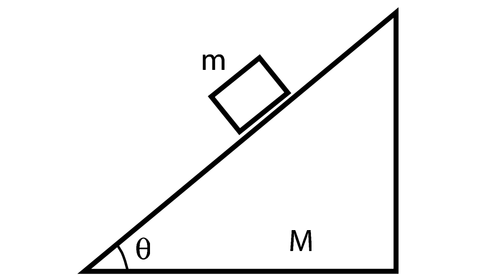
PHY820 Homework Set 2

1. [10 pts] Johnson, problem 1.10. The proper relation to demonstrate is

$$r \cos\big(\sin\theta\,(\phi - \phi_0)\big) = -\frac{\kappa}{\sin\theta}\,.$$

2. [10 pts] A smooth wedge of mass M has a triangular cross section with a side inclined at an angle θ to the horizontal base. The wedge can slide without friction along a horizontal support. Placed on the side of the wedge is a mass m that can slide with no friction along the side. Find vectors of the acceleration for the wedge and for m after the bodies are released from rest. The methodology is up to you.



3. [5 pts] Obtain the Euler-Lagrange equations of motion for a spherical pendulum, i.e. a point mass m suspended by a rigid weightless rod of length ℓ. The mass is free to move over a spherical surface rather than just the circumference of a circle.

4. [5 pts] A system with n degrees of freedom satisfies a set of Euler-Lagrange equations with a Lagrangian L. Show by a direct substitution into the equations, that that system also satisfies the Euler-Lagrange equations with the Lagrangian

$$L' = L + \frac{\mathrm{d}\,F(q_1, \ldots, q_n, t)}{\mathrm{d}\,t}\,,$$

where F is any arbitrary, but differentiable, function of its arguments.

5. [10 pts] A particle is free to move on the surface of a sphere of unit radius under the influence of no forces other than those that constrain the particle to the sphere. It starts at a point q_1 and ends at another point q_2 (without loss of generality, both points may be taken to lie on a meridian of longitude).

(a) Construct the Lagrangian for the particle, employing spherical coordinates.

(b) Choose conveniently the axes for the coordinates and solve the Euler-Lagrange equations. Show that there are many physical paths (i.e. $q(t)$ functions) the particle can take in going from q_1 to q_2 in a given time τ. How many? Under what conditions are there uncountable many?

(c) Calculate the action for each of two possible paths the particle can take and show that they are not in general equal. Now construct two new, nonphysical paths close to the original ones, going from q_1 to q_2 in the same time τ. This can be done by adding to each physical path a small distortion of the form $\eta_k(t)$ such that $\eta_k(0) = \eta_k(\tau) = 0$. Show that for each of the nonphysical paths the action is greater than it is on the neighboring physical one, thus demonstrating that each physical path minimizes the action locally.