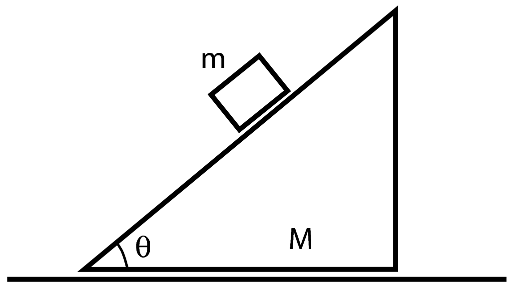
PHY422 Homework Set 2

1. [10 pts] Johnson, problem 1.10. The proper relation to demonstrate is

$$r \cos\big(\sin\theta\,(\phi - \phi_0)\big) = -\frac{\kappa}{\sin\theta}\,.$$

2. [10 pts] A smooth wedge of mass M has a triangular cross section with a side inclined at an angle θ to the horizontal base. The wedge can slide without friction along a horizontal support. Placed on the side of the wedge is a mass m that can slide with no friction along the side. Find vectors of the acceleration for the wedge and for m after the bodies are released from rest. The methodology is up to you.



3. [5 pts] Obtain the Euler-Lagrange equations of motion for a spherical pendulum, i.e. a point mass m suspended by a rigid weightless rod of length ℓ. The mass is free to move over a spherical surface rather than just the circumference of a circle.

4. [5 pts] A system with n degrees of freedom satisfies a set of Euler-Lagrange equations with a Lagrangian L. Show by a direct substitution into the equations, that that system also satisfies the Euler-Lagrange equations with the Lagrangian

$$L' = L + \frac{\mathrm{d}\,F(q_1,\ldots,q_n,t)}{\mathrm{d}\,t}\,,$$

where F is any arbitrary, but differentiable, function of its arguments.

5. [5 pts] Let $q_1, \ldots q_n$ be a set of independent generalized coordinates for a system of n degrees of freedom, with a Lagrangian $L(q,\dot{q},t)$. Suppose we transform to another set of independent coordinates $s_1,\ldots,s_n$ by means of transformation equations

$$q_i = q_i(s_1,\ldots,s_n,t)\,, \qquad i = 1,\ldots,n\,.$$

(Such a transformation is called a *p*oint transformation.) Show that if the Lagrangian function is expressed as a function of s_j, $\dot{s}_j$, and t through the equations of transformation, then L satisfies Lagrange's equations with respect to the s coordinates:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{s}_j}\right) - \frac{\partial L}{\partial s_j} = 0\,.$$

In other words, the form of Lagrange's equations is invariant under a point transformation.