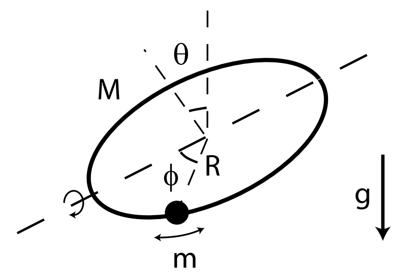
PHY422 Homework Set 5

1. [10 pts] A uniform ring of radius R and mass M is free to rotate about a *horizontal* axle along its diameter. The moment of inertia of the ring about such an axle is $I = MR^2/2$. Mounted onto the ring is a bead of mass m that can slide along the ring without friction. As generalized coordinates use the angle θ of the ring relative to the vertical and the angle ϕ of the bead relative to the axle.

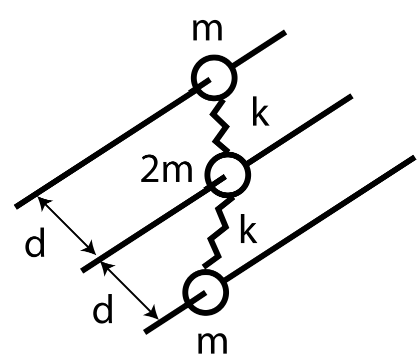


   (a) Determine the kinetic energy of the bead in terms of generalized coordinates and velocities.

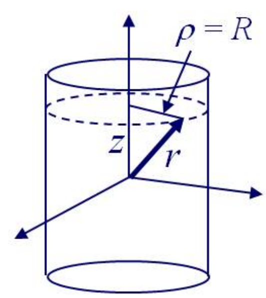
   (b) Account for gravity and kinetic energy of the ring and obtain a Lagrangian for the ring-bead system.

   (c) Obtain Lagrange equations for this system. Divide out redundant factors.

2. [10 pts] Three beads of mass m, $2m$ and m, respectively, are threaded onto three parallel rods, a distance d apart from each other as shown. The beads are connected with springs characterized by a spring constant k. (Assume that the length of unstretched springs is zero.) The beads can move along the rods without friction. Find the normal modes of oscillation of the bead system (frequencies and amplitude vectors - no particular normalization required). Discuss those modes. Note: gravity plays no role in this problem.



3. [10 pts] A particle of mass m is constrained to move on the cylindrical surface described in cylindrical coordinates (ρ, ϕ, z) by the constraint equation $\rho = R$. The *only* force acting on the particle is the central force directed towards the origin, $\vec{F} = -k\,\vec{r}$. Your task will be to obtain and solve the Lagrangian equations for the particle, with and without an explicit reference to the constraint equation.



   (a) At first introduce the constraint inherently and express the Lagrangian for the particle only in terms of the generalized coordinates z and ϕ, and associated velocities, excluding the possibility of the particle leaving the cylindrical surface.

   (b) Obtain and solve the Lagrange equations associated with the coordinates z and ϕ. Describe in words the motion that the particle executes.

   (c) Now restart the problem from scratch and construct the Lagrangian in terms of the coordinates ρ, ϕ and z, allowing at this stage for the possibility of the particle moving outside of the cylindrical surface.

(d) Obtain the Lagrange equations associated with the coordinates ρ, ϕ and z, incorporating explicitly the constraint equation with the accompanying, yet undetermined, multiplier λ.

(e) Solve the Lagrange equations. Arrive at the dependence of the multiplier λ on time, following from the requirement of the particle remaining on the cylindrical surface.

(f) What conclusion can you draw from the Lagrange equations on the normal force that the constraint surface exerts on the particle?

4. [5 pts] To start out with numerical computations, use Mathematica or another mathematical package and carry out the following tasks:

   (a) Plot sine function over three periods. In Mathematica this is accomplished with the command:
   Plot[Sin[phi], {phi,0,6Pi}].

   (b) Solve numerically the harmonic oscillator equation with damping

   $$\ddot{x} = -x - \beta\,\dot{x}\,,$$

   for $x(0) = 1$ and $\dot{x}(0) = -1$, for several values of β, observing underdamped and overdamped motions. Plot your results. In Mathematica, these tasks can be carried out by executing the chain of commands:
   beta = .2;
   Solution = NDSolve[{x″[t] == -x[t] - beta*x′[t], x[0] == 1, x′[0] == -1} , x, {t,0,30}];
   Plot[x[t]/.Solution, {t,0,30 }, PlotRange−>All]

   Attach images of your command sets and of plots with results to your homework. A printer is available in BPS 1248. Alternatively, you can upload your images into the Dropbox on Angel.