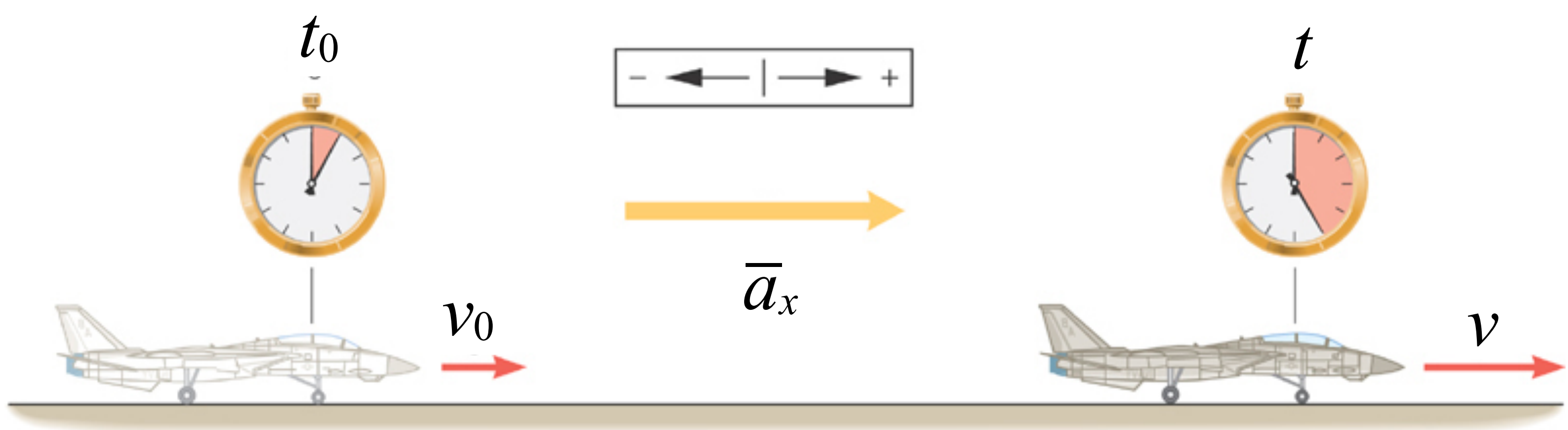
Chapter 2

Kinematics in One Dimension

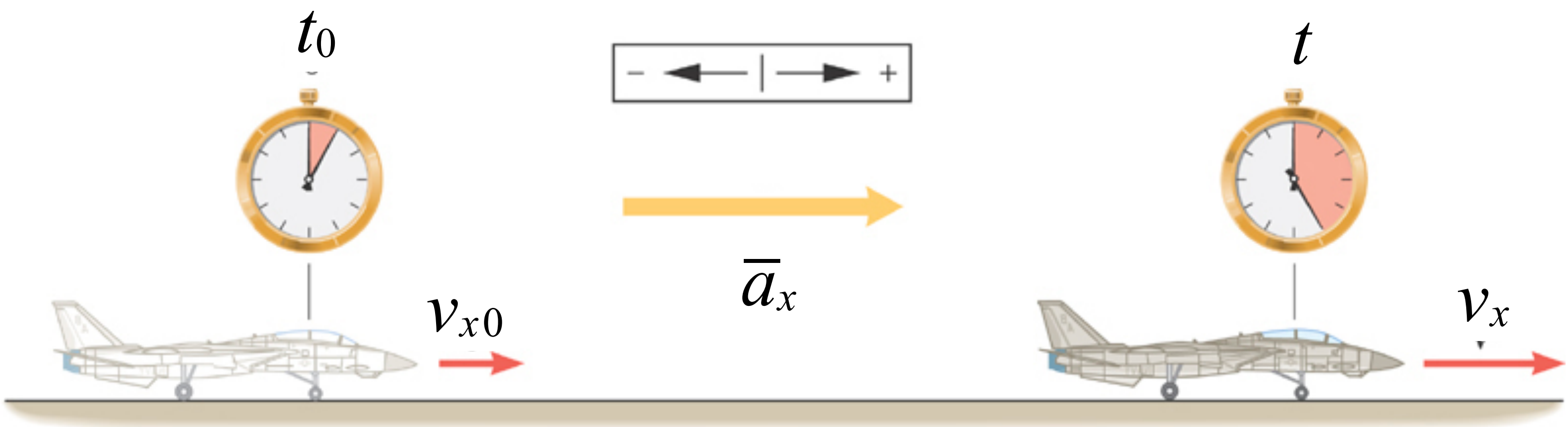
continued

2.3 Acceleration



2.3 Acceleration

Acceleration is the change in velocity divided by the time during which the change occurs.



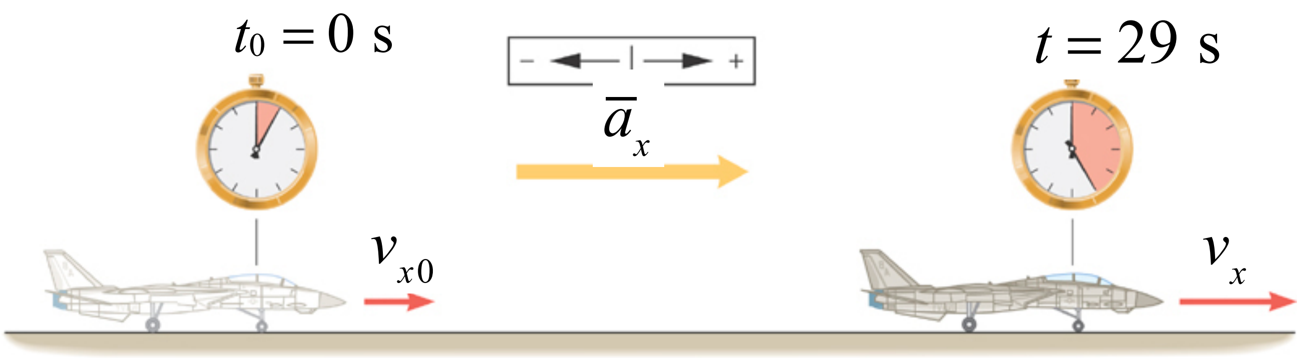
DEFINITION OF AVERAGE ACCELERATION

$$\bar{a}_x = \frac{v_x - v_{x0}}{t - t_0} = \frac{\Delta v_x}{\Delta t} \quad \left(\begin{array}{l}\text{average rate of change} \\ \text{of the velocity}\end{array}\right)$$

Note for the entire course:

$$\Delta(Anything) = \text{Final } Anything - \text{Initial } Anything$$

2.3 Acceleration



Example: Acceleration and increasing velocity of a plane taking off.

Determine the average acceleration of this plane's take-off.

What do we know?

$$t_0 = 0 \text{ s} \qquad t = 29 \text{ s}$$

$$v_{x0} = 0 \text{ m/s} \qquad v_x = 260 \text{ km/h}$$

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_x - v_{x0}}{t - t_0} = \frac{260 \text{ km/h} - 0 \text{ km/h}}{29 \text{ s} - 0 \text{ s}} = +9.0 \frac{\text{km/h}}{\text{s}}$$

This calculation of the <u>average</u> acceleration works even if the acceleration is not constant throughout the motion.

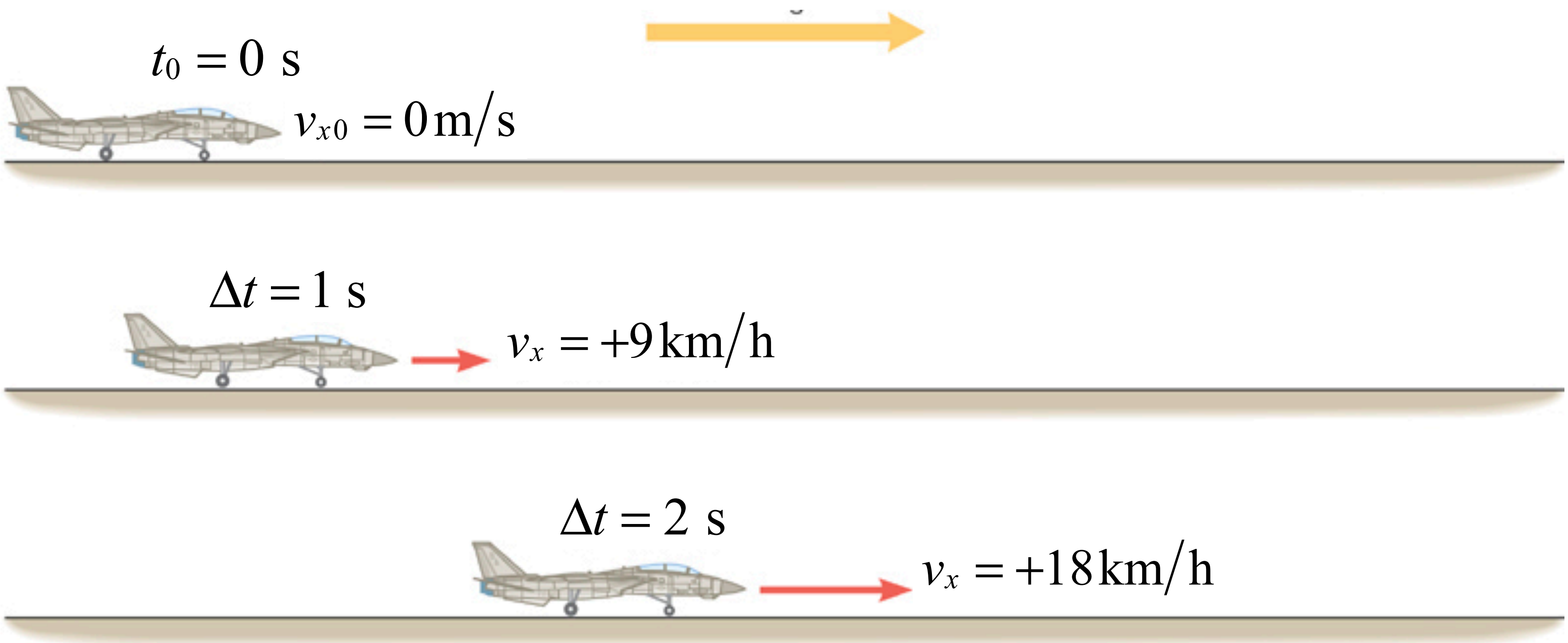
2.3 Acceleration (velocity increasing)

The jet accelerates at $\bar{a}_x = +9.0\dfrac{\text{km/h}}{\text{s}}$

Determine the velocity 1s and 2s after the start. Note: $v_{x0} = 0$

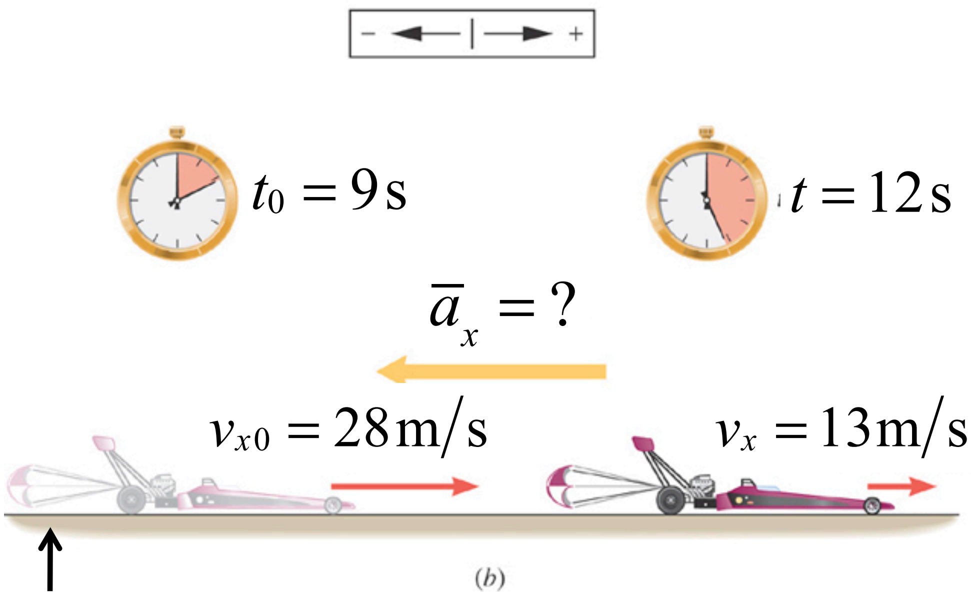
$$\Rightarrow \quad \bar{a}_x = \frac{v_x - v_{x0}}{\Delta t} \quad \Rightarrow \quad v_x = v_{x0} + \bar{a}_x \Delta t \quad \Rightarrow \quad \underline{v_x = \bar{a}_x \Delta t}$$

$$\bar{a}_x = +9.0\frac{\text{km/h}}{\text{s}}$$



2.3 Acceleration

Example: Average acceleration with Decreasing Velocity



Finish
Line

Dragster at the end of a run
Parachute deployed to slow safely.

Determine the acceleration of the dragster

$$\bar{a}_x = \frac{v_x - v_{x0}}{t - t_0} = \frac{13\,\text{m/s} - 28\,\text{m/s}}{12\text{ s} - 9\text{ s}} = -5.0\,\text{m/s}^2$$

Units: L/T^2

Positive accelerations: velocities become more positive.
Negative accelerations: velocities become more negative.
(Don't use the word deceleration)

2.3 Acceleration (velocity decreasing)

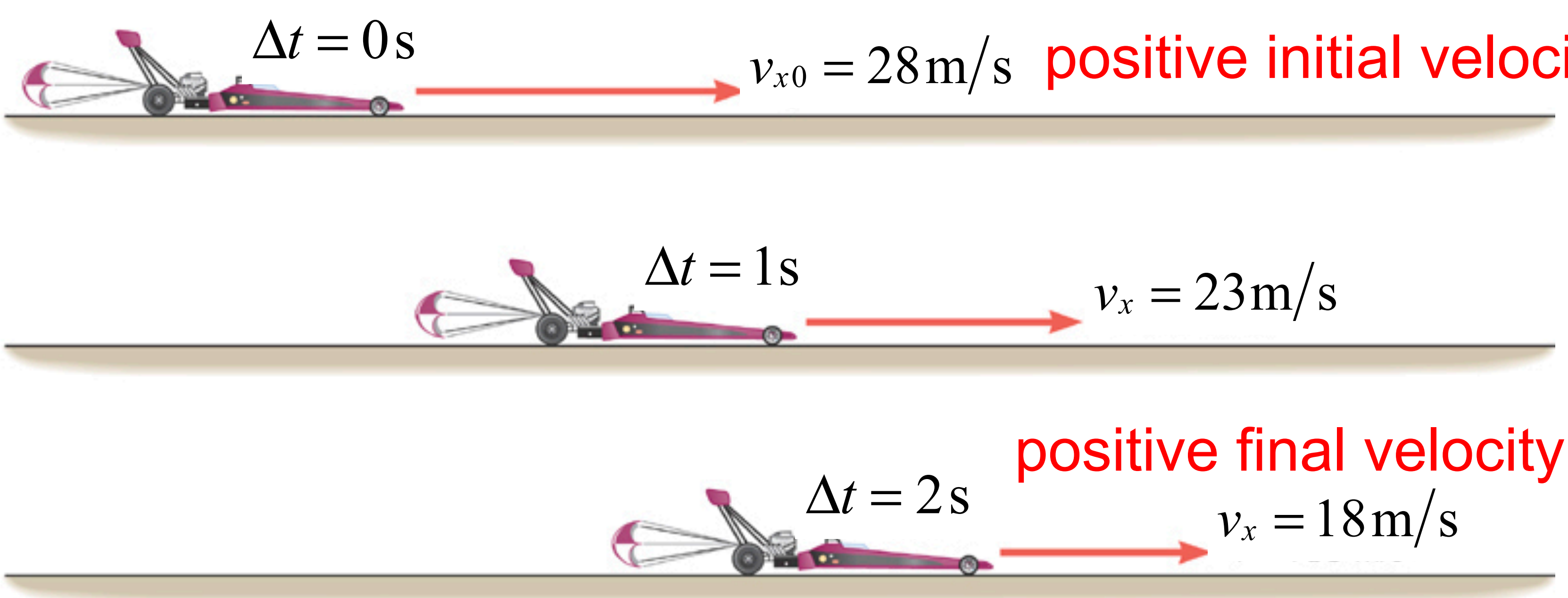
Parachute deployed to slow safely.

Acceleration is $\bar{a}_x = -5.0\,\mathrm{m/s^2}$ throughout.

What is velocity 1s and 2s after deployment?

$$\bar{a}_x = \frac{v_x - v_{x0}}{t - t_0} \quad \Rightarrow \quad \underline{v_x = v_{x0} + \bar{a}_x t}$$

$\bar{a}_x = -5.0\,\mathrm{m/s^2}$ acceleration is negative.



Clicker Question 2.2

A driver of a car applies the brakes when the speed of the car is 30.0 m/s and stops after 10.0 seconds. What was the average acceleration while braking?

a) –3.00 m/s

b) 300. m/s^2

c) –3.00 m/s^2

d) –30.0 m/s^2

e) – 300. m/s^2

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$\boxed{\text{c) –3.00 m/s}^2}$

d) –30.0 m/s^2

e) – 300. m/s^2

$$\bar{a}_x = \frac{\Delta v}{\Delta t} = \frac{v_x - v_{x0}}{t - t_0}$$

$$= \frac{0 - 30 \text{ m/s}}{10 \text{ s}} = -3.00 \text{ m/s}^2$$

2.4 *Equations of Kinematics for Constant Acceleration*

From now on unless stated otherwise

The clock starts when the object is at the initial position.

$$t_0 = 0$$

Simplifies things a great deal

$$\bar{v}_x = \frac{x - x_0}{t - t_0} \Rightarrow \bar{v}_x = \frac{\Delta x}{t}$$

Also, average is (initial + final)/2

$$\bar{v}_x = \frac{v_x + v_{x0}}{2}$$

$$\Downarrow$$

$$\underline{\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right) t}$$

2.4 *Equations of Kinematics for Constant Acceleration*

A <u>constant</u> acceleration (same value at at all times) can be determined at any time *t*.

No average bar needed

$$a_x = \frac{v_x - v_{x0}}{t - t_o}$$

$$a_x = \frac{v_x - v_{x0}}{t}$$

$$a_x t = v_x - v_{x0}$$

$$\underline{v_x = v_{x0} + at}$$

2.4 Equations of Kinematics for Constant Acceleration

Five kinematic variables:

Except for *t*, every variable has a direction and thus can have a positive or negative value.

1. displacement, Δx

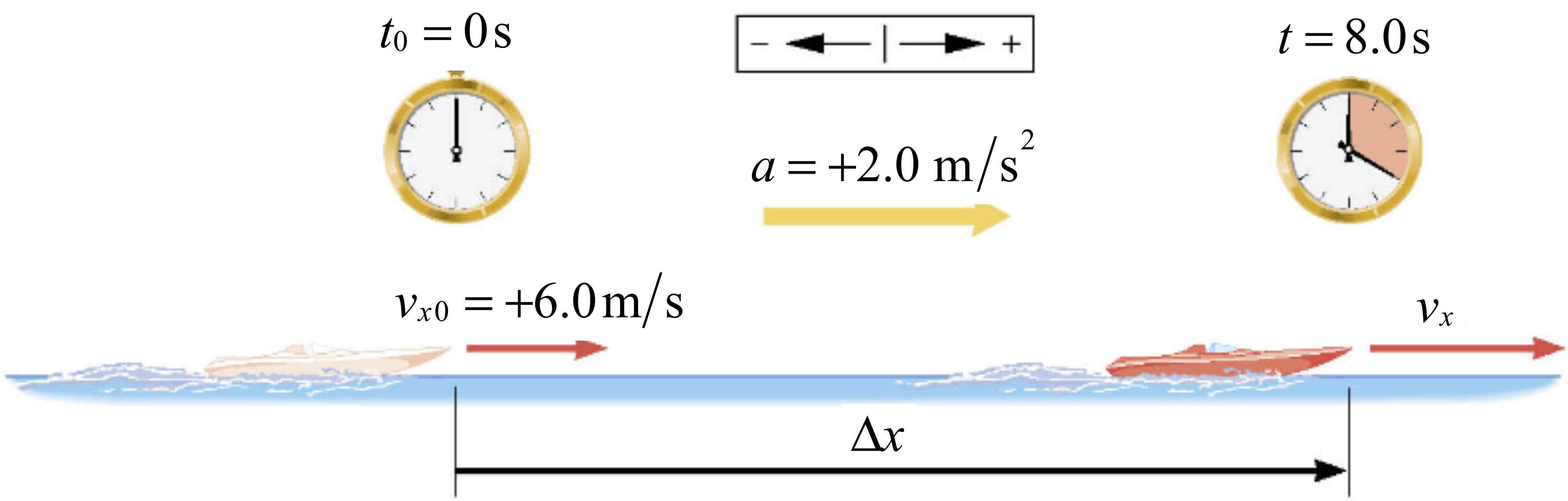
2. acceleration (constant), a_x

3. final velocity (at time *t*), v_x

4. initial velocity, v_{x0}

5. elapsed time, t

2.4 Equations of Kinematics for Constant Acceleration



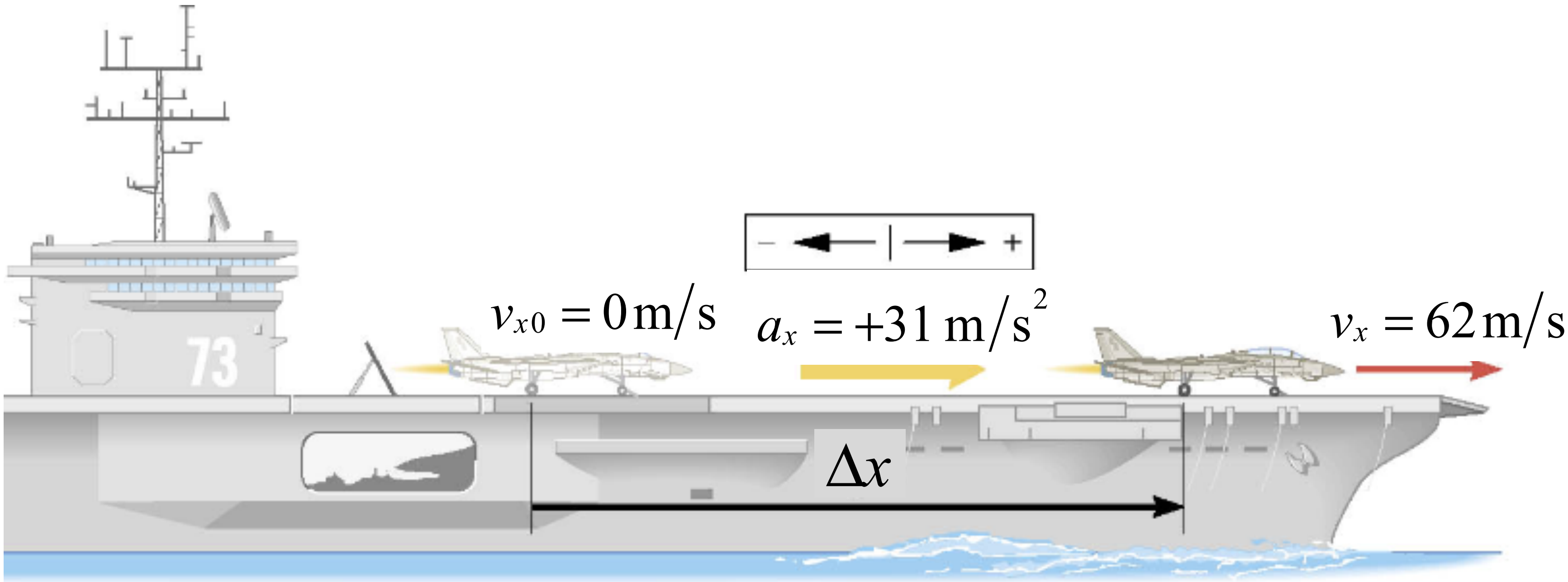
What is displacement after 8s of acceleration?

$$\Delta x = v_{x0}t + \tfrac{1}{2}at^2$$

$$= (6.0\,\text{m/s})(8.0\ \text{s}) + \tfrac{1}{2}(2.0\,\text{m/s}^2)(8.0\ \text{s})^2$$

$$= +110\ \text{m}$$

2.4 *Equations of Kinematics for Constant Acceleration*



(b)

Example: Catapulting a Jet

Find its displacement.

$$v_{x0} = 0\,\mathrm{m/s} \quad v_x = +62\,\mathrm{m/s} \quad a_x = +31\,\mathrm{m/s^2}$$

$$\Delta x = ??$$

2.4 *Equations of Kinematics for Constant Acceleration*

definition of acceleration

$$\boxed{a_x = \frac{v_x - v_{x0}}{t}} \Rightarrow t = \frac{v_x - v_{x0}}{a_x}$$

time that velocity changes

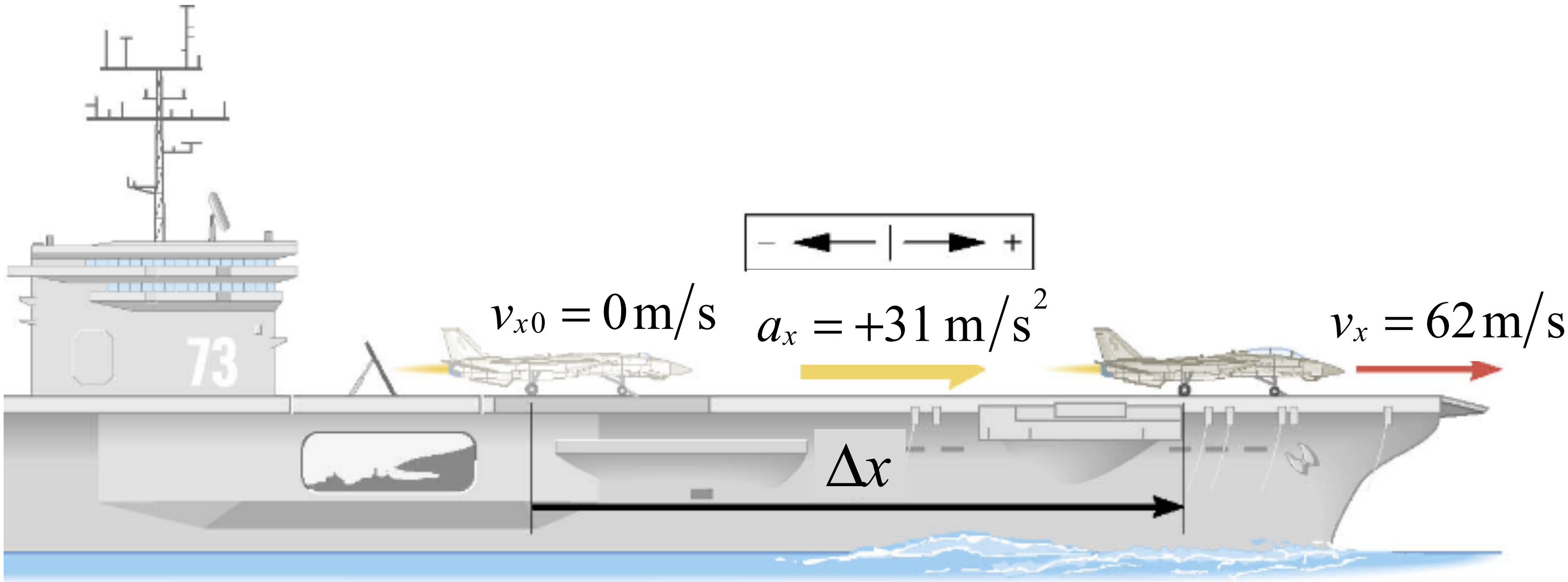
$$\boxed{\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right)t} = \tfrac{1}{2}\left(v_{x0} + v_x\right)\frac{\left(v_x - v_{x0}\right)}{a_x}$$

displacement = average velocity × time

Solve for final velocity

$$\underline{v_x^2 = v_{x0}^2 + 2a_x\Delta x} \Leftarrow \Delta x = \frac{v_x^2 - v_{x0}^2}{2a_x}$$

2.4 Equations of Kinematics for Constant Acceleration



(b)

$$\Delta x = \frac{v_x^2 - v_{x0}^2}{2a_x} = \frac{(62\,\text{m/s})^2 - (0\,\text{m/s})^2}{2(31\,\text{m/s}^2)} = +62\,\text{m}$$

Clicker Question 2.3

A car accelerates from rest to 100 m/s in 200 meters. What was the acceleration of the car in this motion?

Hint: use $v_x^2 = v_{x0}^2 + 2a_x \Delta x$

a) 12.5 m/s^2

b) 4.00 m/s^2

c) 0.25 m/s^2

d) 400 m/s^2

e) 25.0 m/s^2

Clicker Question 2.3

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a) $12.5\ \mathrm{m/s^2}$

b) $4.00\ \mathrm{m/s^2}$

c) $0.25\ \mathrm{m/s^2}$

d) $400\ \mathrm{m/s^2}$

$\boxed{\text{e) } 25.0\ \mathrm{m/s^2}}$

$$v_{x0} = 0; \quad v_x = 100\ \mathrm{m/s}; \quad \Delta x = 200\ \mathrm{m}$$

$$a_x = \frac{v_x^2}{2\Delta x} = \frac{10{,}000\ \mathrm{m^2/s^2}}{2(200\ \mathrm{m})} = 25.0\ \mathrm{m/s^2}$$

2.4 *Equations of Kinematics for Constant Acceleration*

Equations of Kinematics for Constant Acceleration

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right) t$$

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

$$\Delta x = v_{x0} t + \tfrac{1}{2} a_x t^2$$

Except for *t*, every variable has a direction and thus can have a positive or negative value.

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (–).

3. Write down the values that are given for any of the five kinematic variables.

4. Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

Clicker Question 2.4

A car starting at rest maintains a <u>constant acceleration a_x</u>. After a time t, its displacement and velocity are Δx and v_x. What are the displacement and velocity at the time $t' = 2t$?

a) $2\Delta x$ and $2v_x$

b) $2\Delta x$ and $4v_x$

c) $4\Delta x$ and $2v_x$

d) $4\Delta x$ and $4v_x$

e) $8\Delta x$ and $4v_x$

Clicker Question 2.4

A car starting at rest maintains a <u>constant acceleration a_x</u>.

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What are the displacement and velocity at time $2t$?

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b) $2\Delta x$ and $4v_x$

c) $\boxed{4\Delta x \text{ and } 2v_x}$

d) $4\Delta x$ and $4v_x$

e) $8\Delta x$ and $4v_x$

acceleration, a_x, is the same at all times.

$v_{x0} = 0, \quad x, v_x$ at time t, $\quad x', v'_x$ at time $\underline{t' = 2t}$

$$\Delta x = \tfrac{1}{2} a_x t^2 \quad \Big| \quad \Delta x' = \tfrac{1}{2} a_x t'^2$$

$$= \tfrac{1}{2} a_x (2t)^2 = \tfrac{1}{2} a_x (4) t^2$$

$$= (4)(\tfrac{1}{2} a_x t^2) = (4)\Delta x$$

$$v_x = a_x t \quad \Big| \quad v'_x = a_x t'$$

$$= a_x (2t) = (2)(a_x t)$$

$$= (2) v_x$$

2.5 *Freely Falling Bodies*

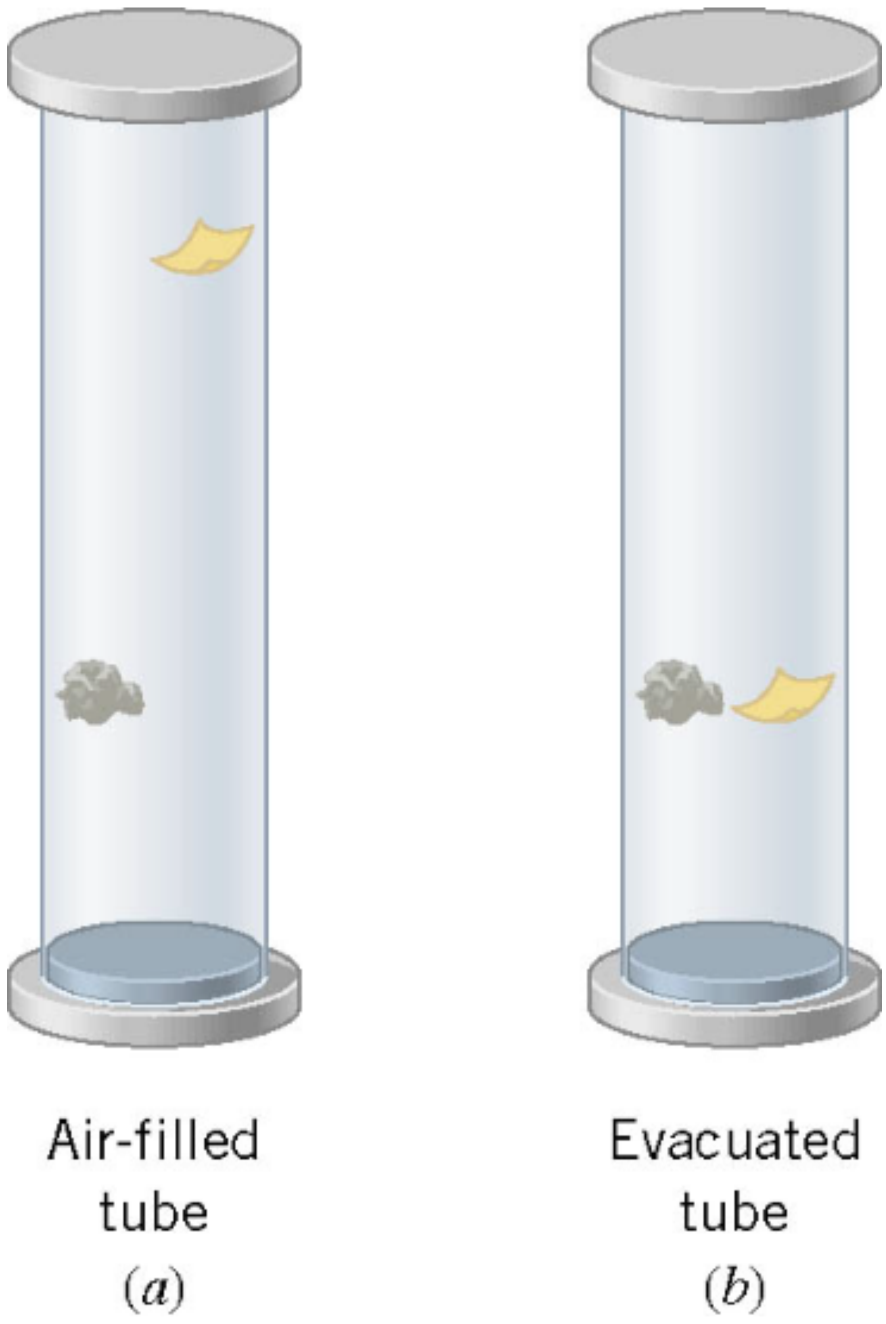
For vertical motion, we will replace the *x* label with *y* in all kinematic equations, and use upward as positive.

In the absence of air resistance, it is found that all bodies at the same location above the Earth fall vertically with the same acceleration. If the distance of the fall is small compared to the radius of the Earth, then the acceleration remains essentially constant throughout the descent.

This idealized motion is called *free-fall* and the acceleration of a freely falling body is called the *acceleration due to gravity*, and the acceleration is downward or negative.

$$a_y = -g = -9.81\,\mathrm{m/s^2} \quad \text{or} \quad -32.2\,\mathrm{ft/s^2}$$

2.5 *Freely Falling Bodies*



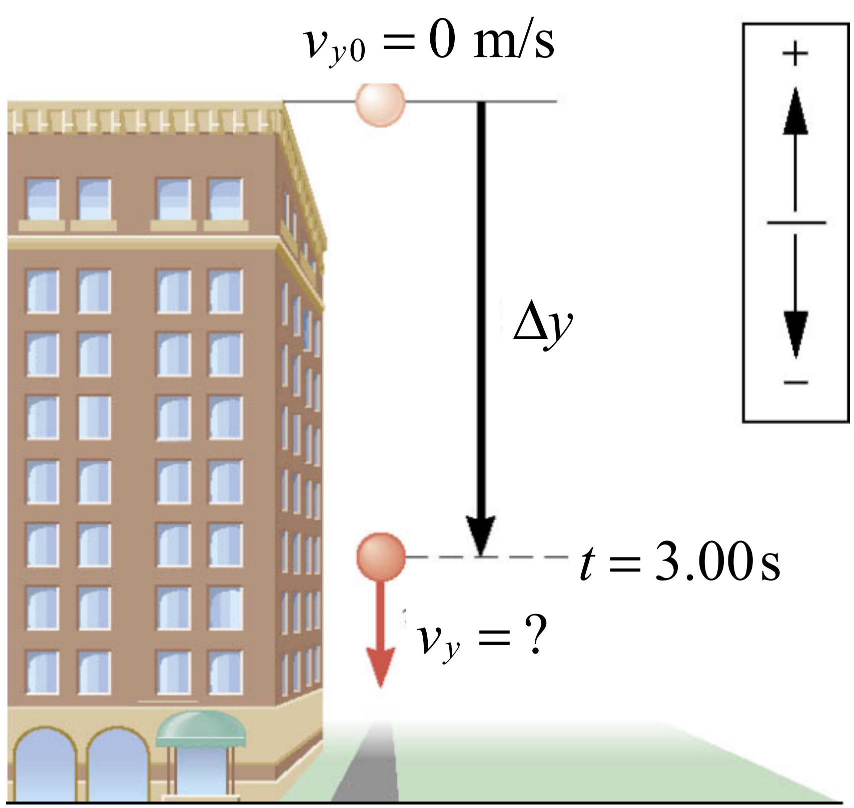
acceleration due to gravity.

$$a_y = -g = -9.80\,\mathrm{m/s^2}$$

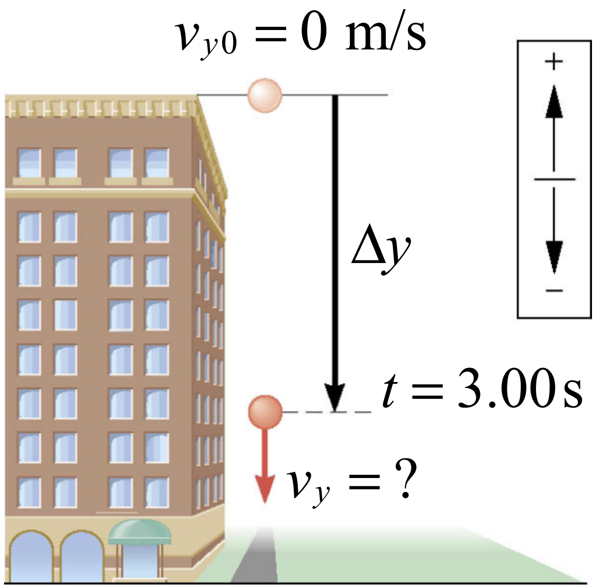
2.5 *Freely Falling Bodies*

Example: A Falling Stone

A stone is dropped from the top of a tall building. After 3.00s of free fall, what is the displacement, Δy of the stone?



2.5 Freely Falling Bodies



Δy	a_y	v_y	v_{y0}	t
?	-9.80 m/s^2		0 m/s	3.00 s

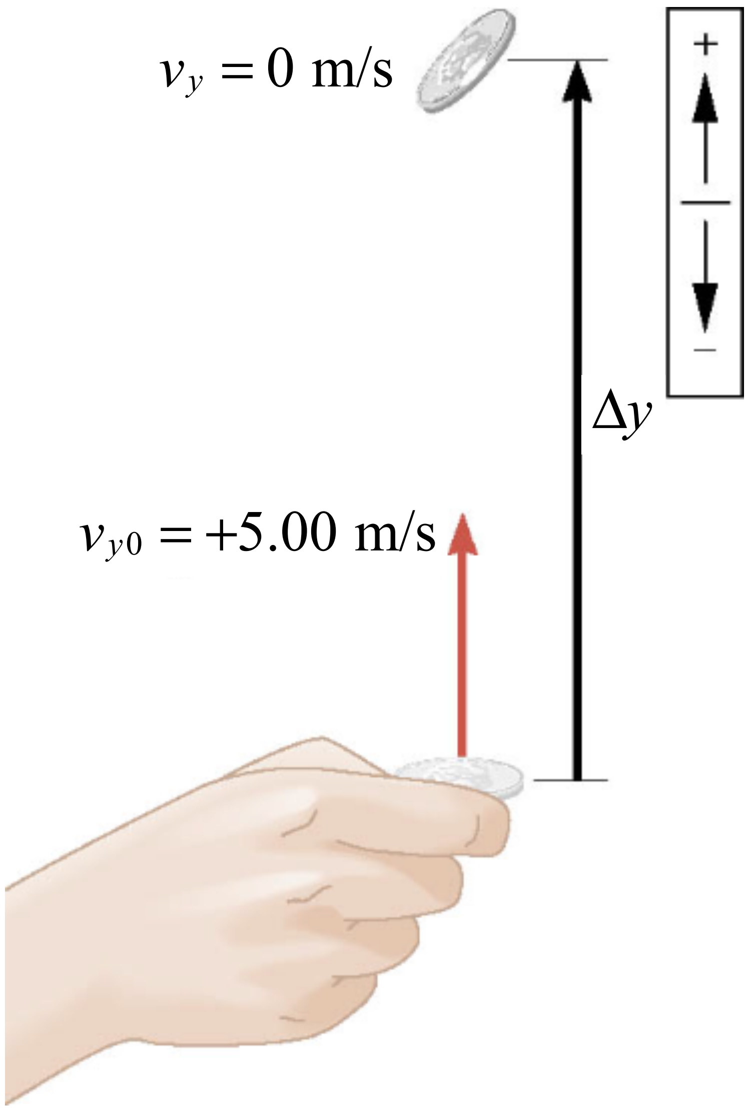
$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

$$= (0\,\text{m/s})(3.00\ \text{s}) + \tfrac{1}{2}(-9.80\,\text{m/s}^2)(3.00\ \text{s})^2$$

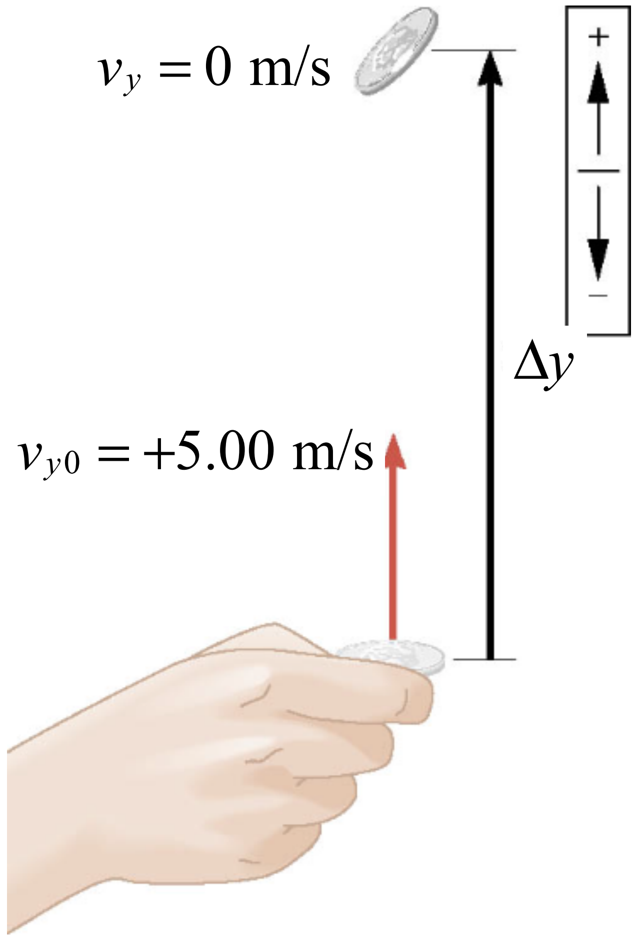
$$= -44.1\ \text{m}$$

Example: How High Does it Go?

The referee tosses the coin up with an initial speed of 5.00m/s. In the absence if air resistance, how high does the coin go above its point of release?



2.5 Freely Falling Bodies



Δy	a_y	v_y	y_{y0}	t
?	−9.80 m/s^2	0 m/s	+5.00 m/s	

$$v_y^2 = v_{y0}^2 + 2a_y\Delta y \quad \Longrightarrow \quad \Delta y = \frac{v_y^2 - v_{y0}^2}{2a_y}$$

$$\Delta y = \frac{v_y^2 - v_{y0}^2}{2a_y} = \frac{(0\,\mathrm{m/s})^2 - (5.00\,\mathrm{m/s})^2}{2(-9.80\,\mathrm{m/s^2})} = 1.28\ \mathrm{m}$$

2.5 *Freely Falling Bodies*

Conceptual Example 14 Acceleration Versus Velocity

There are three parts to the motion of the coin.

1) On the way up, the coin has an upward-pointing velocity with a decreasing magnitude.
2) At that time the coin reaches the top of its path, the coin has an instantaneously zero velocity.
3) On the way down, the coin has a downward-pointing velocity with an increasing magnitude.

In the absence of air resistance, does the acceleration vector of the coin, like the velocity, change from one part to another?

Clicker Question 2.5

During the free flight (no air resistance) of a coin thrown upward (+), what are the values of the acceleration at these times during the motion ?

	On the way up	At the top	On the way down
	$a_y =$	$a_y =$	$a_y =$
a)	-9.80m/s^2	-9.80m/s^2	-9.80m/s^2
b)	$+9.80\text{m/s}^2$	$0.0\ \text{m/s}^2$	$+9.80\text{m/s}^2$
c)	$+9.80\text{m/s}^2$	$0.0\ \text{m/s}^2$	-9.80m/s^2
d)	-9.80m/s^2	$0.0\ \text{m/s}^2$	-9.80m/s^2
e)	$+9.80\text{m/s}^2$	$+9.80\text{m/s}^2$	$+9.80\text{m/s}^2$

Clicker Question 2.5

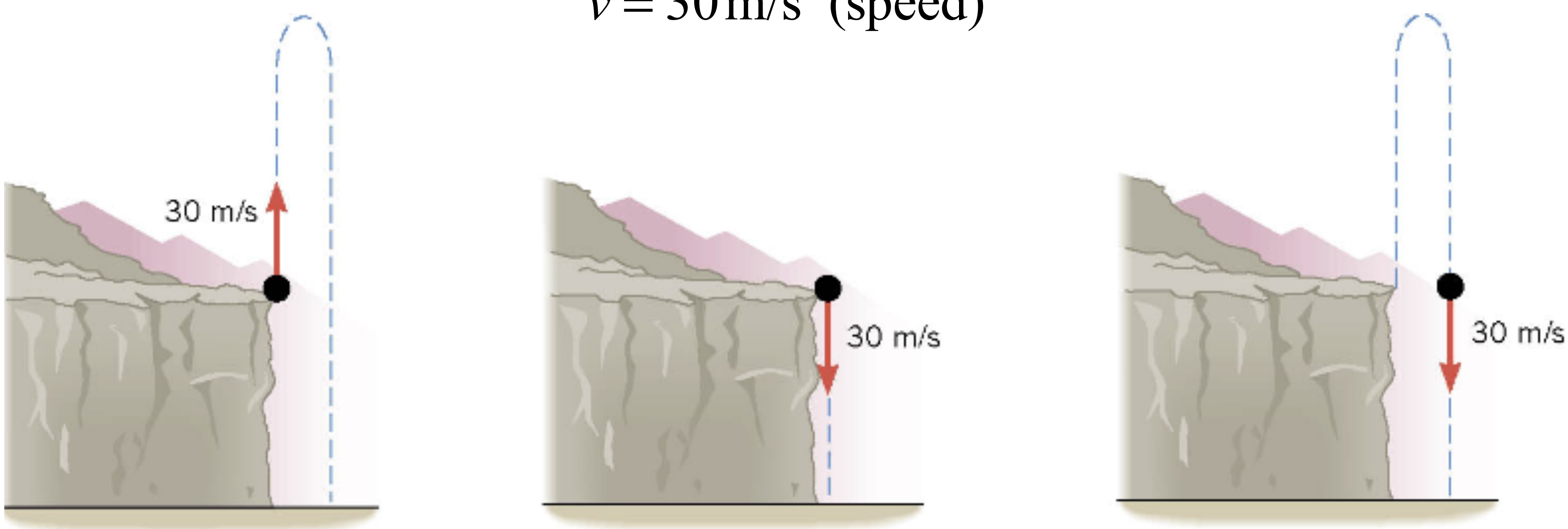
During the free flight (no air resistance) of a coin thrown upward (+), what are the values of the acceleration at these times during the motion ?

	On the way up	At the top	On the way down
	$a_y =$	$a_y =$	$a_y =$
a)	-9.80m/s^2	-9.80m/s^2	-9.80m/s^2
b)	$+9.80\text{m/s}^2$	0.0 m/s^2	$+9.80\text{m/s}^2$
c)	$+9.80\text{m/s}^2$	0.0 m/s^2	-9.80m/s^2
d)	-9.80m/s^2	0.0 m/s^2	-9.80m/s^2
e)	$+9.80\text{m/s}^2$	$+9.80\text{m/s}^2$	$+9.80\text{m/s}^2$

Conceptual Example: Taking Advantage of Symmetry

Does the pellet in part *b* strike the ground beneath the cliff with a smaller, greater, or the same speed as the pellet in part *a*?

$$v = 30\,\mathrm{m/s} \quad \text{(speed)}$$

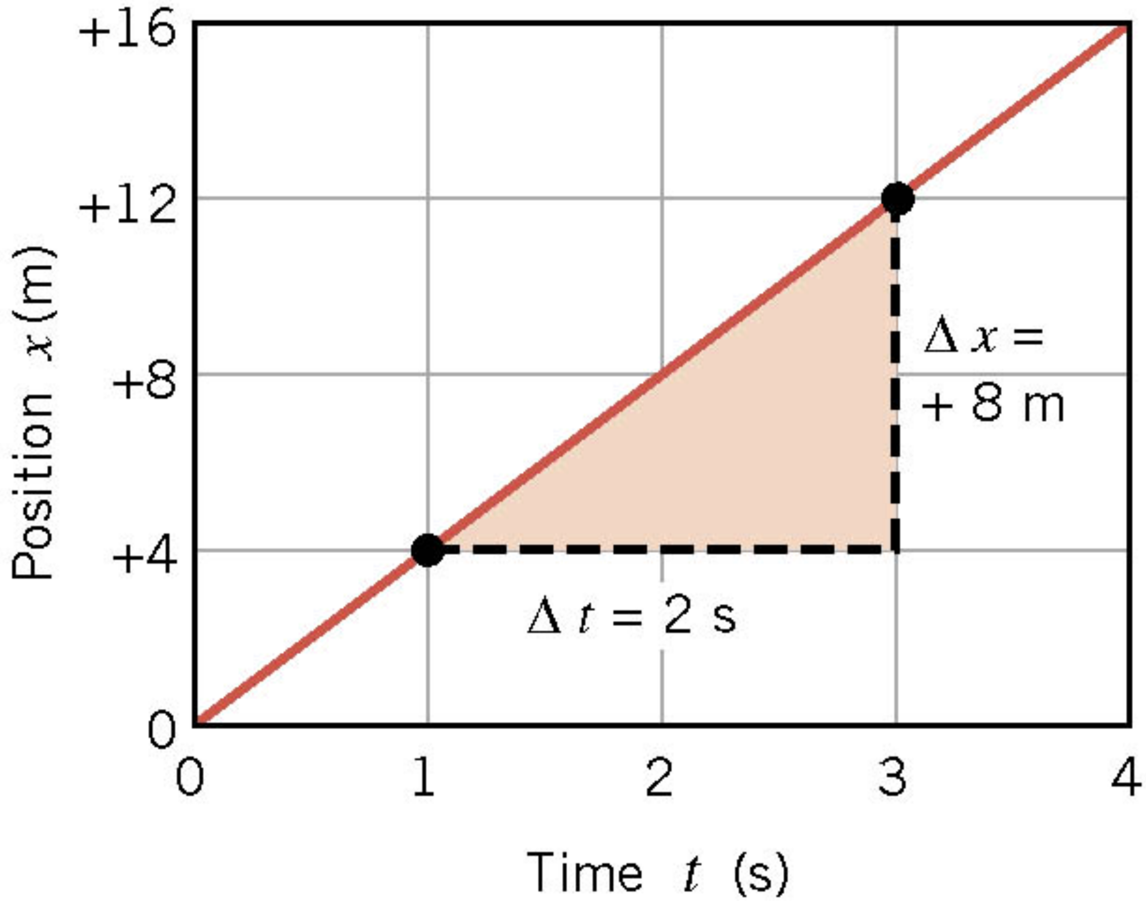


(*a*) (*b*) (*c*)

$$v_{y0} = +30\,\mathrm{m/s} \qquad v_{y0} = -30\,\mathrm{m/s} \qquad v_{y} = -30\,\mathrm{m/s}$$

2.5 Graphical Analysis of Velocity and Acceleration

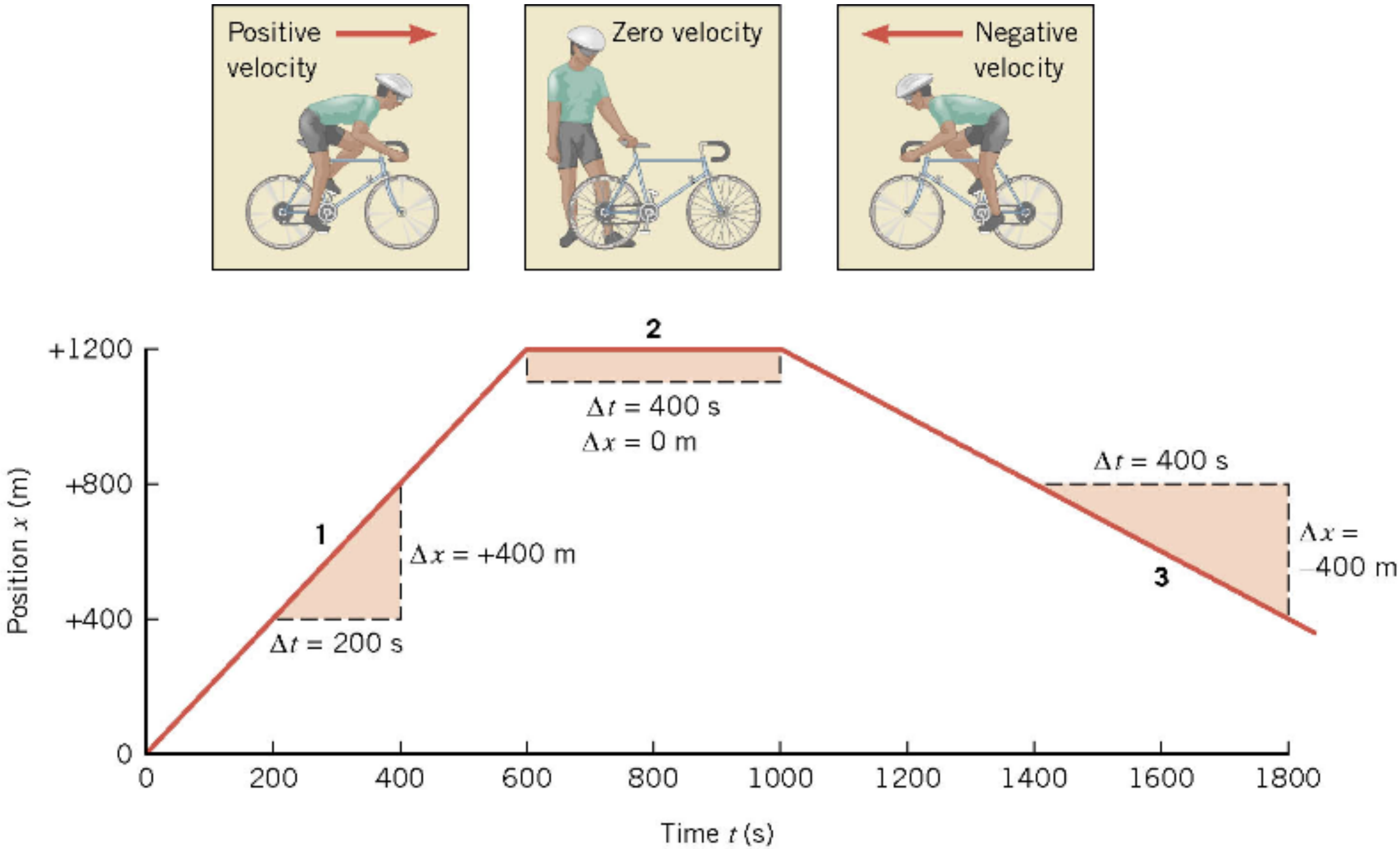
Graph of position vs. time.



$$\text{Slope} = \frac{\Delta x}{\Delta t} = \frac{+8\text{ m}}{2\text{ s}} = +4\,\text{m/s}$$

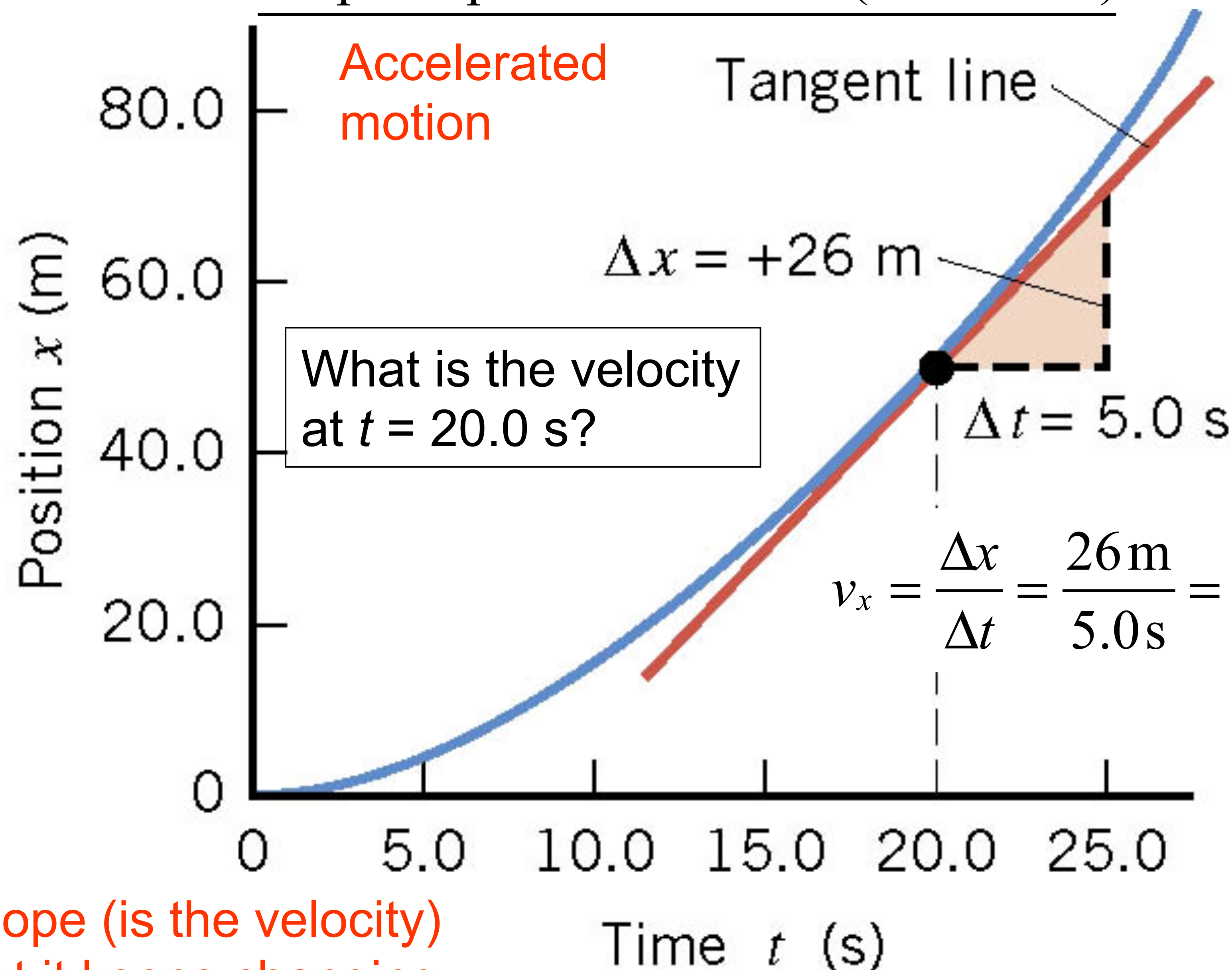
The same slope at all times.
This means constant velocity!

2.5 Graphical Analysis of Velocity and Acceleration



2.5 *Graphical Analysis of Velocity and Acceleration*

Graph of position vs. time (blue curve)

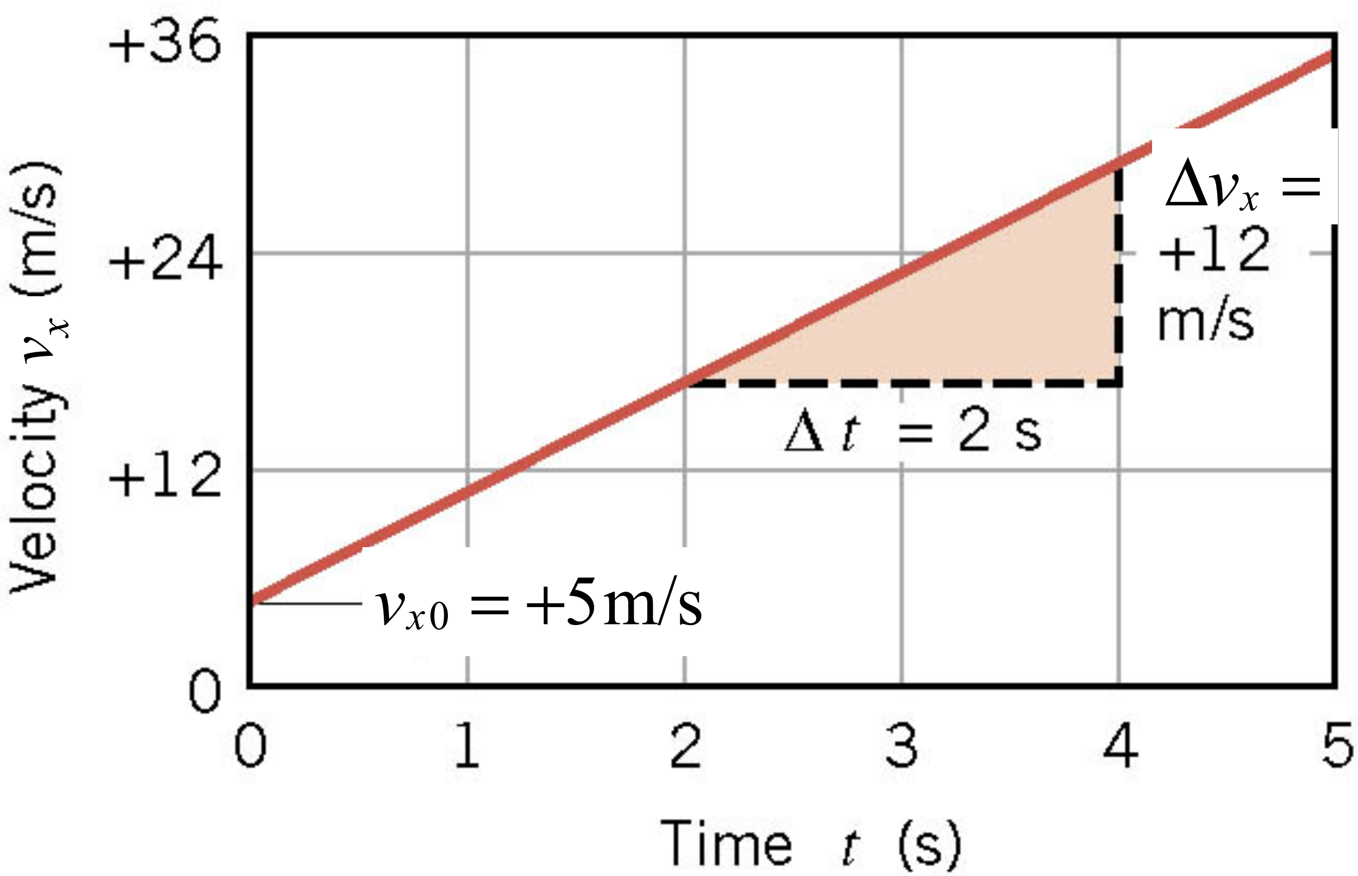


$$v_x = \frac{\Delta x}{\Delta t} = \frac{26\,\text{m}}{5.0\,\text{s}} = 5.2\,\text{m/s}$$

Slope (is the velocity) but it keeps changing.

2.5 Graphical Analysis of Velocity and Acceleration

Graph of velocity vs. time (red curve)



$$\text{Slope} = \frac{\Delta v_x}{\Delta t} = \frac{+12\ \mathrm{m/s}}{2\ \mathrm{s}} = +6\,\mathrm{m/s^2}$$

$$a_x = +6\,\mathrm{m/s^2}$$

The same slope at all times. This means a constant acceleration!

2.5 *Summary equations of kinematics in one dimension*

Equations of Kinematics for <u>Constant Acceleration</u>

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right) t$$

$$v_x^2 = v_{x0}^2 + 2 a_x \Delta x$$

$$\Delta x = v_{x0} t + \tfrac{1}{2} a_x t^2$$

Except for *t*, every variable has a direction and thus can have a positive or negative value.

For vertical motion replace x with y