PHY422 Homework Set 7

1. [5 pts] By considering the respective Lagrangian, determine the integrals of motion for a particle moving in a uniform field $V = -\vec{F} \cdot \vec{r}$.

2. [10 pts] Johnson 4.4. The component of angular velocity along direction 3 is $\omega_3 = \dot{\theta}$ and

$$\ddot{\theta} + \frac{\lambda\,\omega_0^2}{2} \sin 2\theta = 0\,.$$

3. [5 pts] Johnson 4.5. For $v_0 = 8\,\mathrm{m/s}$ and $\mu_k = 0.3$, the distance is about $5\,\mathrm{m/s}$.

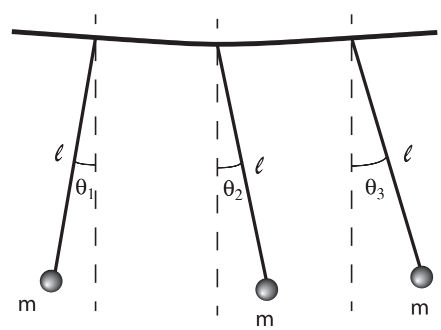
4. [10 pts] Johnson 4.7.

5. [5 pts] (Goldstein) Prove that for a general rigid body motion about a fixed point, the time variation of the kinetic energy T is given by

$$\frac{\mathrm{d}\,T}{\mathrm{d}\,t} = \vec{\omega} \cdot \vec{N}\,,$$

where $\vec{N}$ is the net torque about the fixed point. Note that the tensor of inertia generally *depends on time* in the frame of an observer external to the external body.

6. [10 pts] Consider three identical pendula suspended from a slightly yielding support. Because the support is not rigid, a coupling occurs between the pendula, making the potential energy approximately equal to:



$$U \approx \frac{1}{2}\, m\, g\, \ell\, (\theta_1^2 + \theta_2^2 + \theta_3^2) - \epsilon\, m\, g\, \ell\, (\theta_1\, \theta_2 + \theta_1\, \theta_3 + \theta_2\, \theta_3)\,,$$

where $\epsilon \ll 1$, while the kinetic energy remains equal to

$$T = \frac{1}{2}\, m\, \ell^2\, (\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2)\,.$$

(a) Find the normal frequencies and normal modes for the coupled system. Note: Given the three degrees of freedom, three modes are expected. With the reflection and cyclic symmetries of the system, an individual mode can be expected to be either invariant under a symmetry or get interchanged with another mode. In the latter case, the frequency should not change. Rather than solving the determinant equation, start assuming the amplitude vector to have all components identical. The amplitude vectors for the the two remaining modes must be orthogonal to the vector for the symmetric mode above. Pick the amplitudes for those remaining modes so they indeed transform into each other under reflection.

(b) Next, use Mathematica or another package to find the frequencies. In Mathematica, this can be accomplished with the commands:

```
t = IdentityMatrix[3];
v = {{1,-ep,-ep}, {-ep,1,-ep}, {-ep,-ep,1}};
poly = Det[om0k*v-omk*t];
Roots[poly==0,omk]
```

Attach to the homework images showing the code and results of your calculations or, else, upload them to the Dropbox on Angel.