PHY422 Homework Set 3

1. [10 pts] Johnson, problem 2.5. The proper expression for the total kinetic energy gained by the rocket is
$$\Delta K_r = \frac{1}{2}\, m_0 \left\{ v_1^2\, \mathrm{e}^{-(v_1-v_0)/v_e} - v_0^2 \right\}.$$
The proper expression for the total kinetic energy in the exhaust cloud is
$$\Delta K_c = \frac{1}{2}\, m_0\, v_0^2 + \frac{1}{2}\, m_0\, v_e^2 \left\{ 1 - \mathrm{e}^{-(v_1-v_0)/v_e} \right\} - \frac{1}{2}\, m_0\, v_1^2\, \mathrm{e}^{-(v_1-v_0)/v_e}.$$
The proper expression for the work done in expelling the gas is
$$\Delta W = m_0\, v_e^2 \left\{ 1 - \mathrm{e}^{-(v_1-v_0)/v_e} \right\}.$$

2. [5 pts] Johnson, problem 2.6. The proper result is
$$T = 2\sqrt{\frac{2m}{a}}\, \tan^{-1}\left\{ \frac{2}{b}\sqrt{aE} \right\},$$
and other representations of that result are possible, including one close that in the textbook.

3. [5 pts] Johnson, problem 2.7.

4. [5 pts] Johnson, problem 2.11.

5. [5 pts] Based on Johnson, problem 2.20: A particle is moving in the x direction subject to the following differential equation
$$\ddot{x} = -\beta\, \dot{x} \left(\dot{x}^2 + \omega^2\, x^2 - A^2 \right) - \omega^2\, x,$$
where β, A and ω are positive parameters. In addition to a linear restoring force, there is a velocity-dependent force that may either accelerate or decelerate the particle. The model is an extension of simple harmonic motion which occurs for $\beta = 0$.

    Show that the positive quantity
$$E(t) = \dot{x}^2(t) + \omega^2\, x^2(t)$$
satisfies the relation
$$\dot{E}(t) = -2\beta\, \dot{x}^2 \left(E(t) - A^2 \right).$$
Show that $E(t)$ decreases with time whenever $E(t) > A^2$, and that $E(t)$ increases with time whenever $E(t) < A^2$. Also, note that when $E(t) = A^2$, x must depend on time. In fact, the curve
$$\dot{x}^2(t) + \omega^2\, x^2(t) = A^2$$
represents the limiting behavior of $x(t)$ in the limit $t \to \infty$.

    The parameter ω is the limiting frequency of oscillation and A/ω is the limiting amplitude, whereas β determines the rate of approach to the limit cycle.