PHY820 Homework Set 12

1. [5 pts] (Goldstein) Evaluate approximately the ratio of mass of the Sun to that of Earth, using only the lengths of the year and of the lunar month (27.3 days), and the mean radii of Earth's orbit (1.49×10^8 km) and of the Moon's orbit (3.8×10^5 km).

2. [5 pts] (Goldstein) At perigee (analog of the perihelion for an orbit around Earth) of an elliptic gravitational orbit a particle experiences an impulse S in the radial direction, sending the particle into another elliptic orbit. Determine the new semimajor axis, eccentricity, and orientation in terms of the old. Note: Use of the Runge-Lenz vector can be beneficial.

3. [5 pts] Johnson 6-5. Consider the effective force on a chunk of mass m_0, located at the edge of the orbiting body, within the noninertial frame moving at an acceleration consistent with the gravitational potential due to M at the center of the orbiting body. The Roche limit is

$$r < R_M \left(\frac{2\,\rho_M}{\rho_m} \right)^{1/3},$$

where R_M is the radius of the central body. Note that electromagnetic forces can keep orbiting bodies together. However, when a body is unstable according to the Roche criterion, it cannot reconstitute itself gravitationally after crumbling such as due to a collision.

4. [5 pts] Johnson 6-7.

5. [5 pts] Johnson 6-10. The form of the force from radiation pressure stems from the fact the momentum of absorbed photons is related to the photon energy with $E = pc$, where c is the speed of light. Likewise, the rate of mass accretion stems from the fact that the energy accrued by the particles in absorbing photons is equivalent to a change in particle mass with $\mathrm{d}\,m = \mathrm{d}\,E/c^2$. The corresponding term in particle acceleration is negligible due to the small magnitude of $\dot{r}/c$. Convert the condition on particles getting swept away to a condition on particle radius r_p in terms of particle mass density ρ. Demonstrate the validity of the claim made on the magnitude of radius for particles swept away, by taking the mass density to be about twice as large as for water, $\rho \sim 2000\,\mathrm{kg/m^3}$.

6. [10 pts] Suppose that there are interactions present between particles in a gas, in the form of central forces derivable from a potential $U(r)$, where r is the distance between any pair of particles. Assume further that relative to any given particle

the other particless are on the average distributed in space in such a way that their volume density is given by the Boltzmann factor:

$$\rho(r) = \frac{N}{V}\,\mathrm{e}^{-U(r)/k_B\,T}\,,$$

where N is the total number of particles in a volume V.

(a) Find the addition to the virial of Clausius, from these forces between pairs of particles, and compute the resulting correction to the ideal-gas equation of state. Simplify the result as much as possible. The identity

$$\frac{\mathrm{d}\,U}{\mathrm{d}\,r}\,\mathrm{e}^{-U(r)/k_B\,T} = -k_B\,T\,\frac{\mathrm{d}}{\mathrm{d}\,r}\left(\mathrm{e}^{-U(r)/k_B\,T} - 1\right),$$

may be useful, facilitating a partial integration.

(b) Using the result above, compute the correction for the case of hard-sphere repulsion:

$$U(r) = \begin{cases} \infty, & \text{for } r < a\,, \\ 0, & \text{for } r > a\,, \end{cases}$$

where a is the radius of the potential.