PHY820 Homework Set 7

1. [5 pts] Consider a system of particles described in terms of a lagrangian $L = T - U$, where U is a generalized potential with explicit dependence on velocities $\vec{v}_i$. Demonstrate that when that dynamics is invariant with respect to rotations about an axis $\vec{n}$, then the conserved quantity is not the naively expected $\vec{n} \cdot \vec{L}$, where $\vec{L}$ is total angular momentum, but rather

$$p_{\vec{n}} = \vec{n} \cdot \vec{L} - \vec{n} \cdot \sum_i \vec{r}_i \times \nabla_{\vec{v}_i} U \, .$$

For that combine the Noether's theorem with invariance of the dynamics under the infinitesimal transformation $\vec{r}_i \rightarrow \vec{r}_i^{\,\prime} = \vec{r}_i + \epsilon \, \vec{n} \times \vec{r}_i$. Obtain $p_{\vec{n}}$ for the case of electromagnetic interactions for which the contribution for particle i to the generalized potential is, in SI units, $U_i = q_i \, \Phi_i - q_i \, \vec{v}_i \cdot \vec{A}_i$.

2. [10 pts] Johnson 4.4. The component of angular velocity along direction 3 is $\omega_3 = \dot{\theta}$ and

$$\ddot{\theta} + \frac{\lambda \, \omega_0^2}{2} \sin 2\theta = 0 \, .$$

3. [5 pts] Johnson 4.5. For $v_0 = 8\,\mathrm{m/s}$ and $\mu_k = 0.3$, the distance is about $5\,\mathrm{m/s}$.

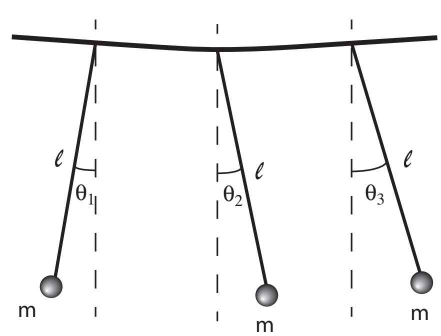
4. [10 pts] Johnson 4.7.

5. [5 pts] (Goldstein) Prove that for a general rigid body motion about a fixed point, the time variation of the kinetic energy T is given by

$$\frac{\mathrm{d}\,T}{\mathrm{d}\,t} = \vec{\omega} \cdot \vec{N} \, ,$$

where $\vec{N}$ is the net torque about the fixed point. Note that the tensor of inertia generally *depends on time* in the frame of an observer external to the external body.

6. [10 pts] Consider three identical pendula suspended from a slightly yielding support. Because the support is not rigid, a coupling occurs between the pendula, making the potential energy approximately equal to:



$$U \approx \frac{1}{2} m \, g \, \ell \, (\theta_1^2 + \theta_2^2 + \theta_3^2) - \epsilon \, m \, g \, \ell \, (\theta_1 \, \theta_2 + \theta_1 \, \theta_3 + \theta_2 \, \theta_3) \, ,$$

where $\epsilon \ll 1$, while the kinetic energy remains equal to

$$T = \frac{1}{2} m \, \ell^2 \, (\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2) \, .$$

(a) Find the normal frequencies and normal modes for the coupled system. Note: Given the three degrees of freedom, three modes are expected. With the reflection and cyclic symmetries of the system, an individual mode can be expected to be either invariant under a symmetry or get interchanged with another mode. In the latter case, the frequency should not change. Rather than solving the determinant equation, start assuming the amplitude vector to have all components identical. The amplitude vectors for the the two remaining modes must be orthogonal to the vector for the symmetric mode above. Pick the amplitudes for those remaining modes so they indeed transform into each other under reflection.

(b) Next, use Mathematica or another package to find the frequencies. In Mathematica, this can be accomplished with the commands:

```
t = IdentityMatrix[3];
v = {{1,-ep,-ep}, {-ep,1,-ep}, {-ep,-ep,1}};
poly = Det[om0k*v-omk*t];
Roots[poly==0,omk]
```

(c) Finally, change the center mass from m to $2m$ and find the frequencies again. Is there any frequency retained relative to the previous case? What type of mode would you expect to retain the frequency? Expand the frequencies to the first order in ϵ, either by hand or by incorporating the expansion into the math package commands.

Attach to the homework images showing your code and results of your calculations or, else, upload them to the Dropbox on Angel.