PHY820 Homework Set 13

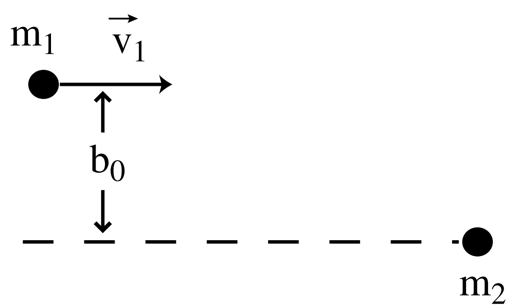
1. [5 pts] A proton of energy 4 MeV scatters elastically off a second proton at rest. One proton comes off at an angle of 30° in the lab system. What is its energy? What is the energy and scattering angle of the second proton?

2. [10 pts] Johnson 7-4.

3. [5 pts] Johnson 7-5.

4. [10 pts] Two particles of masses m_1 and m_2 ($m_1 \neq m_2$) collide. The initial velocity velocity of particle 1 is $\vec{v}_1$, while the particle 2 is initially at rest. The initial impact parameter is b_0, as shown. The particles interact through a repulsive potential $V = V_0/|\vec{r}_1 - \vec{r}_2|^4$. (a) Find the magnitude of net angular momentum and the net energy in the *center of mass*, in terms of provided quantities. (b) Obtain an equation for the rate of change of the separation $r = |\vec{r}_1 - \vec{r}_2|$ in time. Find the distance of closest approach between the particles.

   (c) Consider a situation where b_0 is unknown, but the magnitude v_1^f of the final velocity of particle 1 has been measured in the laboratory frame. Find the angle β between the final laboratory velocities of particles 1 and 2, given v_1^f. (Use conservation laws whenever possible.)



5. [0 pts] Johnson 7-9. PLEASE REFRAIN FROM SOLVING THIS PROBLEM! Fixes that need to be applied erase practical didactic value for this problem.

6. [10 pts] Consider scattering caused by a central force described in terms of potential $V(r)$.

   (a) By manipulating the integral expression for change of polar angle with distance, show that the scattering angle Θ may be expressed as

$$\Theta = \pi - 2 \int_0^{u_{\max}} \frac{\mathrm{d}u}{\sqrt{1 - u^2 - \frac{V(u)}{E}}} .$$

   Here, $u = b/r$, b is the impact parameter and the scattering angle is positive if the projectile comes out on the original side of the scattering plane and negative if on the opposite. In practical measurements normally only absolute value of the scattering angle is determined. Verify that the derived expression yields the expected result in the absence of a force.

(b) Use the derived expression, in combination with a math package, to determine the scattering angle as a function of impact parameter, $\Theta(b)$, numerically for repulsive and for attractive potentials of finite strength:

$$V(r) = \pm V_0 \exp\left(-\frac{r^2}{r_0^2}\right),$$

and for a combination of shorter-range attraction and longer-range repulsion

$$V(r) = -2\,V_0 \exp\left(-\frac{4\,r^2}{r_0^2}\right) + V_0 \exp\left(-\frac{r^2}{r_0^2}\right).$$

Here, V_0 and r_0 are constants. Use impact parameter range of few r_0, such as $b = (0, 3\,r_0)$, and energy slightly in excess of the potential strength, such as $E = 1.2\,V_0$.

Math packages could be stretched to their limits when handling a singularity in the subintegral function, such as above, combined with parametric dependence, so the necessary commands might start to look like those in more rudimentary languages. In Mathematica, in particular, the commands for calculations might take the form:

```
File  Edit  Insert  Format  Cell  Graphics  Evaluation  Palettes  Window  Help

r0 = 1; V0 = 1; EE = 1.2 * V0;
V[r_] := V0 * Exp[-r^2 / r0^2] (*-2*V0*Exp[-4*r^2/r0^2]*);
Plot[V[r], {r, 0, 2 r0}]
Argub[u_, b_] := 1 - u^2 - V[b / u] / EE;
For[i = 0, i < 181, i++,
  b = (.001 + i * (.001 + i / 500) / (18 + i / 200)) * r0;
  Solution = FindRoot[Argub[u, b] == 0, {u, 1}];
  umax = u /. Solution;
  Alf = Re[NIntegrate[1 / Sqrt[Argub[u, b]], {u, 0, umax}]];
  Theta = Pi - 2 * Alf;
  (*Print[b," ",umax," ",Alf," ",Theta];*)
  vecb[i] = b;
  vecT[i] = Theta
 ];
TT = Table[{vecb[i], vecT[i]}, {i, 0, 180}];
ListLinePlot[TT]

100%
```

Plot the potentials $V(r)$ together with the scattering angle $\Theta(b)$ and comment on the findings. Attach the plots together with the printout of your code to your homework or upload them to the Dropbox on Angel.