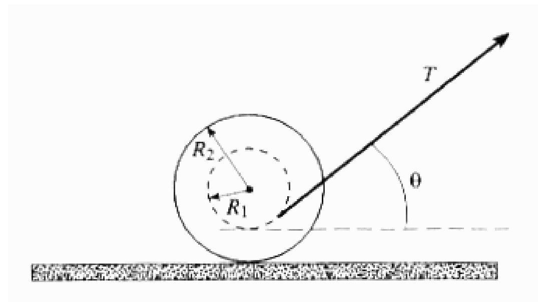
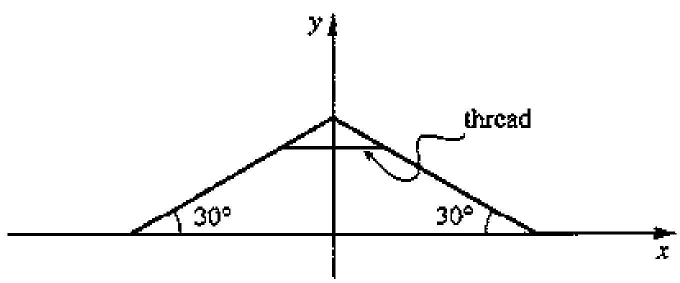
PHY422 Homework Set 8

1. [5 pts] A spool rests on a rough table as shown. A thread wound on the spool is pulled with force T at angle θ. (a) If $\theta = 0$, will the spool move to the left or right? (b) Show that there is an angle θ for which the spool remains at rest. (c) At this critical angle find the maximum T for equilibrium to be maintained. Assume a coefficient of friction μ.

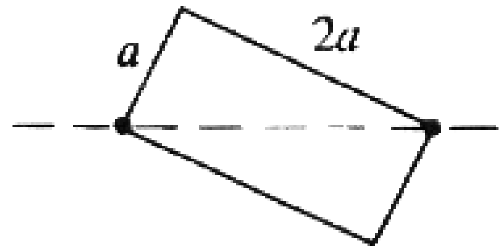


2. [10 pts] (Goldstein) Two thin rods, each of mass m and length ℓ, are connected to an ideal (no friction) hinge and a horizontal thread. The system rests on a smooth surface as shown in the figure. At time $t = 0$, the thread is cut. Neglecting the mass of the hinge and the thread, and considering only motion in the xy plane
   (a) Find the speed at which the hinge hits the floor.
   (b) Find the time it takes for the hinge to hit the floor. You can leave the time proportional to a dimensionless integral.

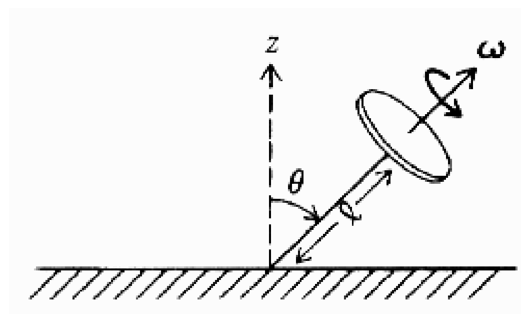


3. [10 pts] Consider a particle of mass m and charge q moving in a uniform constant magnetic field $\vec{B}$ pointing in the $+z$ direction.
   (a) Demonstrate that $\vec{B}$ can be written as $\vec{B} = \vec{\nabla} \times \vec{A}$ with $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$. Prove that equivalently in cylindrical coordinates, (ρ, ϕ, z), $\vec{A} = \frac{1}{2} B\rho\hat{\phi}$.
   (b) Write the Lagrangian for the particle in cylindrical coordinates and find the three corresponding Lagrange equations. Note that this is the case of a Lagrangian constructed using a generalized potential that depends on velocity.
   (c) Describe in detail those solutions of the Lagrange equations in which ρ is a constant. Sketch a particle trajectory following those solutions.

4. [5 pts] A flat rectangular plate of mass M and sides a and $2a$ rotates with angular velocity ω about an axle through two diagonal corners, as shown. The bearings supporting the plate are mounted just at the corners. Follow Euler's equations and find the force on each bearing due to rotation. Only two principal moments of inertia are relevant.



5. [5 pts] A heavy axially symmetric gyroscope is supported at a pivot, as shown. The mass of the gyroscope is M, and the moment of inertia about its symmetry axis is I. The initial angular velocity about its symmetry axis is ω. Follow an approximate solution of the equation of motion for the system, under the assumption that ω is very large and obtain the angular frequency Ω of gyroscopic precession. Show that the approximation requires that $\omega \gg \sqrt{g/\ell}$, when ℓ takes the role of an overall size scale with all moments of inertia taken to be roughly $M\,\ell^2$.



6. [5 pts] Investigate the motion of a heavy axially symmetric top of Sec. 4.5 in Johnson for the case when the top is started at $\Theta = 0$ with low $\dot{\Theta}$. By considering the effective potential around $\Theta = 0$, show that the motion is stable or unstable in that vicinity depending on whether $I_3\,\omega_3$ is greater or lesser than $2\sqrt{I_\perp\, m\, g\, \ell}$. Sketch the effective potential in the two cases. If the top is set spinning in the stable configuration, what is the effect as friction gradually reduces ω_3? Do not use the approximate potential claimed in Johnson, but rather derive one yourself. *Hint:* Start out from the definitions of p_Ψ and p_Φ and demonstrate that these two momenta become identical when the top is set at $\Theta = 0$. The latter ensures a good behavior of $U_{ef}(\Theta)$ around $\Theta = 0$.