Notes on Optical Pumping Procedure & Theory

Pre-lab

1. Why is the experiment called optical pumping? What is pumped?

2. What is the experimental signature of having cancelled all magnetic fields in the sample cell?

3. What determines the maximum rate at which the B field should be modulated?

4. What are the different roles of the RF and visible sources?

5. What are the advantages of studying ^{87}Rb instead of the isotopically more abundant ^{85}Rb?

6. What is the nature of the double-quantum transitions in the high-power RF measurements?

Procedure

1. The manual for this experiment is not very well written. So dealing with this manual should be an interesting learning experience that will build character!

2. Turn on the electronics and wait for the cell to warm up: ~ ½ hour. The oven should already be set to 50 °C—don't change this setting. You can leave the black cover off the apparatus. Set the front panel settings to the values outlined in Section (g) on p. 5-3 of the manual. Once the cell is warmed up, look for pinkish glow from the lamp and a signal from the detector. Move the two lenses around to maximize the signal. Underneath the optical components is a cm scale that you should use to record the positions of these various optical components, since other students may change these settings for their experiment. Then check that the Linear Polarizer and ¼-wave plate are properly aligned. You can determine the degree of circular polarization of the light by placing a second linear polarizer in front the detector and rotating its axis of polarization. The detector signal should change minimally as you rotate the linear polarizer. If you wish, you can use the procedure on p. 3-8 & 3-9 of the manual to maximize the circular polarization of the light. Now remove second linear polarizer, put the black cover on and shut its door to put the apparatus in the dark. Note what happens to the detector signal.

3. Start with experiment 4B, p. 4-6. You should do a careful job of canceling the vertical and horizontal magnetic fields, as evidenced by a narrow zero-field transition. At this time, make sure that the main horizontal-field coil is not connected to its current source. For the vertical coil, the red banana plug should be plugged into the red receptacle on its current source in order to cancel out the vertical component of "earth's field." For the horizontal-field sweep coil, you'll have to figure this out for yourself. Use the compass to determine the direction of the horizontal component of the earth's field and then apply a current to the sweep coil of appropriate polarity so that the earth's field is opposed. Next apply a current to the main horizontal-field coil and see what polarity is needed to produce a horizontal field in the same direction as that of the sweep coil—this will be useful later when you do the quadratic Zeeman Effect. Now disconnect the main horizontal coil. It is a good idea to use a portable DVM to measure the current through the sweep-field coil from the voltage across a 1-Ω resistor in series with this coil.

Do a sweep of the horizontal field manually to observe the zero-field resonance. Note that the signal is "negative"—being on resonance corresponds to minimum transmission of light to the detector. You will need to adjust the vertical field to minimize the width of the resonance. The best procedure is to (1) adjust the horizontal sweep field so the detector output is sitting halfway between its maximum and minimum values and then (2) adjust the vertical field until the output is a maximum. Then repeat (1) & (2) until you cannot narrow the resonance any further. Also use another DVM to measure the current through the vertical-field coil. The final step is to repeat (1) but rock the S end of the apparatus E & W until you get a maximum signal. Here you are aligning the apparatus along the direction of the horizontal magnetic field in the lab. Once you have nulled out both components of the earth's field, you should calculate the magnitude and direction of the earth's field in the lab, using the approximate coil calibrations given in the table on p. 3-14. Do you get a reasonable result?

Using the DVM and sweeping manually, measure the-full-width-at-half-maximum of the zero-field transition in Gauss—it should be ~ a few mG. Now you can repeat this sweep using the Labview program for automatic taking of the data, so you can generate a graph.

4. Next you must carefully calibrate the sweep-field and main-field coils as outlined on p. 4-8 to 4-11. Set the RF amplifier gain on ×4 and the oscillator output to near 0 db. For the lowest-frequency 10-kHz Zeeman resonance, you can use the Labview program to generate a graph like Figure 4B-2 on p. 4-7 for your notebook. However, for generating the frequency vs sweep-field-current graph shown on p. 4-10, you should use the DVM to record the sweep-field current at each Zeeman resonance. On p. 2-15 you are given the useful quantity: $\mu_o/h = 1.3996$ MHz/Gauss.

For the main-field calibration, you should use two DVMs: one for sweep-field coil and another for the main-field coil. It is a good idea to adjust the sweep-field current so that its field and the horizontal component of the earth's field cancel. This way your calibration of the main coil is almost decoupled from your calibration of the sweep-field coil. Of course, you will need to adjust the sweep-field current slightly away from this zero-total-field state to attain a Zeeman resonance for a given frequency, because the main-coil current knob is rather coarse. You will need to make up a table like the one on p. 4-11. Your plot of main-coil field vs. current should pass close to (0,0).

5. For the quadratic Zeeman effect, keep the RF-amplifier gain on ×4 and vary the output of the oscillator. To avoid detecting the double-photon resonances, keep oscillator output as small as possible, consistent with having reasonable signal. Use the Labview program to record the single-photon resonances as a function of the sweep field, a plot that should like Fig. 4C-1 on p. 4-13. You will need to use both DVMs for the calibration: either manually recording the sweep-field current at each resonance or interpolating off the graph you recorded with Labview. Use the MathCad program to obtain the expected values of B at each resonance and carefully compare them with your data.

To record qualitatively what happens when double-photon absorption occurs, sweep the field with higher oscillator output—like Fig. 4C-3 on p. 4-14.

6. For the Transient Effects, there are all sorts interesting experiments to do using a digital oscilloscope—see 4D on p. 4-17.

(1) After the RF modulation is turned on, the frequency of the transient oscillations ("Rabi oscillations") should depend linearly on the amplitude of the RF magnetic field ($\propto$ I_{RF}, the amplitude of the RF current through the RF coil). See if you can confirm this. Although you'll probably be recording the oscillator output voltage in db, you can devise an experiment to determine I_{RF} from the db outputs by looking at the discussion on p. 3-19. You can use the known geometry of this RF coil to calculate the field produced by a given current and compare this result with the actual RF field amplitude determined from the frequency of the Rabi oscillations.

(2) You can determine how the recovery rate for reestablishing pumping (after the RF modulation is turn off) depends on the intensity of the light passing through the cell. Place an additional linear polarizer between the linear polarizer of the circular polarizer and the interference filter and rotate the axis of this new polarizer to vary the light intensity. Also record the DC output signal of the light detector to see if detector signal obeys Malus' Law. The following theory section will give you a hint on how to analyze your recovery-rate results.

Theory of Optical Pumping

1. From your Quantum Mechanics course (or the Optical Pumping instruction manual), you know that the total angular momentum of an electron (J) obeys $J = L + S$, where L is orbital angular momentum vector and S ($\pm \frac{1}{2}$) is spin angular momentum vector. Recall that the values of J range over: L + S, L + S −1, … |L - S|. For the ground state of Rb (L=0), one has $^2S_{J= (0 + ½) = ½}$; and for the first excited state, $^2P_{J=|1 - ½| = ½}$.

2. The precursor to Equation 2B-1 is:

$$\mathcal{H} = A\boldsymbol{I}\cdot\boldsymbol{J} - \boldsymbol{\mu_J}\cdot\boldsymbol{B} - \boldsymbol{\mu_I}\cdot\boldsymbol{B} \tag{1}$$

For the two Zeeman-Effect terms (electron and nucleus), the lowest energy is when moment $\boldsymbol{\mu}$ is parallel to magnetic field $\boldsymbol{B}$. The term $A\boldsymbol{I}\cdot\boldsymbol{J}$ is the hyperfine coupling between the electron and its nucleus.

Because the electron has a negative charge, its moment $\boldsymbol{\mu_J}$ is opposite to its total angular momentum vector $\boldsymbol{J}$. Thus $\boldsymbol{\mu_J} = - (|\boldsymbol{\mu_J}|/|\boldsymbol{J}|)\boldsymbol{J}$.

Thus Eq. 2B-1 should read:
$$\mathcal{H} = A\boldsymbol{I}\cdot\boldsymbol{J} + (|\boldsymbol{\mu_J}|/|\boldsymbol{J}|)\boldsymbol{J}\cdot\boldsymbol{B} - (|\boldsymbol{\mu_I}|/|\boldsymbol{I}|)\boldsymbol{I}\cdot\boldsymbol{B} \tag{2}$$
where it is assumed for the nucleus that $\boldsymbol{\mu_I}$ is parallel to $\boldsymbol{I}$, which is the case for ^{87}Rb and ^{85}Rb.

Next, Eq. 2B-5 should be corrected to read: W = + $g_J\cdot\mu_0\cdot B\cdot M_J$, where μ_0 is the Bohr magneton, M_J is the component of $\boldsymbol{J}$ along $\boldsymbol{B}$ and g_J = 2 for the ground state $^2S_{1/2}$.

Eq. 2B-7 should really read W = + $g_F\cdot\mu_0\cdot B\cdot M_F$ where M_F is the component of angular momentum vector $\boldsymbol{F}$ along $\boldsymbol{B}$. Note that $\boldsymbol{F}$ is defined to be: $\boldsymbol{F} = \boldsymbol{J} + \boldsymbol{I}$, and the values of F range over J + I, J+I − 1, … |J - I|. For ^{87}Rb, one has I = 3/2, giving F = J + I = 1/2 + 3/2 = 2 and F = |J - I| = |1/2 - 3/2| = 1 for both the $^2S_{1/2}$ and $^2P_{1/2}$ states. For F = 2, one has M_F = -2, -1, 0, +1, +2; and for F = 1, M_F = -1, 0, +1.
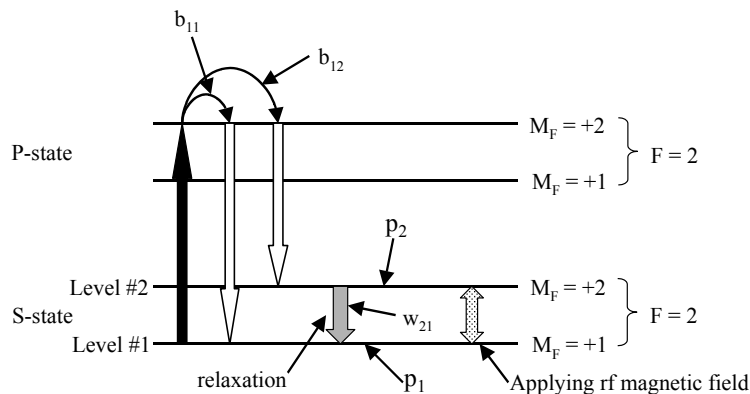
Finally, Eq. 2B-8 is correct if you let M = M_F.

3. Understanding Fig. 2B-3—the $^2S_{1/2}$ ground state of ^{87}Rb: Note that the order of M_F with increasing energy is not monotonic— (+1, 0 ,-1, -2, -1, 0 , +1, +2). This can be understood by going to the large-B limit where $g_J \cdot \mu_0 \cdot B \cdot M_J >> A I \cdot J$, where $M_J = \pm 1/2$. In this limit, one can treat the vectors J and I as being independent, with projections along B of M_J and M_I, respectively. Then our Eq. (2) becomes

$$\mathcal{H} = A \cdot M_J \cdot M_I + g_J \cdot \mu_0 \cdot B \cdot M_J;$$

we ignore the nuclear spin term because it's small. Thus one can categorize these levels by $|M_J, M_I\rangle$ where $M_F = M_J + M_I$. Obviously, the four lowest-energy levels at high B are associated with $M_J = -\frac{1}{2}$. Since the $A \cdot M_J \cdot M_I$ term is negative (due to $M_J = -\frac{1}{2}$), then the lowest-energy hyperfine-split level must have the largest positive M_I (=3/2), giving $M_F = -1/2 +3/2 = +1$. The next largest energy level must have $M_I = \frac{1}{2}$, as shown in the table, etc.

M_J	M_I	M_F
+1/2	+3/2	= 1/2 + 3/2 = +2
+1/2	+1/2	= +1/2 + 1/2 = +1
+1/2	-1/2	= 1/2 = -1/2 = 0
+1/2	-3/2	= 1/2 + -3/2 = -1
-1/2	-3/2	= -1/2 + -3/2 = -2
-1/2	-1/2	= -1/2 + -1/2 = - 1
-1/2	+1/2	= -1/2 + 1.2 = 0
-1/2	+3/2	= -1/2 + 3/2 = +1

4. A better understanding of Fig. 2D-3. Let's assume the circular polarization of the light is such that $\Delta M_F = + 1$. Further, let us focus only upon the two highest-energy F = 2 levels ($M_F = +1$ & +2) for the $^2S_{1/2}$ and $^2P_{1/2}$ states, as shown below. Since $\Delta M_F = + 1$, the only excitation (upward solid arrow) allowed by incident light is from the $M_F = +1$ of $^2S_{1/2}$ to $M_F = +2$ of $^2P_{1/2}$. We are ignoring the possibility of lower-energy states of $^2S_{1/2}$ feeding into the $M_F = +1$ & +2 states of $^2P_{1/2}$. No upward transition from $M_F = +2$ state of $^2S_{1/2}$ is allowed because there is no $M_F = +3$ state for $^2P_{1/2}$. For $^2S_{1/2}$, all that the $M_F = +2$ state of can do is relax back (downward gray arrow) to the lower-energy $M_F = +1$ state. Such spontaneous relaxation usually occurs due to scattering of the Rb atoms by (1) other Rb atoms, or (2) the walls of the cell. Spontaneous emission of photons (downward open arrows) can occur when an electron jumps from the $M_F = +2$ state of $^2P_{1/2}$ to either the $M_F = +1$ or $M_F = +2$ state of $^2S_{1/2}$. Such emission of P-to-S state photons is not specifically directed toward the detector, so such emission looks like a reduction in transmission of light between the lamp and the detector.

For the S-state:

b$_{12}$ = probability per unit time for #1 → #2 transition via P state

b$_{11}$ =   "   "   "   "   " #1 → #1 transition via P state

$w_{21} =$ " " " " " #2 → #1 relaxation

$p_1 =$ probability of occupation of #1 state

$p_2 =$ " " " " #2 state

Note: $p_1 + p_2 = 1$

Modifying Eq. 2D-1 for this simpler case (where the b_{11} transition can be ignored because it does not affect the value of p_1), we get

$$\dot{p}_1 = -b_{12} \cdot p_1 + w_{21} \cdot p_2 \qquad (3)$$

Note: #2 → #1 transition via P state is not possible ($b_{21} = 0$), and #1 → #2 "relaxation" in S state is not possible ($w_{12} = 0$).

Eliminating p_2, we get

$$\dot{p}_1 = -b_{12} \cdot p_1 + w_{21} \cdot (1 - p_1), \text{ and}$$

$$\dot{p}_1 + (b_{12} + w_{21}) \cdot p_1 = w_{21} \qquad (4)$$

The solution to Eq. (4) is the following, where $p_1 = 1$ at time $t = 0$:

$$p_1 = \frac{b_{12} \exp[-(b_{12} + w_{21})t] + w_{21}}{b_{12} + w_{21}} \quad \& \quad p_2 = \frac{b_{12}(1 - \exp[-(b_{12} + w_{21})t])}{b_{12} + w_{21}} \qquad (5)$$

If one takes the $t \to \infty$ limit of Eq. (5), then one obtains

$$p_1 = \frac{w_{21}}{b_{12} + w_{21}} \quad \& \quad p_2 = \frac{b_{12}}{b_{12} + w_{21}} \qquad (6)$$

If $w_{21} = 0$ (no relaxation), then $p_1 = 0$ and $p_2 = 1$. That is, the #2 state becomes fully occupied and the #1 state becomes empty—strong pumping situation. If $w_{21} < b_{12}$, then Eq. (6) gives $p_1 < 1/2$ and $p_2 > 1/2$. The #2 level is still occupied, but the #1 level is not completely depleted—a weaker pumping situation.

If the magnetic field B = 0, then the #2 → #1 relaxation become very rapid because the #1 and #2 levels are degenerate. We can approximate this case with $w_{21} \to \infty$ ($w_{21} \gg b_{12}$), and Eq. (6) now becomes

$$p_1 \to \frac{w_{21}}{w_{21}} = 1 \text{ and } p_2 \to \frac{b_{21}}{w_{21}} = 0$$

Now suppose we apply a rf transverse magnetic field that is in resonance for a particular value of B, as shown in the cartoon above as a double-headed arrow connecting the #1 and #2 S states. If the rf-field-induced transition probabilities (per unit time) for #1 ⇔ #2 are much larger than b_{12} and w_{21}, then the derivation on pages 2-19 & 2-20 of the manual applies. In the $t \to \infty$ limit, one has $p_1 = \frac{1}{2}$ and $p_2 = \frac{1}{2}$.

So let's summarize the results in the $t \to \infty$ limit:

   (a) If $w_{21} = 0$, then $p_1 = 0$ & $p_2 = 1$, B ≠ 0 situation with no relaxation

(b) If $w_{21} < b_{12}$, then $p_1 < \frac{1}{2}$ & $p_2 > \frac{1}{2}$, $B \neq 0$ situation with weak relaxation

(c) If $w_{21} = \infty$, then $p_1 = 1$ & $p_2 = 0$, $B = 0$ situation with strong relaxation

(d) If one applies a strong rf magnetic field in resonance, $p_1 = \frac{1}{2}$ & $p_2 = \frac{1}{2}$.

Now let us relate this summary to the transmission of light to the detector. First define the probability per unit time of photon absorption (and re-emission), P_{abs}:

$$P_{abs} = (b_{11} + b_{12}) \cdot p_1$$

As we stated earlier, such emission of P-to-S state photons is not specifically directed toward the detector, so such emission looks like a reduction in transmission of light between the lamp and the detector. Thus a higher P_{abs} corresponds to a lower transmission. Case (c) corresponds to maximum p_1 (= 1) which means P_{abs} is a maximum, giving a minimum transmission (zero-field resonance). Cases (a) & (b) correspond to the "pumping" situation where $p_1 < 1/2$ and P_{abs} is smaller, giving an increased transmission. Case (d), where a resonant rf magnetic field is applied, relates to an intermediate situation where $p_1 = \frac{1}{2}$, meaning that P_{abs} could be between those values for case (c) and cases (a) & (b). Thus for case (d) the transmission at rf resonance should be between the zero-field case and the pumping situation. Perhaps this is the reason that the transmission reduction for the zero-field resonance is stronger than that for the rf field resonance.

Note that p_2 in Eq. (5) has the form $p_2 \propto [1 - \exp(-\alpha t)]$ where $\alpha = (b_{12} + w_{21})$. Thus when the rf field is suddenly turned off, the optical-pumping state recovers at rate α which is proportional to b_{12}. Try to observe this recovery-rate dependence on light intensity ($\propto b_{12}$).