PHY820 Homework Set 10

1. [5 pts] (Goldstein) A carousel (counter-clockwise merry-go-round) starts from rest and accelerates at a constant angular acceleration of 0.02 revolutions/s^2. A girl sitting on a bench on the platform 7.0 m from the center is holding a 3.0 kg ball. Calculate the magnitude and direction of the force she must exert to hold the ball 6.0 s after the carousel starts to move. Give the direction with respect to the line from the center of rotation to the girl.

2. [5 pts] Consider the problem of the Foucault pendulum, following different strategy than that in the Johnson's textbook, namely identifying the angular momentum component of the pendulum around its vertical axis and connecting that component to the angular velocity of azimuthal precession for the pendulum. Either rewrite the Lagrangian in Johnson in polar coordinates (r, ϕ) or consider explicitly an interplay of the angular momentum component and the vertical component of the torque due to Coriolis force. Ultimately show that the plane of the pendulum rotates uniformly $2\pi \cos\theta$ per day, where θ is colatitude.

3. [5 pts] Johnson 3.12.

4. [10 pts] Johnson 5.18. For the critical case, show that the quantity that is constant is
$$\dot{\theta}^2 + \frac{\omega^2\,\theta^4}{4} = \frac{\omega^2\,\epsilon^4}{4}\,.$$
Moreover, show that the period of small oscillations of small amplitude ϵ about $\theta = 0$ is
$$T = \frac{2}{\epsilon\,\omega}\,B(1/4, 1/2)\,,$$
where B is the Euler Beta function.

5. [10 pts] Johnson 5.6. In this problem r is the distance from the z axis, within the cylindrical coordinate system. The Jacobi integral is
$$J = \dot{r}\,\frac{\partial L}{\partial \dot{r}} - L\,.$$
For $\omega^2 = ga$ show that for $t \to \pm\infty$, the bead position approaches
$$r^2 \to \frac{2}{a}\sqrt{\frac{2J}{m}}\,|t|\,.$$
For $\omega^2 > ga$, show that the bead moves towards infinity with asymptotic speed that satisfies
$$v_\infty^2 = \frac{\omega^2 - g\,a}{a^2}\,.$$