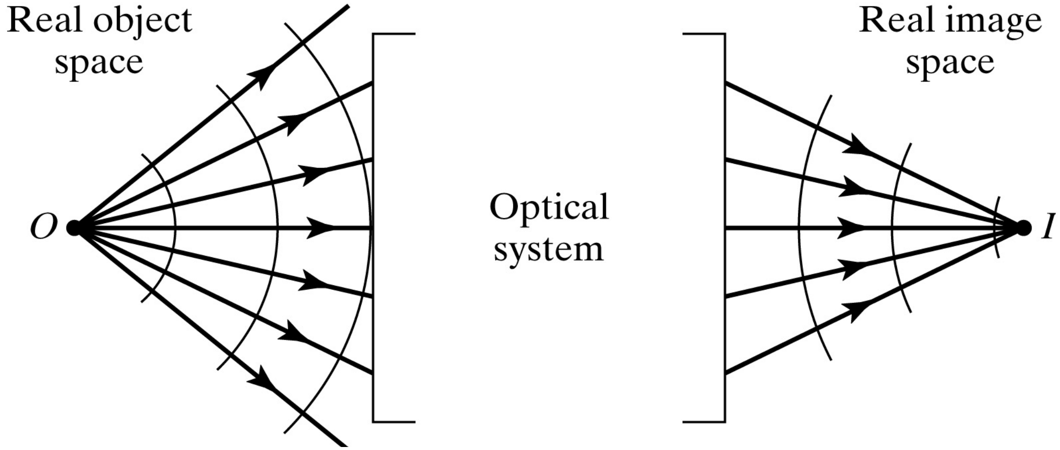
Reflection and refraction at a flat dielectric interface

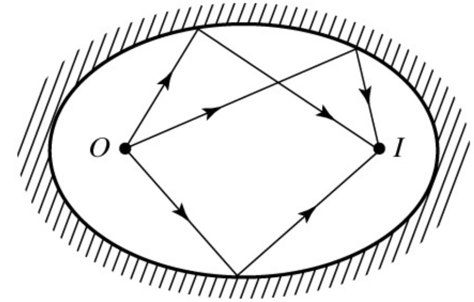


Quiz: What is the refractive index of this dielectric medium? What is it likely to be?

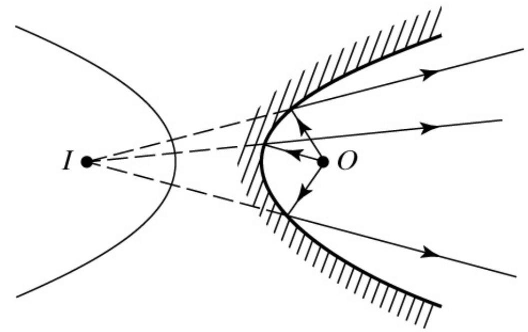
Imaging

Imaging by an Optical System

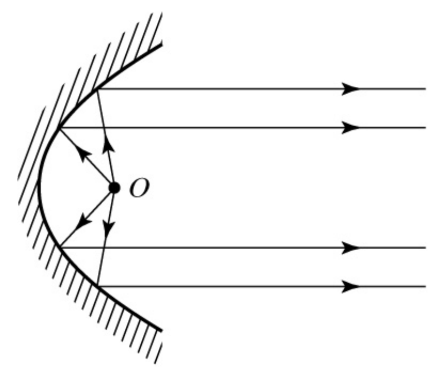




(a) Ellipsoid



(b) Hyperboloid



(c) Paraboloid

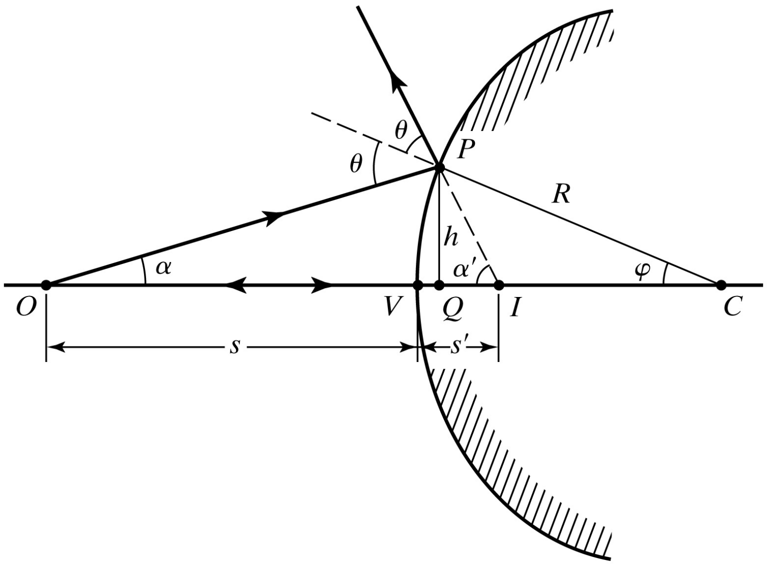
Key words: object point, image point, object space, image space, wavefronts.
Rays spread out radially in all directions from object point O. A Ray diagram deals with selected major rays.

Nonideal images are formed in practice because of
(1) light scattering,
(2) aberrations, and
(3) diffraction.

Trading off *fabrication (grinding)* and *spherical aberrations*.

Molded aspheric lenses are more commonly nowadays.

Reflection at a Spherical Surface



$$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R}$$

$$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \cdots$$

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \cdots$$

$$\varphi \to 0 \,(\text{small angle})$$

$$\sin\varphi \cong \varphi \quad \text{and} \quad \cos\varphi \cong 1$$

Sing Convention
(assuming the light propagates from left to right)

The object distance
s is positive when O is to the left of V (real object).
s is negative when O is to the right of V (virtual object).
The image distance
s' is positive when I is to the left of V (real image).
s' is negative when I is to the right of V (virtual image).
The radius of curvature
R is positive when C is to the right of V (a convex mirror).
R is negative when C is to the left of V (a concave mirror).

positive object and image distances ➔ real objects and real images
convex mirrors ➔ positive radii of curvature (eg. a silver spoon)

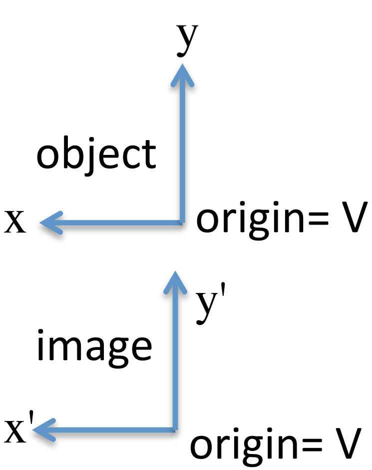
Ray Diagrams for Spherical Mirrors

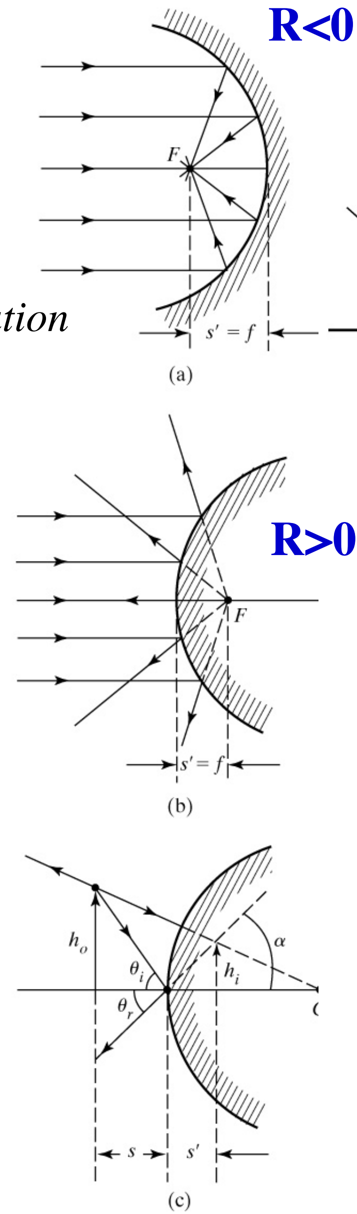
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}, \quad f=-\frac{R}{2}$$

$$m=-\frac{s'}{s} \quad \textit{lateral magnification}$$

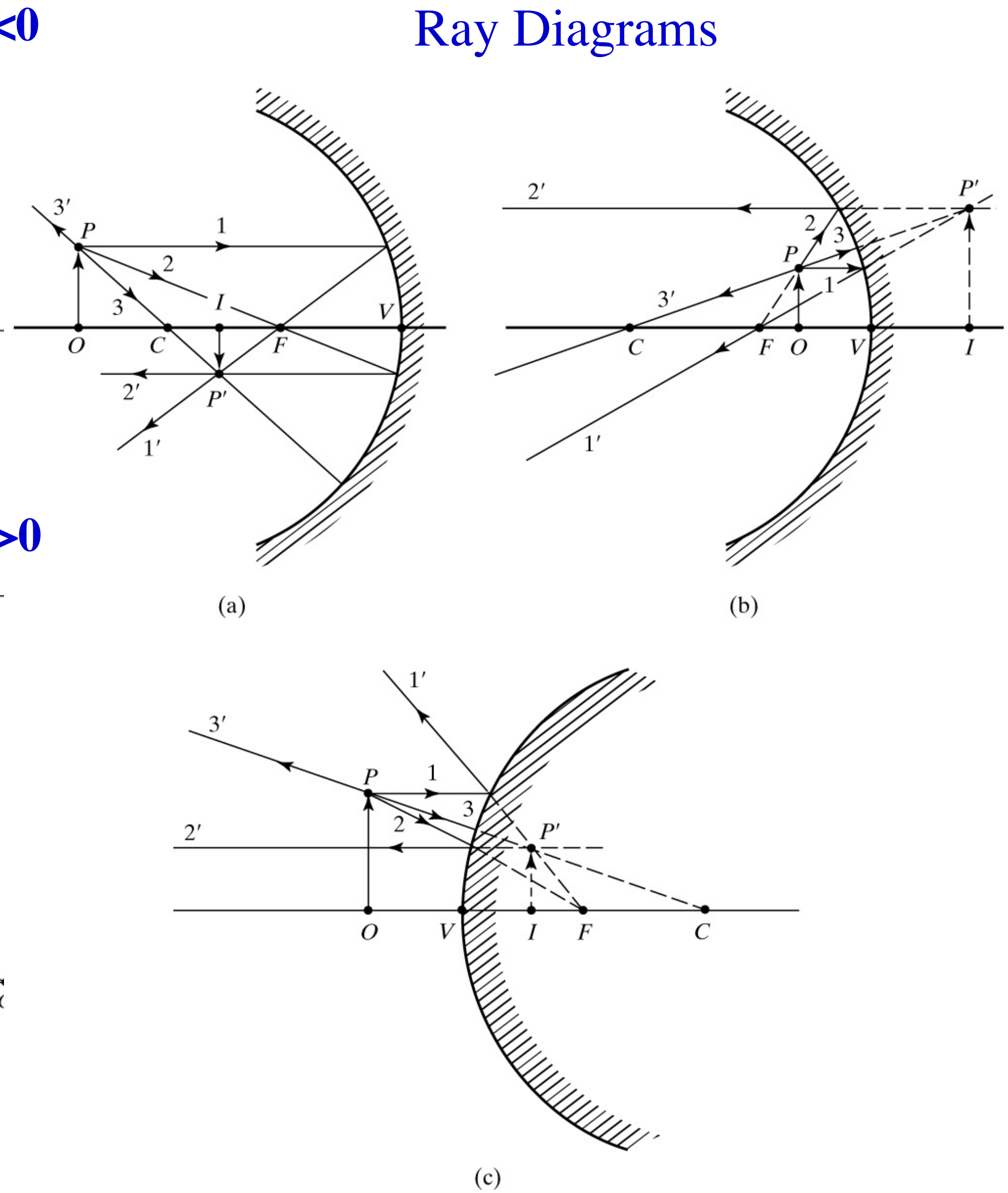
$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}, \quad f=-\frac{R}{2}$$

$$m=\frac{h_i}{h_o}=-\frac{s_i}{s_o}$$

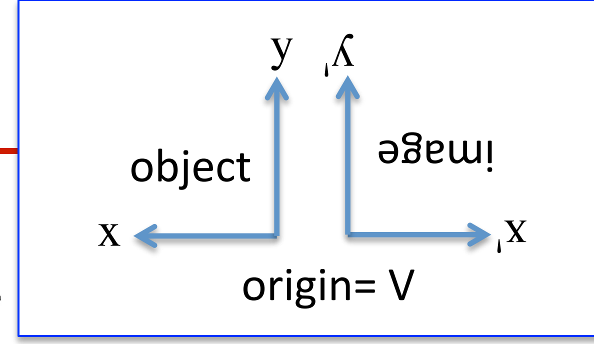


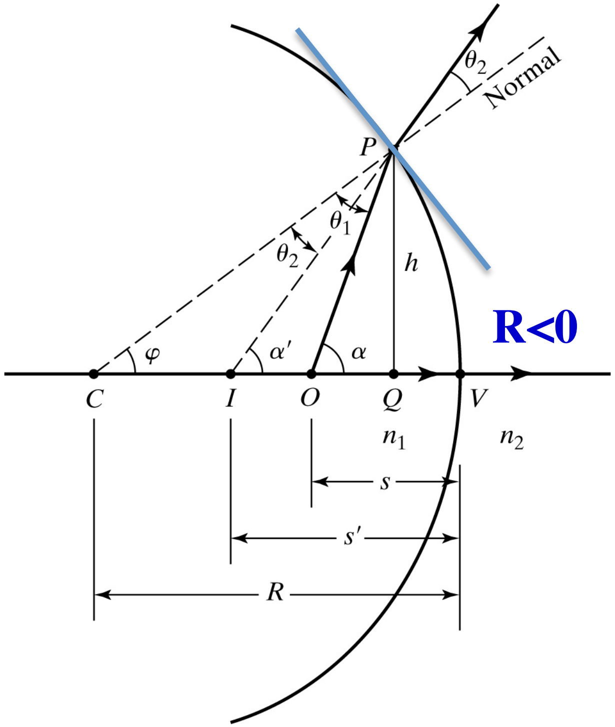


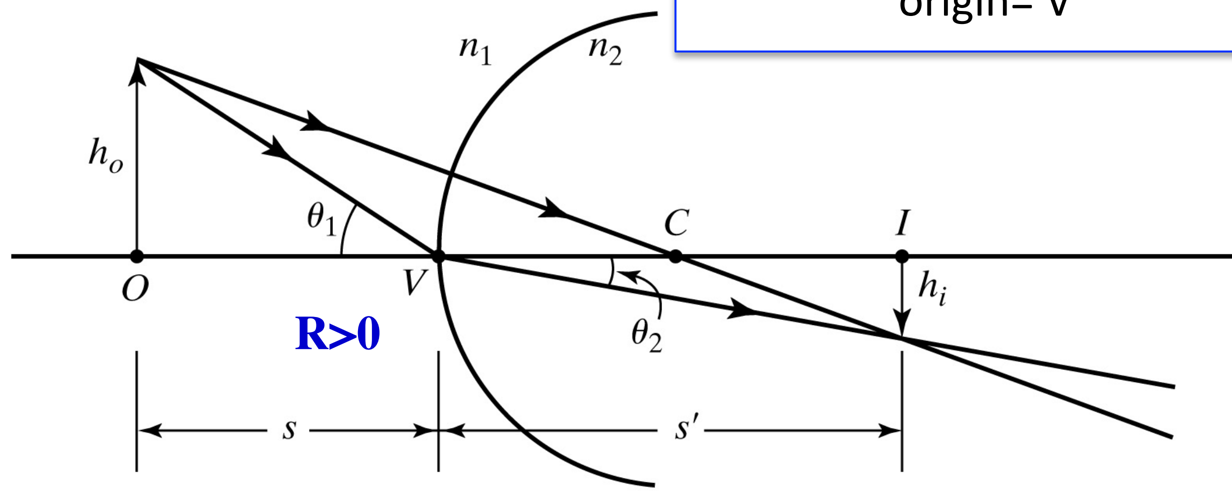
Ray Diagrams



Refraction at a Spherical Surface







$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

$$m = \frac{h_i}{h_o} = -\frac{n_1 s'}{n_2 s}$$

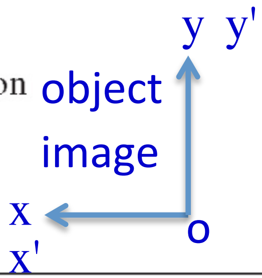
When $R \to \infty$ (i.e. a plane surface)

$$s' = -\left(\frac{n_2}{n_1}\right)s$$

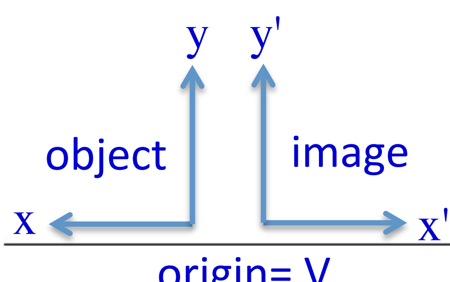
$$m = +1$$

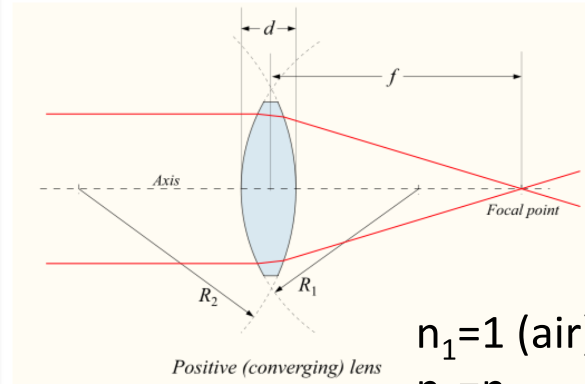
TABLE 2-1 SUMMARY OF GAUSSIAN MIRROR AND LENS FORMULAS

	Spherical surface	Plane surface
Reflection	$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f},\ f = -\frac{R}{2}$ $m = -\frac{s'}{s}$ Concave: $f > 0, R < 0$ Convex : $f < 0, R > 0$	$s' = -s$ $m = +1$
Refraction Single surface	$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$ $m = -\frac{n_1 s'}{n_2 s}$ Concave: $R < 0$ Convex : $R > 0$	$s' = -\frac{n_2}{n_1} s$ $m = +1$
Refraction Thin lens	$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ $\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ $m = -\frac{s'}{s}$ Concave: $f < 0$ Convex : $f > 0$	



the lensmaker's formula





n_1=1 (air)
n_2=n

The lensmaker's formula

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

A "Thin" lens → d is negligible

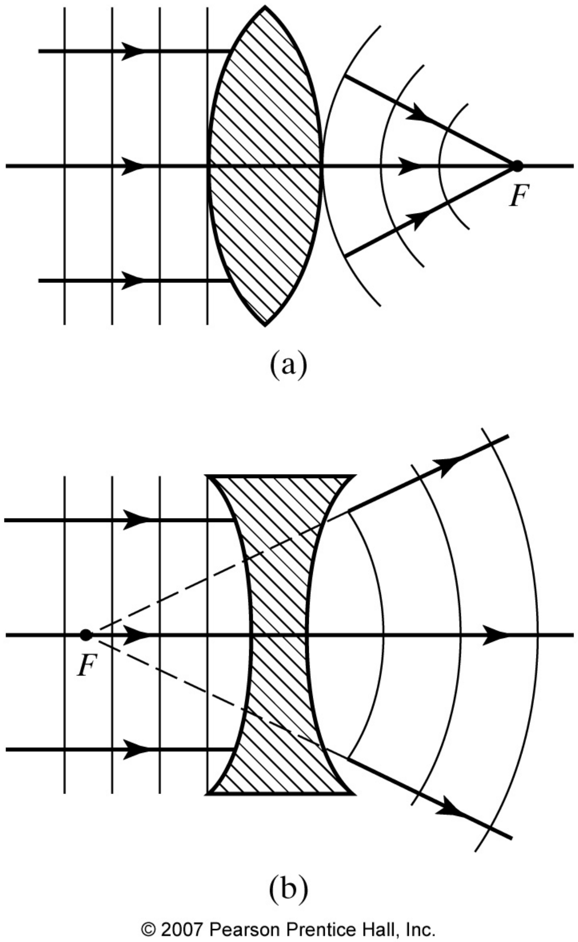
$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

The refractive power of a lens of focal length f

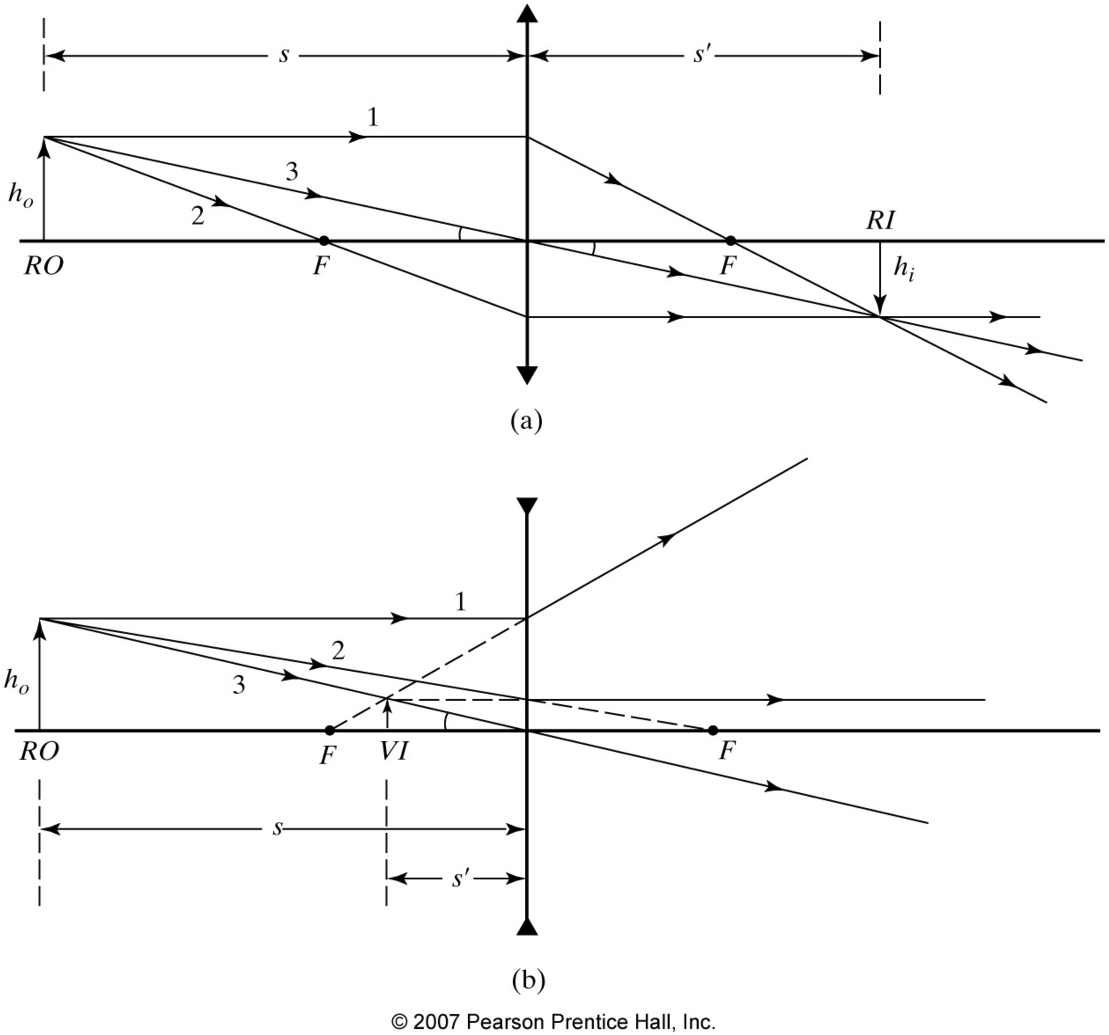
$$D\,[\text{diopters}] = \frac{1}{f\,[\text{in meters}]}$$

Thin Lenses

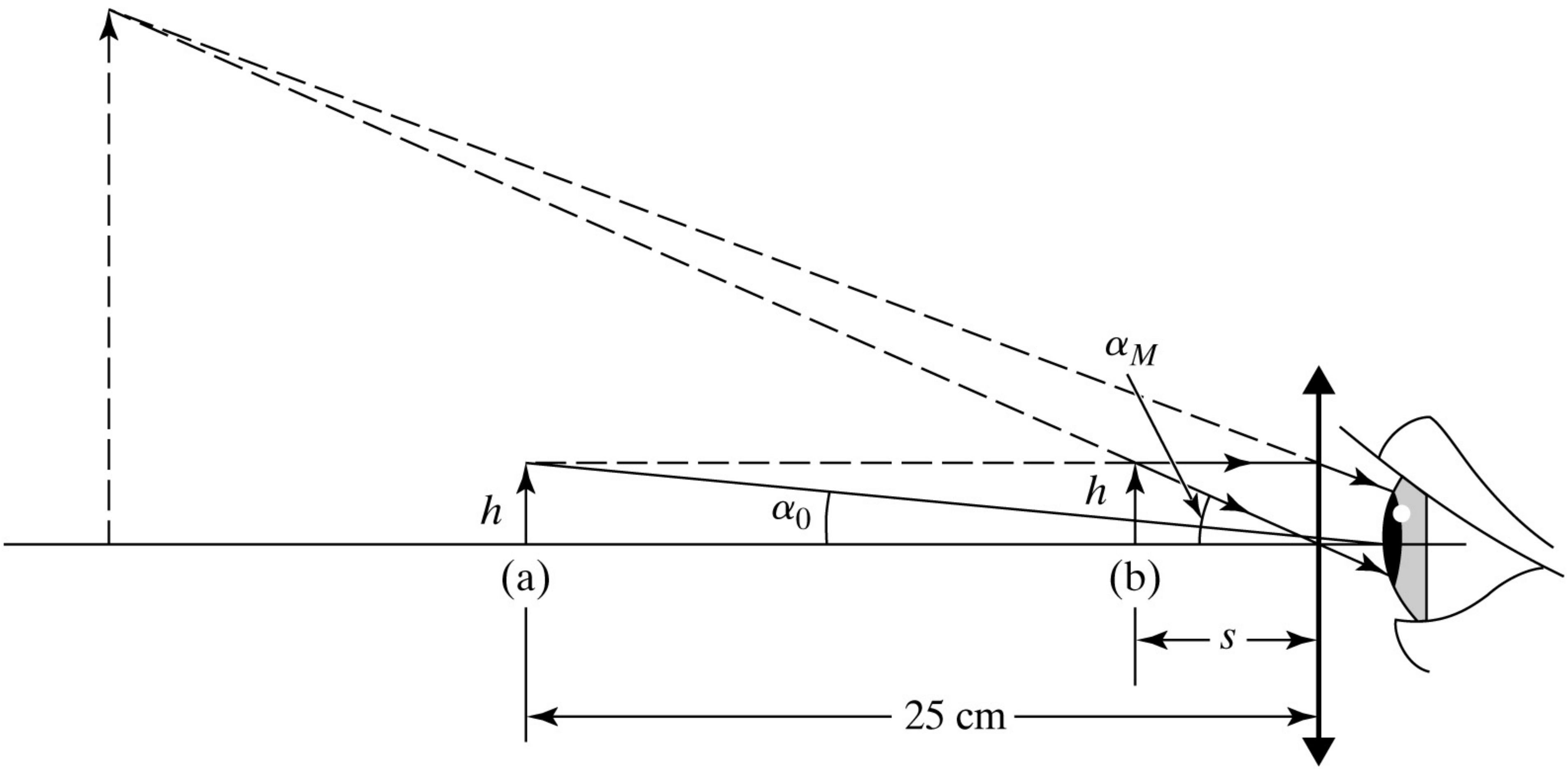
Lens action on plane wave-fronts of light.

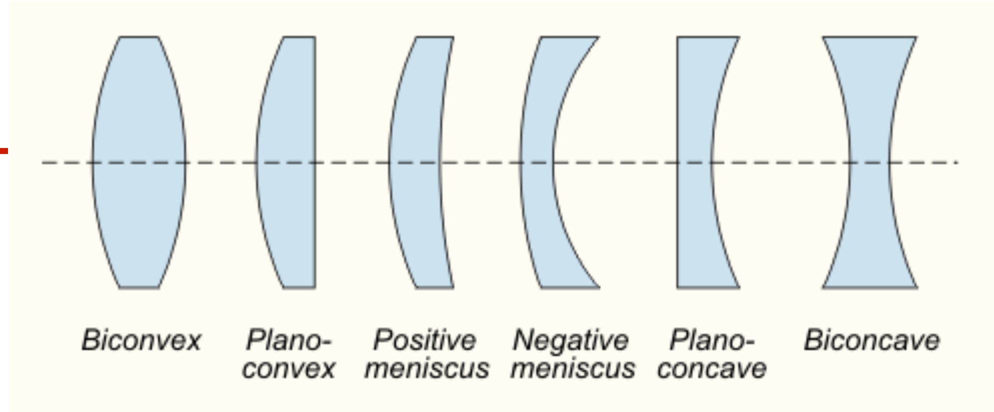


Ray Diagrams



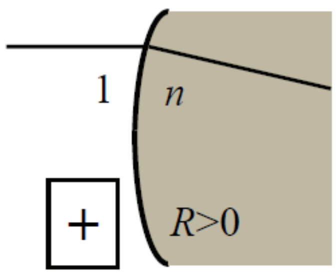
Simple Magnifiers



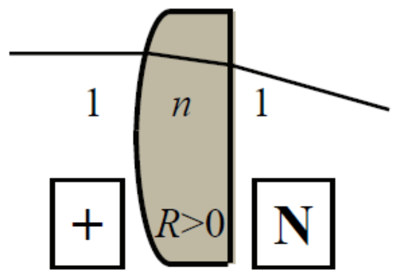


$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right].$$

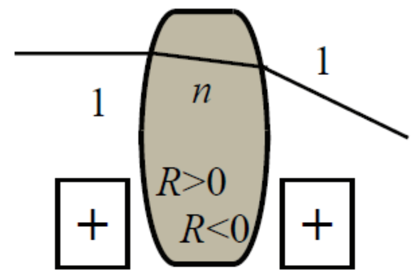
Positive power : exiting rays converge



Simple spherical refractor (positive)

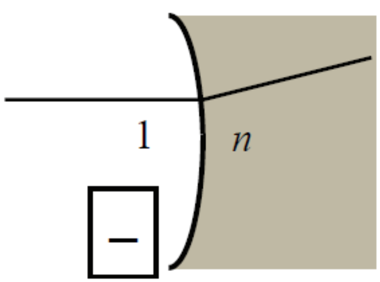


Plano-convex lens

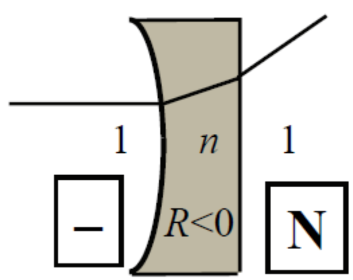


Bi-convex lens

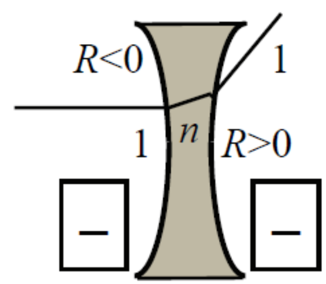
Negative power : exiting rays diverge



Simple spherical refractor (negative)

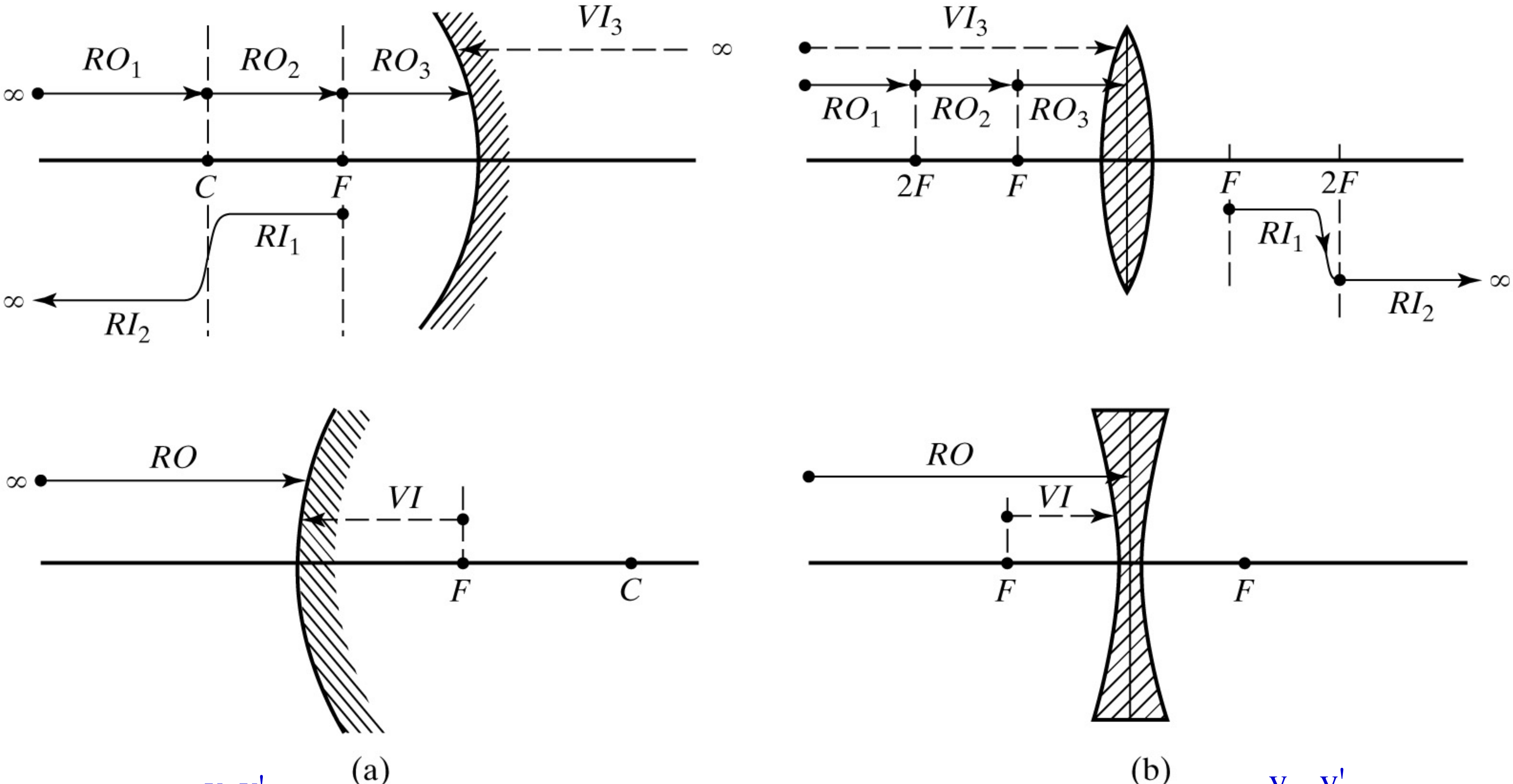


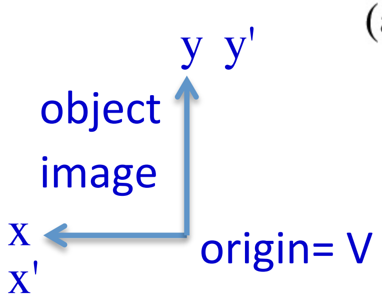
Plano-concave lens

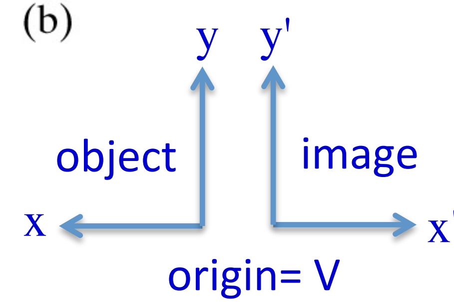


Bi-concave lens

Summary of Image Formation: Spherical Mirrors & Thin Lenses







Summary : Real and Virtual Images

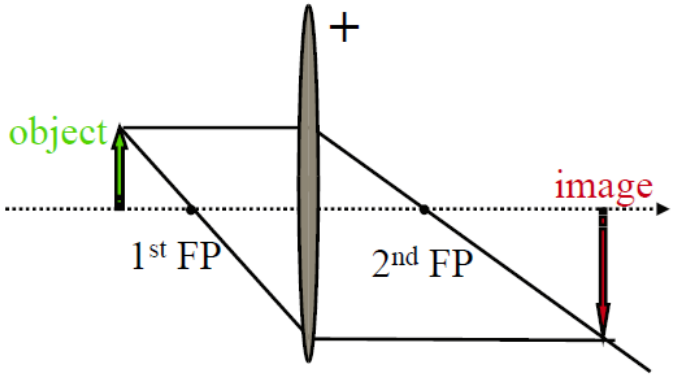


image: real & inverted; $M_T<0$

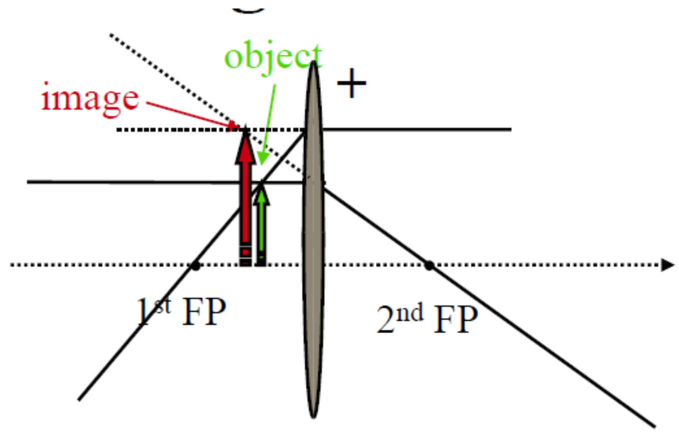


image: virtual & erect; $M_T>1$

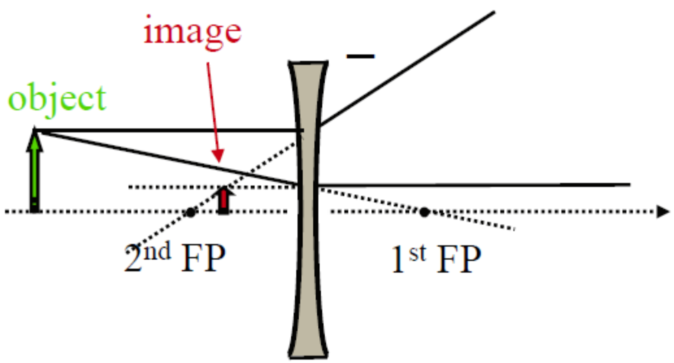


image: virtual & erect; $0<M_T<1$

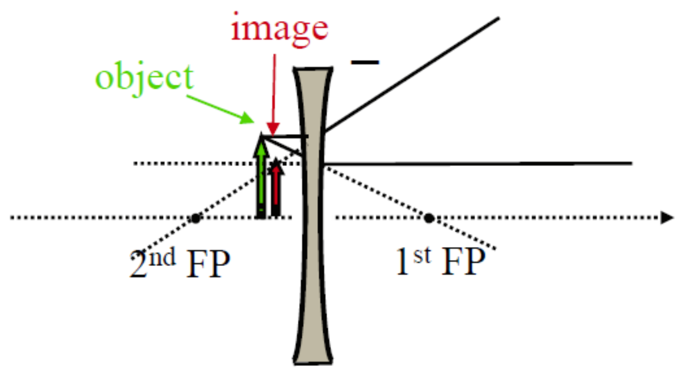
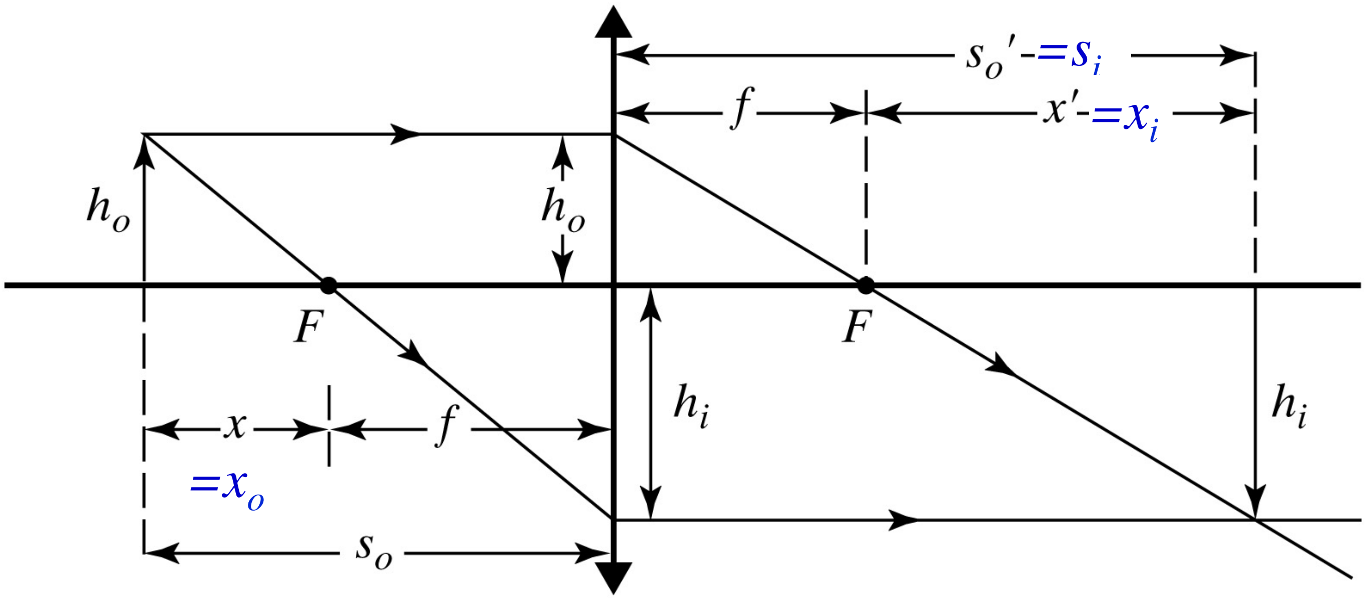


image: virtual & erect; $0<M_T<1$

Newtonian Equation for the Thin Lens



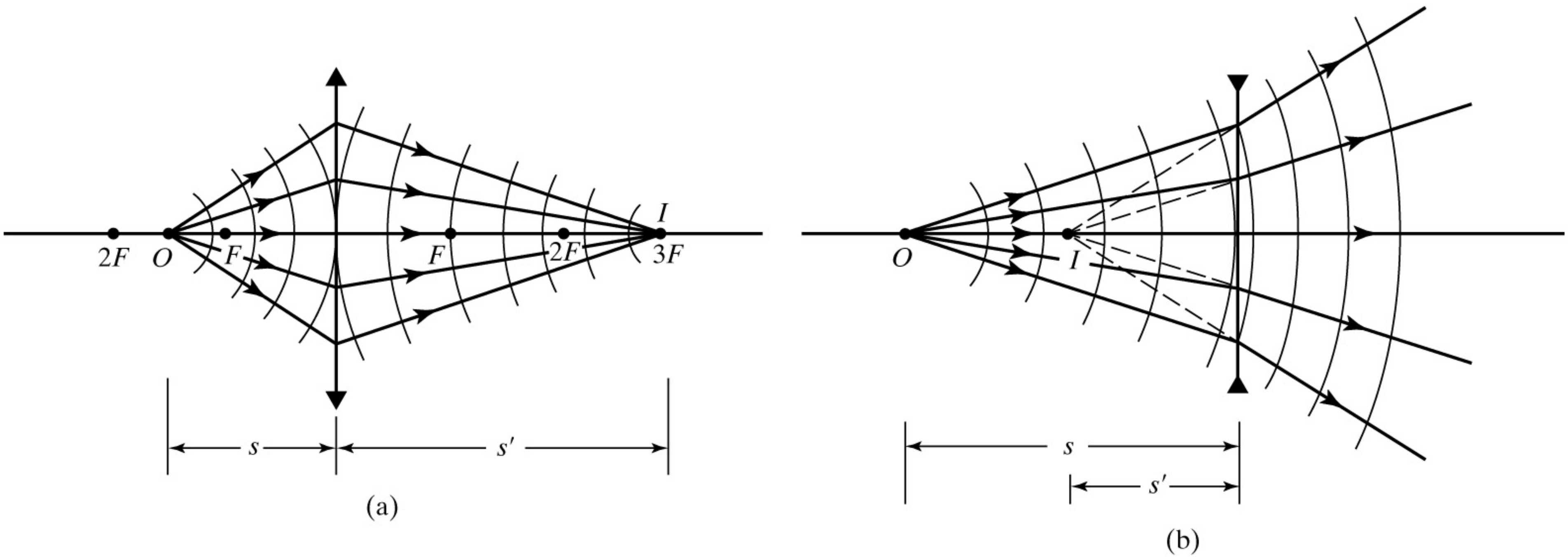
$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_i}$$

$$x_0 x_i = f^2$$

$$M_L \equiv \frac{y_i}{y_0} = -\frac{s_i}{s_o}$$ *lateral magnification*

$$M_T \equiv \frac{dx_i}{dx_0} = -\frac{f^2}{x_0^2}$$ *transverse magnification*

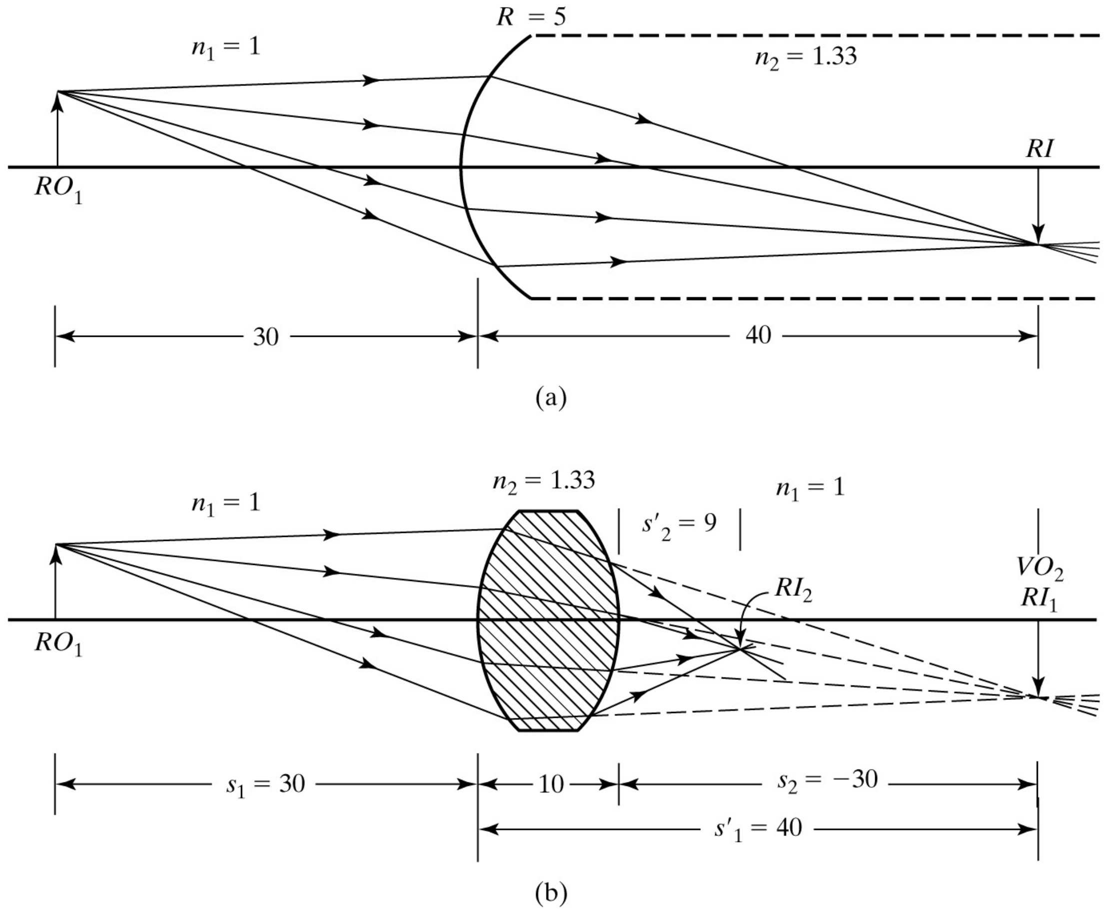
Change in curvature of wavefronts by a thin lens



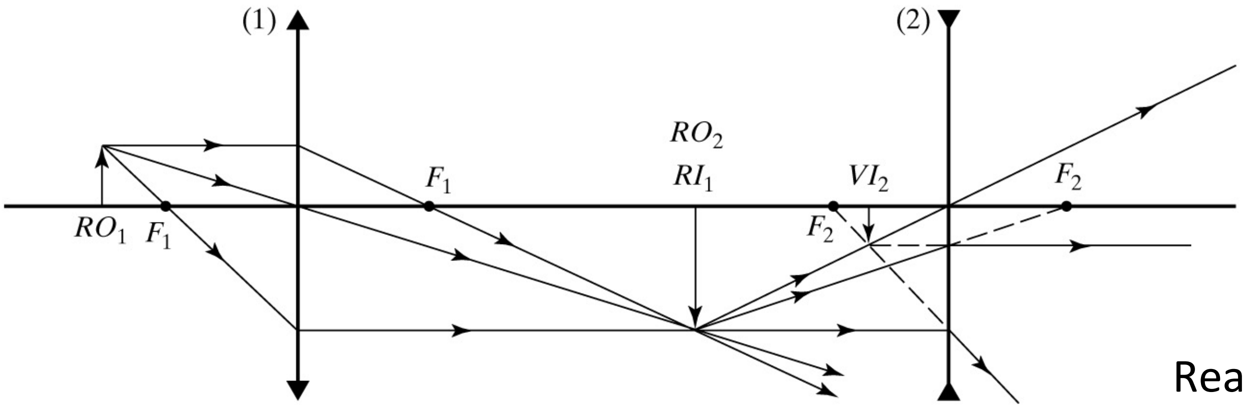
Example of refraction by spherical surfaces

Extra Credit (0.25 pt)

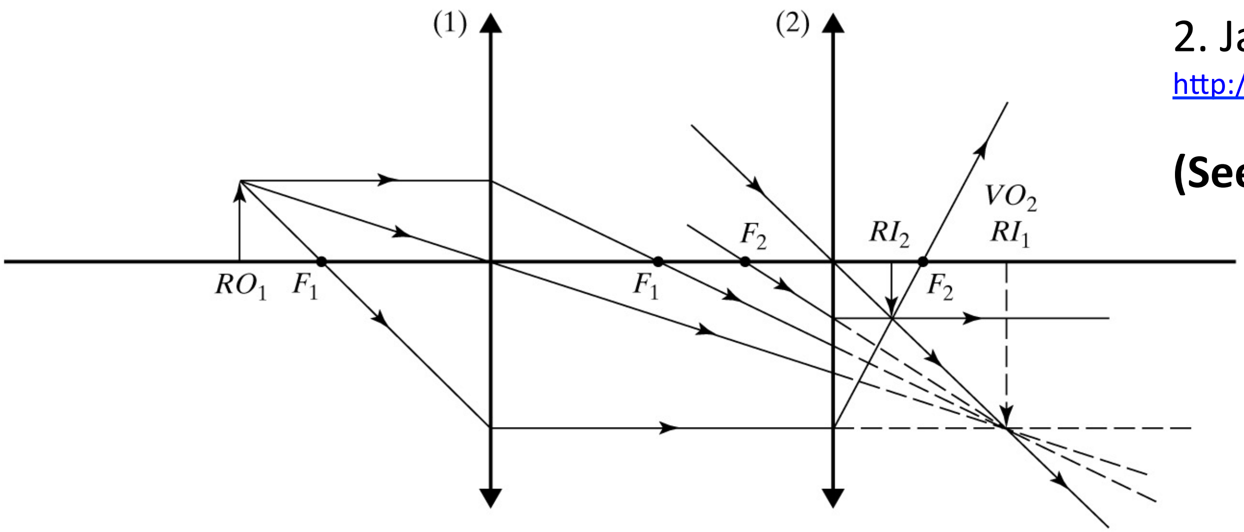
Explain the imaging formation by a cylinder filled with water. Be specific (i.e. use 'real' values'), See Example 2-2 (page 34)



Thin Lens Combination ➔ Sequential Imaging



(a)



(b)

Reading Assignments

1. Example 2-3 (page 38)

2. Java Applets
http://silver.neep.wisc.edu/~shock/tools/ray.html

(See also Lab #1 Appendix)