PHY422 Homework Set 4

1. [10 pts] Johnson, problem 3.6. The Lagrangian should be

$$L = \frac{1}{2}\,(m_1 + m_2)\,\dot{x}^2 + \frac{1}{2}\,m_2\,(\ell^2\,\dot{\theta}^2 + 2\,\ell\,\dot{x}\,\dot{\theta}\cos\theta) + m_2\,g\,\ell\cos\theta\,.$$

   The equation of the ellipse should be

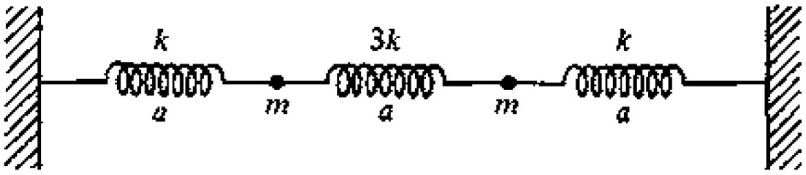
$$\frac{(x - X)^2}{[m_2\,\ell/(m_1 + m_2)]^2} + \frac{y^2}{\ell^2} = 1\,.$$

2. [10 pts] Johnson, problem 3.8. The Lagrangian should be

$$L = \frac{m}{8}\,(\dot{x}_1 + \dot{x}_2)^2 + \frac{m}{24}\,(\dot{x}_1 - \dot{x}_2)^2 + \frac{mg}{2}\,(x_1 + x_2) - \frac{k}{2}\,(x_1^2 + x_2^2)\,.$$

3. [5 pts] Johnson, problem 3.14

4. [5 pts] (Goldstein) Two particles move in one dimension at the junction of three springs, as shown in the figure. The springs all have unstretched lengths equal to a, and the force constants and masses are shown.



   Find the eigenfrequencies and normal modes of the system.

5. [10 pts] A thin hoop of radius R and mass M oscillates in its own plane hanging from a single fixed point on its circumference. Moving along the hoop, without friction, is a small bead also of mass M. Consider only small oscillations around equilibrium and determine the normal frequencies for the system.