PHY820 Homework Set 9

1. [5 pts] (Goldstein) A homogeneous cube of side ℓ 1s initially at rest in unstable equilibrium with one edge in contact with a horizontal plane. The cube is given a small angular displacement and allowed to fall. What is the angular velocity of the cube when one face contacts the plane if:

   (a) the edge in contact with the plane cannot slide?

   (b) the plane is frictionless so the edge can slide?

2. [10 pts] Apply Euler's equations to the problem of the heavy symmetrical top, Sec. 4.5 in Johnson, expressing ω_i there in terms of the Euler angles. Show that the two conserved general momenta can be obtained directly from the Euler's equations cast in this form.

3. [5 pts] (Goldstein) A particle is thrown up vertically with initial speed v_0, reaches a maximum height and falls back to ground. Show that the Coriolis deflection, when it again reaches the ground, is opposite in direction, and four times greater in magnitude, than the Coriolis deflection when it is released at rest (relative to the surface of Earth) from the same maximum height. In solving the problem, be concerned with the effects of uniform gravity and Coriolis force only.

4. [5 pts] Johnson 5.1. The projectile stays for more, not less, time in the air due to the Coriolis effect. The southward displacement is

$$\frac{4\,v_0^2}{g^2}\,\omega\sin\theta\sin^2\alpha\cos\alpha\,.$$

5. [10 pts] Consider the motion of a symmetric heavy top.

   (a) From the Lagrangian in terms of the Euler angles, obtain the Euler-Lagrange equation associated with the Euler polar angle Θ, of the symmetry axis 3 of the top relative to the vertical axis z.

   (b) By using the conservation laws for the angular momentum components ℓ_z and ℓ_3, eliminate the angular velocities $\dot{\Phi}$ and $\dot{\Psi}$ from the equation above.

   (c) Now turn to numerical solutions of the Euler-Lagrange equations for the top, with Mathematica or with another math package, employing exemplary values for the constants in the equations, i.e. $m\,g\,d/I_\perp$ (in s^{-2}), $\ell_z/I_\perp$ and $\ell_3/I_\perp$ (in s^{-1}). First put the conserved angular momentum components to zero and solve the differential equation for Θ, now amounting to the equation for a physical pendulum. Note that Θ is here measured from the top rather than the bottom position of the pendulum. Plot $\Theta(t)$ for a range of times sufficient to show periodicity of the motion. In Mathematica, the set of commands might have the form:

mg=0.5;
Solution=NDSolve[{Theta"[t] == mg Sin[Theta[t]], Theta[0] == Pi/4, Theta'[0] == 0}, {Theta}, {t,0,20}];
Plot[Evaluate[Theta[t]]/.Solution], {t,0,20}]

(d) Next, set finite values for $\ell_z/I_\perp$ and $\ell_3/I_\perp$ and solve the Euler-Lagrange equation for Θ again, now including the nonzero inertial terms. The commands might be:
mg=0.5; lz=0.5; l3=3;
Solution=NDSolve[{Theta"[t] == ..., Theta[0] == 1.3, Theta'[0] == 0}, {Theta}, {t,0,20}];
Plot[Evaluate[Theta[t]/.Solution], {t,0,20}]
How does the range of Θ covered compare with the case of the physical pendulum pendulum? How is the periodicity for Θ changed? Try out different starting positions in Θ.

(e) Now add a solution of the differential equation for Φ and plot the angular trajectory $\Theta(\Phi)$ for the top. The set of commands might have the form:
Solution=NDSolve[{Theta"[t] == ..., Theta[0] == 1.3, Theta'[0] == 0, Phi'[t] == ..., Phi[0] == 0}, {Theta, Phi}, {t,0,20}];
ParametricPlot[Evaluate[Phi[t], Theta[t]/.Solution], {t,0,20}]
Again try out different starting positions in Θ.

Attach to the homework images showing the code and results of your calculations or, else, upload them to the Dropbox on Angel.