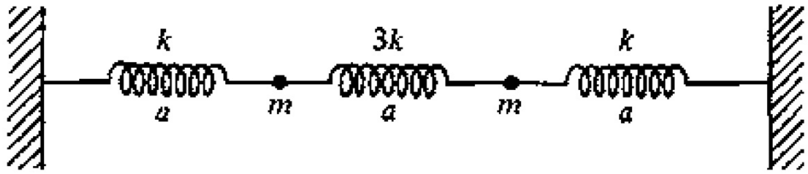
PHY820 Homework Set 5

1. [10 pts] Two particles move in one dimension at the junction of three springs, as shown in the figure. The springs all have unstretched lengths equal to a, and the force constants and masses are shown.



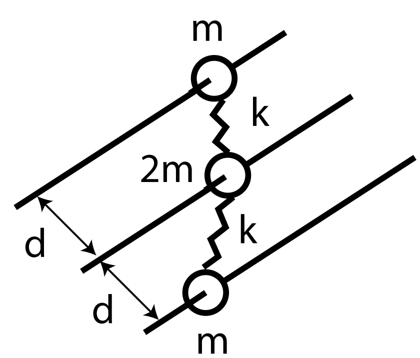
   (a) Find the eigenfrequencies and normal modes of the system.

   (b) Determine the particle positions as a function of time, if, at $t = 0$,

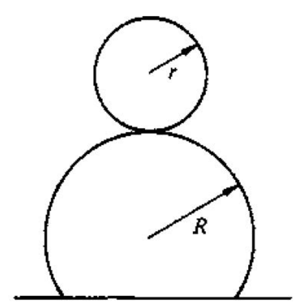
      i. the displacements and the velocity of the second particle are zero while the first particle moves at a velocity v,

      ii. the velocities and the displacement of the second particle are zero while the first particle is displaced by $+d$.

2. [10 pts] Three beads of mass m, $2m$ and m, respectively, are threaded onto three parallel rods, a distance d apart from each other as shown. The beads are connected with springs characterized by a spring constant k. (Assume that the length of unstretched springs is zero.) The beads can move along the rods without friction. Find the normal modes of oscillation of the bead system (frequencies and amplitude vectors - no particular normalization required). Discuss those modes. Note: gravity plays no role in this problem.



3. [10 pts] (Goldstein) A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R as shown in the figure. The only external force is that of gravity. If the smaller cylinder starts rolling from rest on top of the bigger cylinder, use the method of Lagrange multipliers to find the point at which the hoop falls off the cylinder.



4. [10 pts] To start out with numerical computations, use Mathematica or another mathematical package and carry out the following tasks:

   (a) Plot sine function over three periods. In Mathematica this is accomplished with the command:
   Plot[Sin[phi], {phi,0,6Pi}].

(b) Solve numerically the harmonic oscillator equation with damping

$$\ddot{x} = -x - \beta\,\dot{x}\,,$$

for $x(0) = 1$ and $\dot{x}(0) = -1$, for several values of β, observing underdamped and overdamped motions. Plot your results. In Mathematica, these tasks can be carried out by executing the chain of commands:
beta = .2;
Solution = NDSolve[{x″[t] == -x[t] - beta*x′[t], x[0] == 1, x′[0] == -1} , x, {t,0,30}];
Plot[x[t]/.Solution, {t,0,30 }, PlotRange−>All]

(c) Change the form of the friction force from $-\beta\,\dot{x}$ to $-\beta\,\mathrm{sign}(\dot{x})$. Again generate and plot results for different β. What is the qualitative difference between the results for the two forms of friction force?

Attach images of your command sets and of plots with results to your homework. A printer is available in BPS 1248. Alternatively, you can upload your images into the Dropbox on Angel.