Experiment 4

RC Circuits

4.1 Objectives

- Observe and qualitatively describe the charging and discharging (decay) of the voltage on a capacitor.

- Graphically determine the time constant τ for the decay.

4.2 Introduction

We continue our journey into electric circuits by learning about another circuit component, the capacitor. Like the name implies, "capacitors" have the physical capability of storing electrical charge. Many things can be accidental capacitors. Most electrical components have some amount of capacitance within them, but some devices are specifically manufactured to do the sole job of being capacitors by themselves.[1] The capacitors in today's lab will lose their charge rather quickly, but still slowly enough for humans to watch it happen. Capacitors in electrical circuits can have very different characteristic times for charging and discharging.

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[1]Batteries, in fact, are actually capacitors that discharge very, very slowly (they take a while to lose their charge) and can lose their overall effectiveness through that loss.

4.3   Key Concepts

As always, you can find a summary online at HyperPhysics[2]. Look for keywords: electricity and magnetism (capacitor, charging of a capacitor)

To play with building circuits with capacitors, or to get a head start on trying out the circuits for today, run the computer simulation at http://phet.colorado.edu/en/simulation/circuit-construction-kit-ac.

4.4   Theory

The Capacitor

A capacitor is a device that stores electrical charge. The simplest kind is a "parallel-plate" capacitor: two flat metal plates placed nearly parallel and separated by an insulating material such as dry air, plastic or ceramic. Such a device is shown schematically in Fig. 4.1.

Here is a description of how a capacitor stores electrical energy. If we connect the two plates to a battery in a circuit, as shown in Fig. 4.1, the battery will drive charges around the circuit as an electric current. When the charges reach the plates they can't go any further because of the insulating gap so they collect on the plates, one plate becoming positively charged and the other negatively charged. This slow buildup of electric charge actually begins to resist the addition of more charge as a voltage

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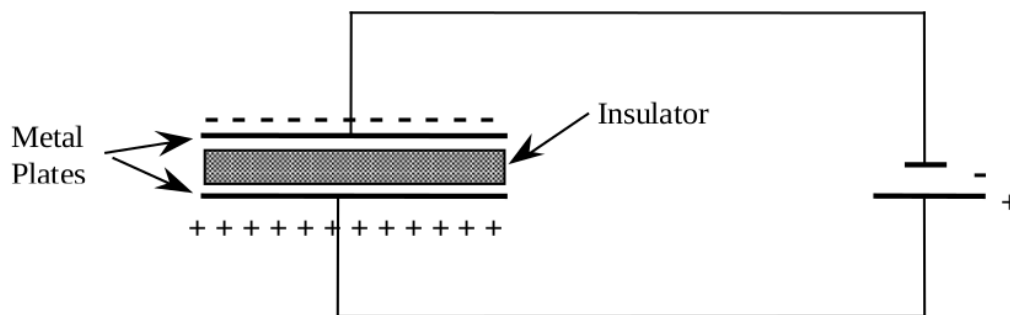[2]http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

Figure 4.1: Schematic of a capacitor in a circuit with a battery.

begins to build across the plates, thus opposing the action of the battery. As a consequence, the current flowing in the circuit gets less and less (i.e. it decays), falling to zero when the "back-voltage" on the capacitor is exactly equal and opposite to the battery voltage.

If we were to quickly disconnect the battery without touching the plates, the charge would remain on the plates. You could literally walk around with this "stored" charge. Because the two plates have different signs of electric charge, there is a net electric field between the two plates. Hence, there is a voltage difference across the plates. If, some time later, we connect the plates again in a circuit, say this time with a light bulb in place of the battery, the plates will discharge through the bulb: the electrons on the negatively charged plate will move around the circuit through the bulb to the positive plate until all the charges are equalized. During this short discharge period a current flows and the bulb will light up. The capacitor stored electrical energy from its original charging by the battery and then discharged it through the light bulb. The speed with which the discharge process (and conversely the charging process) can take place is limited by the resistance R of the circuit connecting the plates and by the capacitance C of the capacitor (a measure of its ability to hold charge).

RC Circuit

An RC circuit is a circuit with a resistor and a capacitor in series connected to a voltage source such as a battery.

As with circuits made up only of resistors, electrical current can flow in this RC circuit with one modification. A battery connected in series with a resistor will produce a constant current. The same battery in series with a capacitor will produce a time-varying current, which decays gradually to zero as the capacitor charges up. If the battery is removed and the circuit reconnected without the battery, a current will flow (for a short time) in the **opposite** direction as the capacitor "discharges." A measure of how long these transient currents last in a given circuit is given by the **time constant** τ.

The time it takes for these transient currents to decay depends on the resistance (R) and capacitance (C). The resistor resists the flow of current; it thus slows down the decay. The capacitance measures "capacity" to hold charge: like a bucket of water, a larger capacity container takes longer to empty than a smaller capacity container. Thus, the time constant of

the circuit gets larger for larger R and C. In detail, using the units of capacitance which are "farads",

$$\tau(\text{seconds}) = R(\text{ohms}) \times C(\text{farads}) \tag{4.1}$$

Isn't it strange that ohms times farads equals seconds? Like many things in the physical world, it is just not intuitive. We can at least show this by breaking the units down. From Ohm's Law, $R = V/I$. Current is the amount of charge flowing per time, so $I = Q/t$. Capacitance is proportional to how much charge can be stored per voltage applied, or $C = Q/V$. So,

$$
\begin{aligned}
RC &= R \times C \\
&= \frac{V}{I} \times \frac{Q}{V} \\
&= \frac{Q}{I} \\
&= \frac{Q}{Q/t} \\
&= t
\end{aligned}
$$

The current does not fall to zero at time τ. Instead, τ is the time it takes for the voltage of the discharging capacitor to drop to 37% of its original value. It takes 5 to 6 τs for the current to decay to essentially zero amps. Just as it takes time for the charged capacitor to discharge, it takes time to charge the capacitor. Due to the unavoidable presence of resistance in the circuit, the charge on the capacitor and its stored energy only approaches an essentially final (steady-state) value after a period of several times the time constant of the circuit elements employed.

(a) Charging

(b) Discharging

Figure 4.2: Schematics of charging and discharging a capacitor.

Charging and discharging the RC circuit

Charging

Initially, a capacitor is in series with a resistor and disconnected from a battery so it is uncharged. If a switch is added to the circuit but is open, no current flows. Then, the switch is closed as in Fig. 4.2(a). Now the capacitor will charge up and its voltage will increase. During this time, a current will flow producing a voltage across the resistor according to Ohm's Law, $V = IR$. As the capacitor is charging up the current is actually decreasing due to the stored charge on the capacitor producing a voltage that increasingly opposes the current. Since the current through the resistor (remember the resistor and capacitor are in series so the same current flows through both) is decreasing then by Ohm's law so is the resistor's voltage. Fig. 4.3 graphs the behavior of the voltage across the capacitor and resistor as a function of the time constant, τ, of the circuit for a charging capacitor. Notice that as the capacitor is charging, the voltage across the capacitor increases but the voltage across the resistor decreases.

While the capacitor is charging, the voltages across the capacitor, V_C, and resistor, V_R, can be expressed as

$$V_C(t) = V_0 \left(1 - e^{\frac{-t}{\tau}}\right) \qquad (4.2) \qquad\qquad V_R(t) = V_0 e^{\frac{-t}{\tau}} \qquad (4.3)$$

where e is the base of the natural logarithm and V_0 is the initial voltage. The value of e is approximately 2.718. Remember that the time constant τ of a circuit depends on capacitance and resistance as $\tau = RC$. When the time t is exactly equal to 1 time constant τ then $t = \tau$ and the previous equations become

$$V_C = V_0 \left(1 - e^{-1}\right) \qquad (4.4) \qquad V_R = V_0 \left(e^{-1}\right) \qquad (4.5)$$
$$\approx 0.63V_0 \qquad\qquad\qquad \approx 0.37V_0$$

This means that after $t = \tau$ seconds, the capacitor has been charged to 63% of its final value and the voltage across the resistor has dropped to 37% of its peak (initial) value. After a very long time, the voltage across the capacitor will essentially be equal to the battery's voltage, V_0, and the voltage across the resistor will be (for all practical purposes) zero.
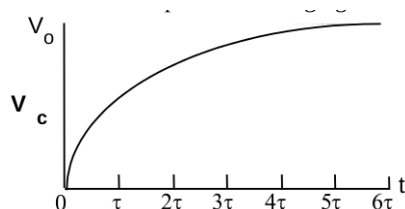
(a) Voltage across the capacitor V_C.
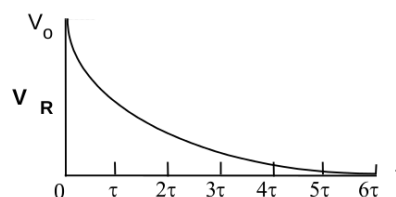
(b) Voltage across the resistor V_R.

Figure 4.3: Voltage in RC circuit components as a function of time for a charging capacitor where the time constant $\tau = RC$.

Discharging

If we flip the switch to the position shown in Fig. 4.2(b), so that the battery is no longer included in the circuit, we will discharge the capacitor. Now the charge stored on the capacitor is free to leave the plates and will cause a current to flow. The current will be the largest at the beginning, $t = 0$, and will decay away as charge leaves the capacitor's plates. Since the current is decreasing the voltage difference across the resistor is also decreasing (Ohm's law again). Fig. 4.4 graphs the behavior of the voltage across the capacitor and resistor as a function of the time constant, τ, of the circuit for a discharging capacitor. For the case when the capacitor is discharging notice that both the voltage across the capacitor and the resistor are decaying to zero. It is critical to remember that the total voltage between the capacitor and the resistor must add up to the applied voltage (Kirchhoff's loop law). When the circuit is disconnected from the power supply, then
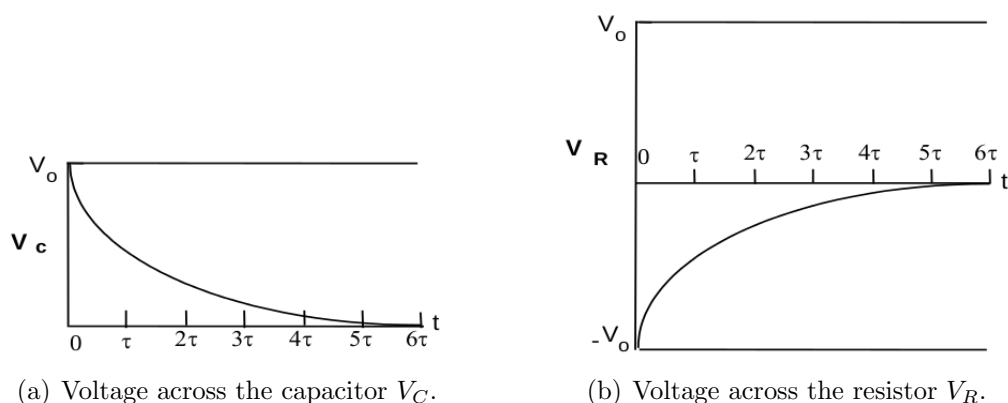
(a) Voltage across the capacitor V_C.

(b) Voltage across the resistor V_R.

Figure 4.4: Voltage in RC circuit components as a function of time for a discharging capacitor where the time constant $\tau = RC$.

the sum of the voltages must be zero so the graph of the voltage across the resistor must be increasing from $-V_0$ to zero.

For the case when the capacitor is discharging, the voltages across the capacitor, V_C, and resistor, V_R, are given by

$$V_C = V_0 e^{\frac{-t}{\tau}} \qquad (4.6) \qquad\qquad V_R = -V_0 e^{\frac{-t}{\tau}} \qquad (4.7)$$

Compare these equations to the ones given for when the capacitor is charging and make sure you understand the differences.

It is usually easier to interpret a graph when the plot gives a straight line. For this lab we want to plot our voltages versus time, however, all of our equations for V_C and V_R involve exponentials. In order to get a straight line on our graphs we will use the logarithm function to find an equation that looks like a straight line. First, we divide the voltage across the capacitor, V_C, by the initial voltage, V_0, giving

$$\frac{V_C}{V_0} = e^{\frac{-t}{\tau}} \qquad\qquad (4.8)$$

Then we calculate the natural logarithm[3]

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[3]The natural logarithm of x, $\ln(x)$, asks the question, "e raised to what power equals x?" For example, to find $\ln\left(e^2\right)$, we ask, "e raised to what power equals e^2?" e raised to the power 2 equals e^2, so $\ln\left(e^2\right) = 2$. We can thus use the natural logarithm to get just the power of e in an equation.

of both sides, yielding

$$\ln\left(\frac{V_C}{V_0}\right) = \ln\left(e^{\frac{-t}{\tau}}\right)$$

$$\ln\left(\frac{V_C}{V_0}\right) = \frac{-t}{\tau} \tag{4.9}$$

This last equation is a straight line, even though it may not look like one at first glance. If $y = mx + b$ is the equation of our straight line, then: 1) time t is the independent variable x, 2) the term with the voltage V_C that changes with time is the dependent variable y, and 3) the slope m is everything that is multiplied by the independent variable t. In this case, there is no term added to the t term, so the y-intercept b is zero. (You'll need this for Question 1).

Here is a more visual comparison between the equation for a straight line and Eq. 4.9 which is described in words above.

$$y = mx + b$$
$$\ln\left(\frac{V_C}{V_0}\right) = \frac{-t}{\tau} + 0$$

Matching up variables we see that

$$y = \ln\left(\frac{V_C}{V_0}\right) \qquad\qquad m = -\frac{1}{\tau}$$

$$x = t \qquad\qquad b = 0$$

Repeated charging and discharging

If we now repeat the charging and discharging process by alternating the switch position every 6τ seconds, the voltages will look like Fig. 4.5. Notice that the voltage across the capacitor does not immediately match the voltage at the power supply, but rather it has some delay to get there. On the
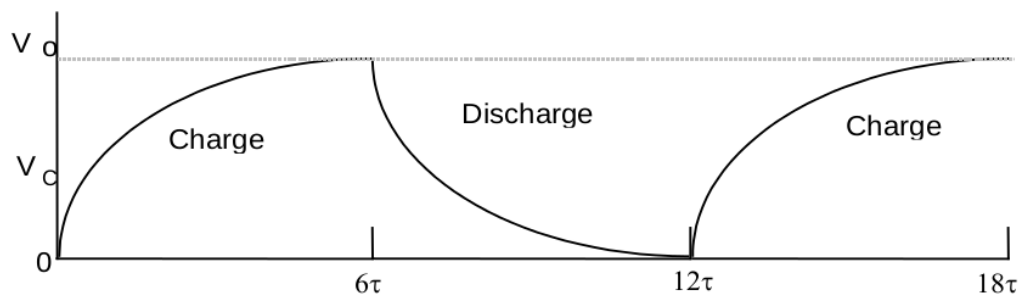
other hand, the voltage across the resistor changes very quickly to match the power supply voltage, but then dies down over time (as the capacitor gains charge and slows the current down).
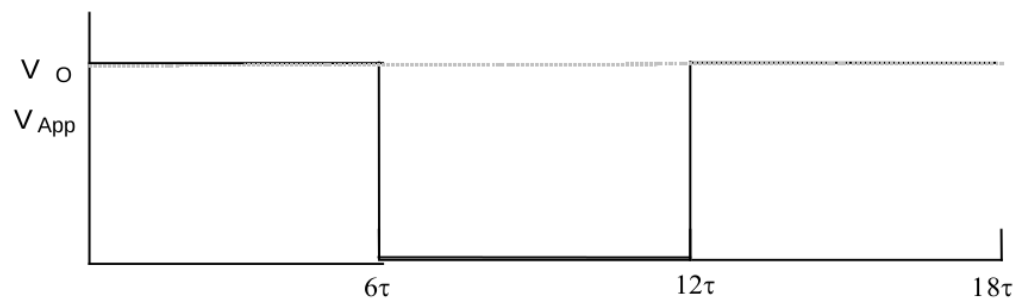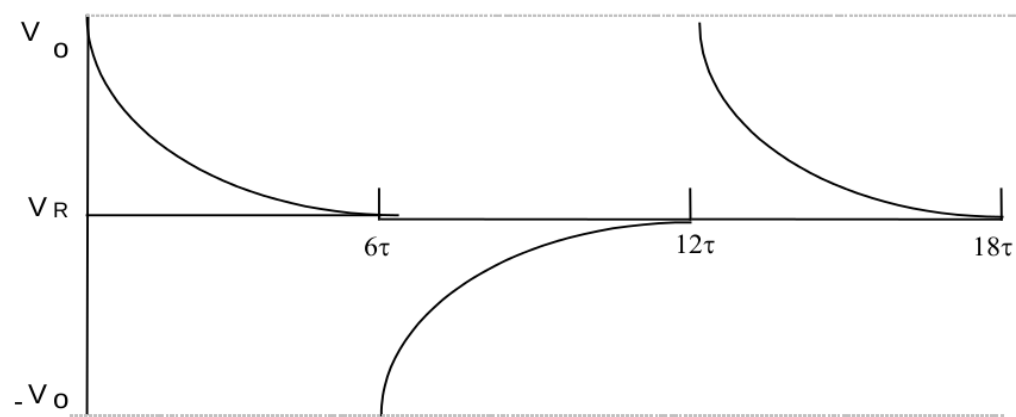
(a) Alternating voltage on and off at the power supply.

(b) Voltage across the capacitor.

(c) Voltage across the resistor.

Figure 4.5: Voltage during repeated cycles of charging and discharging for (top to bottom) the battery (which is constant at V_0 or off), the capacitor, and the resistor.

4.5   In today's lab

In this particular experiment, we are hooking up a signal (or wave) generator to an RC circuit, which allows us to reverse the applied voltage. In effect, it will allow us to drive the circuit with alternating $+V_0$ and $-V_0$ as input voltage like in Fig. 4.6. The voltages as a function of time for the resistor and capacitor are shown in Fig. 4.6. Then to see visually what is happening, we'll install a two-color LED that changes color when the current going through it changes directions.
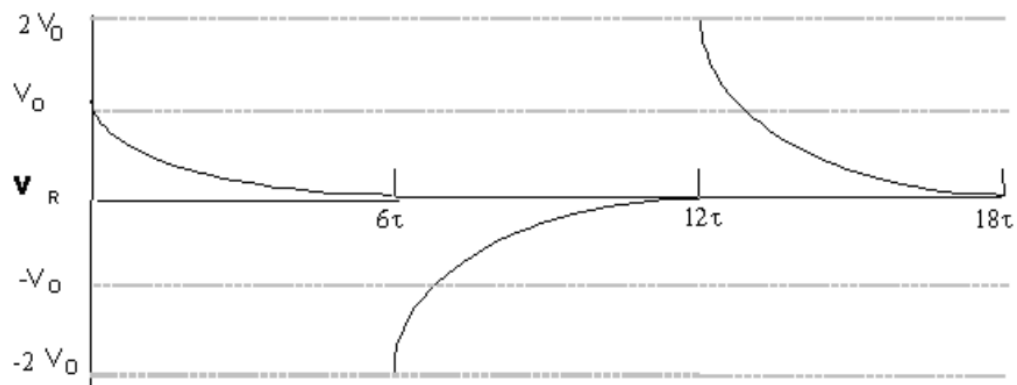
(a) Alternating direction of voltage from signal generator.
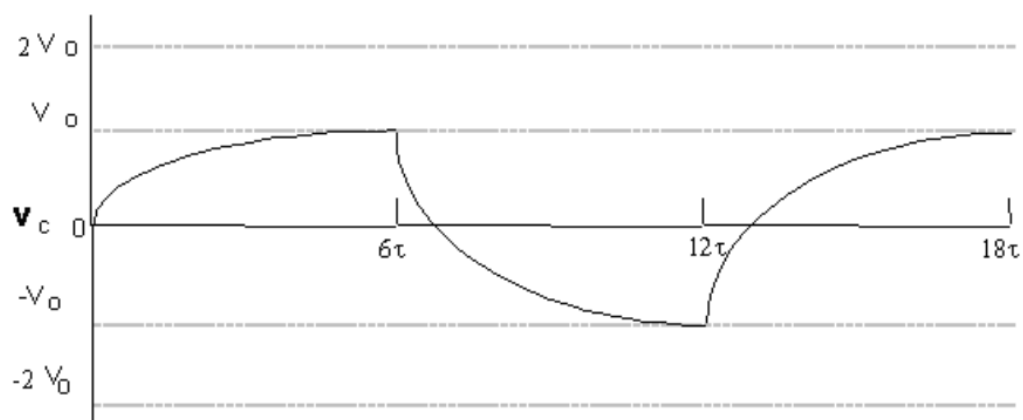
(b) Voltage across the capacitor.

(c) Voltage across the resistor.

Figure 4.6: Voltages for repeated charging and discharging with a signal generator.

4.6 Equipment

- Signal generator (see Figure 4.10)

- DC power supply

- Desktop timer (or a timer on the internet)

- Resistors and capacitors

- Two-color Light Emitting Diode (LED) (see Figure 4.11)

- Circuit breadboard

Safety Tips

- When plugging or unplugging wires, first **turn off all electronics that are connected** or will become connected to the circuit. Today there is one instance where you will unplug first before turning off the power supply. Be sure to not touch the ends of the wires while or after you are unplugging them.

## 4.7 Procedure

Measuring the time constant

First we want to measure the time constant τ. This means we need to charge the capacitor fully, then let it discharge through a resistor while we measure the voltage as it changes over time.

1. Build the charging circuit shown in Fig. 4.8(a). Note that the resistor should only be attached at one end — it's dangling here. We place it in the circuit so that it is convenient to convert to the discharging circuit in Fig. 4.8(b). Be sure to set the multimeter to read DC voltage. Use the DC power supply, a 100 kΩ resistor, and a 1 000 μF capacitor. Check the color code of the resistor to verify that it is 100 kΩ and check its tolerance. Your instructor will tell you the tolerance of the capacitor, which is likely to be ±20%. The resistor codes can be found in Fig. 4.7.

Standard EIA Color Code Table

■ **4 BAND: ±2%, ±5%, AND ±10%**
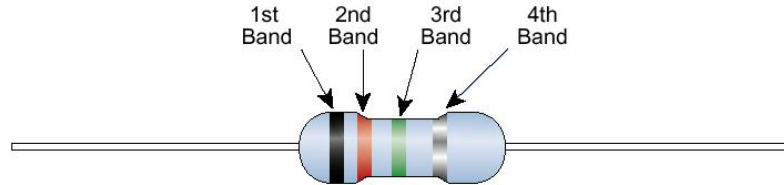
Color	1st Band (1st figure)	2nd Band (2nd figure)	3rd Band (multiplier)	4th Band (tolerance)
Black	0	0	10^0	
Brown	1	1	10^1	
Red	2	2	10^2	±2%
Orange	3	3	10^3	
Yellow	4	4	10^4	
Green	5	5	10^5	
Blue	6	6	10^6	
Violet	7	7	10^7	
Gray	8	8	10^8	
White	9	9	10^9	
Gold			10^{-1}	±5%
Silver			10^{-2}	±10%

Figure 4.7: Standard EIA (the Electronic Industries Alliance) resistor color codes for 4-band resistors. (from http://www.denizyildirim.org/mylibrary/)
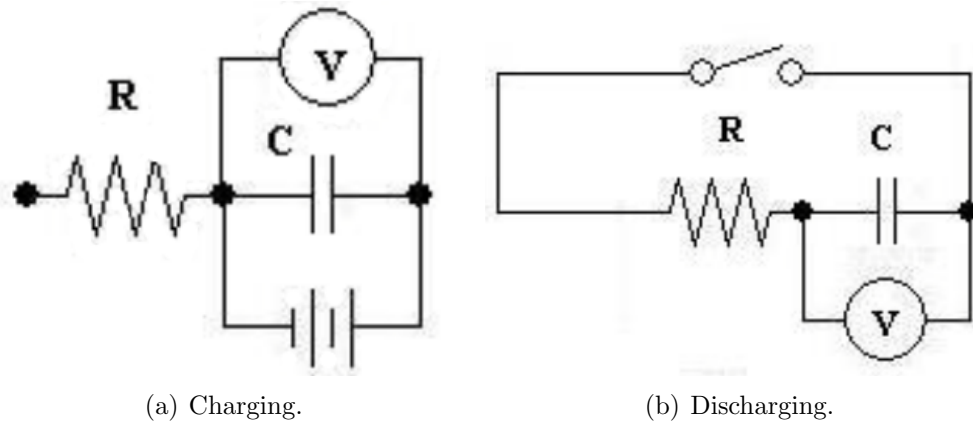

(a) Charging.   (b) Discharging.

Figure 4.8: Circuit schematics for the measurement of the time constant τ.

2. Use the power supply to charge the capacitor to approximately 12–13 V. Then disconnect the power supply from the circuit **by unplugging the wires from the power supply without turning it off**. Notice that the voltage across the capacitor slowly decreases. What could possibly cause this effect (see Question 3)?

3. Build the discharging circuit shown in Fig. 4.8(b). Find a banana plug from a bucket in the front of the room. In this circuit, it will clip to another wire connector and make an effective [4] switch. After building the circuit shown in this figure with your "switch" unconnected (i.e "open") the capacitor should still be charged from the previous step but you should notice that the voltage is slowly decreasing.

4. Close the switch by connecting the banana plug and notice the voltmeter starts to register that the charge on the capacitor is decreasing as it discharges through the resistor. Let it go until the voltage across the capacitor has dropped to about 10 or 11 volts, then start the timer and record the time and voltage in Table 1 of your spreadsheet. Continue recording the voltage across the capacitor once every 10 seconds until your timer has reached 300 seconds.

5. Have Excel calculate $\frac{V_C}{V_0}$ and $\ln\left(\frac{V_C}{V_0}\right)$. V_0 is the initial voltage across the capacitor when you start taking data so $V_0 = V_C$ at $t = 0$. Then, import your data into Kaleidagraph.

6. Make a plot of the voltage ratio $\frac{V_C}{V_0}$ versus time. Does it look similar to Fig. 4.4(a)? Does your graph fall almost to 0 after 5 or 6 τs?

7. Now make a plot of $\ln\left(\frac{V_C}{V_0}\right)$ versus time. Have Kaleidagraph fit your graph with a best-fit line. Use the curve fit parameters to determine the time constant of your circuit and its uncertainty (Questions 1–2). You'll have to think about the slope of the line as you plotted it and the time constant as it appears in the formula Eq. 4.9.

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[4]We physicists use "effective" to mean "has the same effect as" rather than "useful". So here, an "effective switch" is a setup that has the same effect as a switch would. This makes for some good wordplay sometimes.
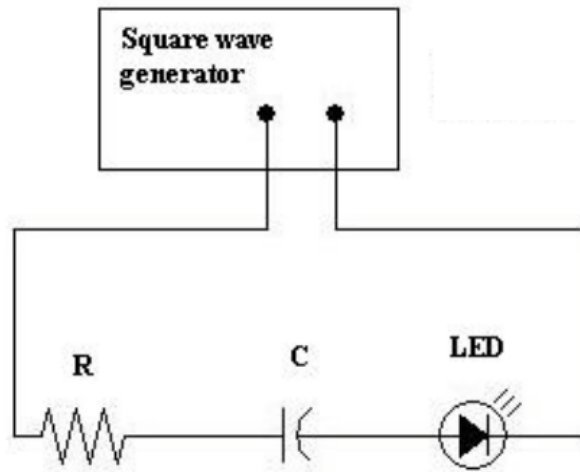
Figure 4.9: Schematic for an oscillating RC circuit with an LED.

Visualizing the RC circuit

In this part of the experiment you will build the circuit in Fig. 4.9. You will use a square wave generator (shown in Fig. 4.10) and a two-color light emitting diode, called an "LED", (shown in Fig. 4.11) to observe an RC circuit. The color which the LED displays depends on the direction of the current flowing through it. The brightness of the LED depends on how much current is flowing through it. The LED glows brighter for higher currents.

1. Use a **100 Ω** resistor (this is a different resistor from the first part of the lab!), a 1 000 μF capacitor, an LED, and a signal generator to build the circuit shown in Fig. 4.9.

2. When you turn on the the signal generator, it will default to a sine wave. Change it to a square wave. Set the amplitude of the signal generator to maximum and set the frequency to 0.500 Hz. Record your observations (Question 4).
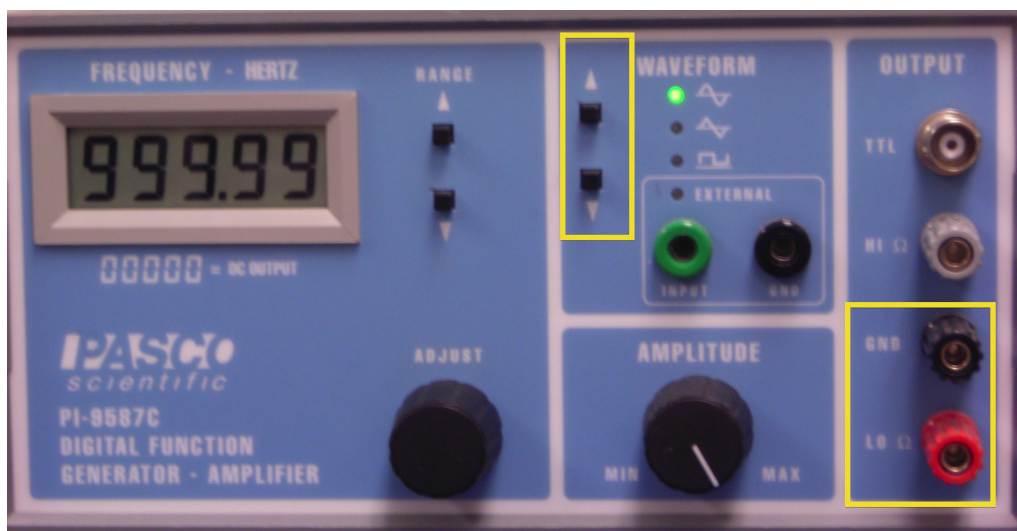
Figure 4.10: The Signal Generator produces output of a frequency you can control with the "Adjust" knob in the middle, reading out the frequency in Hertz in the window. It also generates signals of different shapes: sine wave, triangle wave, and square wave. You switch from one to the other with the buttons in the top yellow rectangle. You connect to the output of the Signal Generator at the red and black connectors shown in the lower right yellow rectangle.

Figure 4.11: The little white object on the left is a light-emitting diode (an "LED") for the second part of the lab. The blue object on the right is the capacitor that we'll use in this lab. Both are in the plastic bucket of parts.

4.8 Questions

Uncertainties

From the multiplication rule for uncertainties from Appendix A,

$$\delta(\tau) = \delta(RC) = RC\left(\frac{\delta R}{R} + \frac{\delta C}{C}\right). \tag{4.10}$$

The uncertainty in the time constant τ obtained from the graph of $\ln\left(\frac{V_C}{V_0}\right)$ vs. time is given by

$$\delta\tau = \left|\tau\frac{\delta(\text{slope})}{\text{slope}}\right| \tag{4.11}$$

1. Use Eq. 4.9 to determine τ and $\delta\tau$ from your plot of $\ln\left(\frac{V_C}{V_0}\right)$ vs. time. Show all your work for the calculations, including formulas and units.

2. Discuss the consistency between your measurement of the time constant τ from the slope of your graph and $\tau = RC$ calculated from your circuit values R and C.

3. After charging the capacitor and disconnecting the power supply, you observed that the voltage measured by the voltmeter across the capacitor slowly decreased. What are possible explanations for this observation?

4. What are your observations of the RC circuit involving the LED? Be specific about the LED's brightness, color, and any changes in color and brightness compared with the wave generator's signal.

5. Qualitatively, how does your observation of the apparent brightness of the LED compare with your findings when you actually measured the time constant τ in the earlier part of the lab? Discuss the brightness of the LED compared to current in the circuit and the voltage across the capacitor.