PHY820 Homework Set 4

1. [10 pts] (after Goldstein) Obtain the normal modes of vibration for the double pendulum shown in Fig. 3.4 of Johnson, assuming equal lengths, but not equal masses. Show that when the lower mass is small compared to the upper one, the two resonant frequencies are almost equal. If the pendula are set in motion by pulling the upper mass slightly away from the vertical and then releasing it, show that subsequent motion is such that at regular intervals one pendulum is at rest while the other has its maximum amplitude This is the familiar phenomenon of "beats."

2. [10 pts] Johnson, problem 3.8. The Lagrangian should be

$$L = \frac{m}{8}\left(\dot{x}_1 + \dot{x}_2\right)^2 + \frac{m}{24}\left(\dot{x}_1 - \dot{x}_2\right)^2 + \frac{mg}{2}\left(x_1 + x_2\right) - \frac{k}{2}\left(x_1^2 + x_2^2\right).$$

3. [5 pts] Johnson, problem 3.14

4. [10 pts] Johnson, problem 3.16. The z axis points down and starts at the equilibrium position of the mass m, distance b below the points of suspension of the two springs. The condition for equilibrium to be demonstrated is

$$m\,g = 2\,k\,b\left(1 - \frac{\ell}{\sqrt{a^2+b^2}}\right).$$

   The frequencies of oscillation to be demonstrated are

$$\begin{aligned}
\omega_x^2 &= \tfrac{2k}{m}\left(1 - \tfrac{\ell\, b^2}{(a^2+b^2)^{3/2}}\right), \\
\omega_y^2 &= \tfrac{2k}{m}\left(1 - \tfrac{\ell}{\sqrt{a^2+b^2}}\right), \\
\omega_z^2 &= \tfrac{2k}{m}\left(1 - \tfrac{\ell\, a^2}{(a^2+b^2)^{3/2}}\right),
\end{aligned}$$

5. [10 pts] Three beads are mounted on a ring and connected by three identical springs, as shown. Two beads are of mass m and one of mass $2m$. The ring radius is R. The spring constant is k and the spring mass may be neglected. The masses and springs are free to move around the ring. (a) Find a Lagrangian for the system of beads and springs, in terms of suitably chosen coordinates. (b) Find frequencies of normal vibrations for the system. (Can you find any shortcuts, bypassing standard procedures, relying on physical or symmetry considerations?)