PHY422 Homework Set 6

1. [5 pts] Two particles, characterized by charge q_1 and q_2, respectively, and by mass of m_1 and m_2, move under the influence of each other in an external uniform electric field $\vec{E}$. Examine the Lagrangian for the particles with external and mutual Coulomb potential terms and demonstrate that the particle motion may be studied by considering *separately* the motion of the center of mass and the motion in the particle relative separation.

2. [10 pts] A ladder of length L and mass M rests against a smooth wall and slides without friction on the wall and the floor. Assume that the ladder is initially at rest at an angle α_0 with respect to the floor. Use the method of Lagrange undetermined multipliers to find the angle α_1 at which the ladder leaves the wall.

3. [5 pts] (Goldstein) What is the height-to-diameter ratio of a right cylinder such that the inertia ellipsoid at the center of the cylinder is a sphere?

4. [5 pts] (Goldstein) Three equal masses are located at $(a, 0, 0)$, $(0, a, 2a)$, $(0, 2a, a)$. Find the principal moments of inertia about the origin and a set of principal axes.

5. [10 pts] Consider the equation of the quartic oscillator:

$$\ddot{x} = -x - 2\,x^3\,.$$

   (a) What is the expression for the quantity that plays the role of energy for this oscillator?

   (b) What is the period for this oscillator in the limit of a vanishing amplitude of oscillations? Note that in the lecture we derived an expression for the period of a quartic oscillator in the lowest order of perturbation in anharmonicity.

   (c) Calculate the period for the oscillator numerically, for different amplitudes of oscillations x_F. Compare the results to those from the perturbation expansion. In Mathematica the proper chain of commands is:
   xf=.02;
   tp=2Pi*(1-.75*xf*xf);
   t=4*NIntegrate[1/. . . ,{x,0,xf},Method− >"GlobalAdaptive"];
   Print[xf," ",tp," ",t]
   In the above, the dots, . . . , need to be replaced by an expression for the velocity. The choice of the method for integration thereafter helps Mathematica handle the singularity at the edge of integration, where the velocity approaches zero. Attach to the homework images showing the code and results of your calculations or, else, upload them to the Dropbox on Angel.