RC and RL Circuits

RC Circuits

Within this part of the lab we study a simple circuit with a resistor and a capacitor, following two perspectives, of time and of frequency. The time perspective relies on a differential equation. The differential equation demonstrates that the RC circuit acts as an approximate integrator or an approximate differentiator. The frequency perspective perceives the RC circuit as a filter, either low-pass or high-pass. For each experiment, starting with 2, make a copy of the screen showing sample input and output waveforms and place that copy in your lab notebook.

Experiment 1. Storage of charge by a capacitor

Set up the circuit below to charge the capacitor to 5 volts. Note that the scope is connected in parallel with the capacitor, with the scope ground connected to the circuit ground. Disconnect the power supply and watch the trace decay on the scope screen. In order to freeze a slowly varying non-periodic signal on the oscilloscope screen you can use the RUN/STOP button. Measure decay time τ within the exponential decay law: $V = V_0\, e^{-t/\tau}$. For an RC combination, it will be shown that this decay time is equal to τ = RC, where R is the resistance in ohms and C is the capacitance in farads. Relying on the latter result and on your measurement, calculate an approximate value for the effective resistance in parallel with the capacitor. (This resistance is the parallel combination of the intrinsic leakage resistance within the capacitor and of the input impedance of the scope; expectation is that the decay time is of the order of 1 s.)

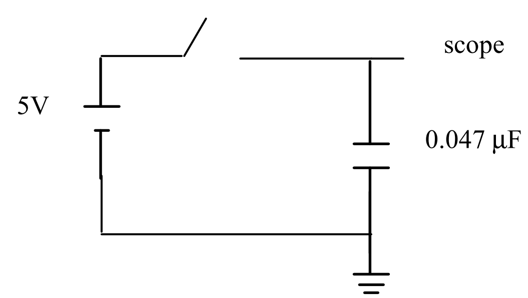

Figure 1: Capacitor charging circuit.

Next, replace the 0.047 µF capacitor by a 1000µF electrolytic capacitor [pay attention to the polarity of that capacitor!] and watch the voltage across the capacitor, after you disconnect the power supply. While you are waiting for something to happen, calculate the expected decay time. Come to a decision about whether you want to wait for something to happen. Act accordingly.

Experiment 2. RC integrator in time

Consider the RC circuit in Figure 2 below:

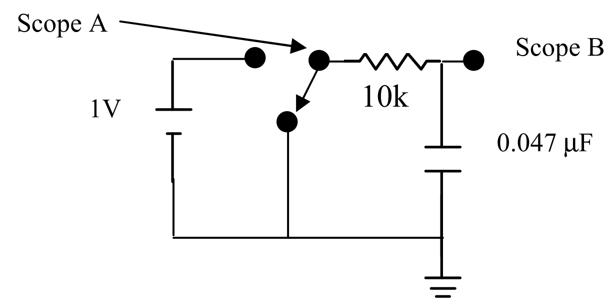

Figure 2: RC Circuit.

In lecture you learned that this circuit can be described by a differential equation for q(t) which is the charge on the capacitor at time t. Solution to that equation gives the voltage on the capacitor, V_C= q(t)/C, set to be consistent with no initial charge on the capacitor and equal then to $V_C = V_0(1-e^{-t/\tau})$, where τ = RC and V_0 represents the initial voltage.

Now build such a circuit, replacing the battery and the switch by a square wave generator. That is, the 10k resistor should be connected directly to the output of the generator. (Note: The square wave generator has positive and negative outputs, but this is the same as switching the battery with an added constant offset and a scale factor adopted for amplitude.)

Set the frequency to 900Hz, observe the capacitor voltage and verify that the circuit indeed acts as an approximate integrator. Sketch the response expected from a true integrator acting on a square-wave input.

Next, set the square wave frequency to 200 Hz, and observe the capacitor voltage. Use the vertical cursors of your DPO to measure the time required for the voltage to rise from V_- by $(V_+ - V_-)(1-e^{-1})$. Accuracy in this measurement is improved, if the pattern nearly fills the screen. This rise time is expected to be equal to τ. Compare the measurement with the calculated value of τ.

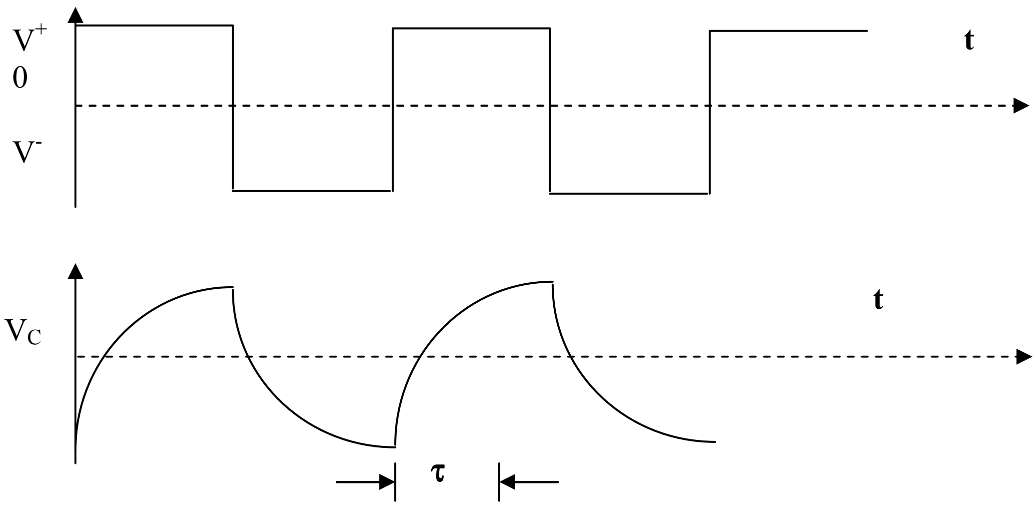

Figure 3: Square Wave and RC Integrator Output.

Experiment 3. RC low-pass filter

The low-pass filter is simply the integrator circuit above, but connected to another source. Specifically, we replace the source by a sine oscillator so that we can measure circuit's response at a single frequency. (The sine wave is, in fact, the only waveform that represents a single frequency.) We define the transfer function for a filter as the complex output-to-input signal-voltage ratio, $H(\omega) = V_{out}/V_{in}$ (or equivalently the ratio of complex voltage amplitudes), at the angular frequency ω of the sinusoidal input voltage.

The transfer function in this case is given by:

$$H(\omega)=1/(1+j\omega\tau) ,$$

where j is the imaginary number $\sqrt{-1}$.

Calculate the frequency, $f = \omega/(2\pi)$, of the so called "half-power point." This is simply the frequency where the magnitude of output voltage amplitude is equal to the magnitude of input voltage amplitude divided by $\sqrt{2}$.

Calculate the phase shift at this frequency, $\Phi = \tan^{-1}(\text{Im}(H(\omega)/\text{Re}(H(\omega)))$.

Build the discussed circuit and measure the frequency for half power. Use the "measure" utility of your DPO to find the phase shift at that frequency and compare that shift with calculations. (Note: The phase shift is related to the time, Δt, between associated zero-crossings for the input and output signals, through the formula

$$\Delta Ø = 2 \pi f \Delta t,$$

or

$$\Delta Ø = 360 f \Delta t,$$

for radians or degrees, respectively. We will use the convention that the phase shift is positive, if the output leads the input. Does the output lead the input for this filter?)

Next, instead of the half-power point, we consider the half-voltage point, where the magnitude of output amplitude is half the magnitude of input amplitude. Show that this reduction is encountered when $\omega\tau = \sqrt{3}$. Calculate the half-voltage frequency for the low-pass filter above. Show that the phase shift at this point is -60 degrees.

Change the oscillator frequency to find the half-voltage point. Compare frequencies and phase shifts with your calculations.

Experiment 4. RC differentiator in time

Consider the RC circuit in Figure 4 below:

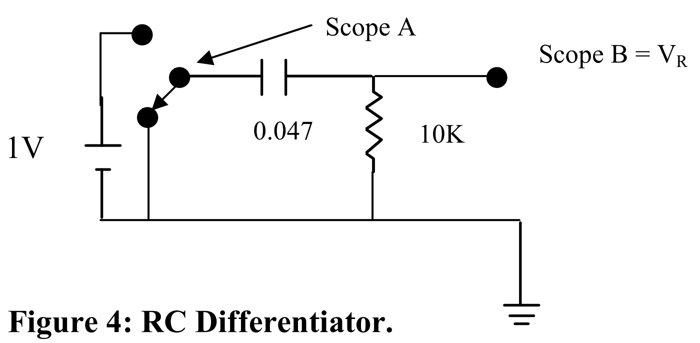

Figure 4: RC Differentiator.

The output is the voltage across the resistor, which is the current, or *dq/dt*, multiplied by the resistance R. Show that the solution for this voltage, consistent with no initial charge on the capacitor, is $V_R = V_0 e^{-t/\tau}$, where $\tau = RC$.

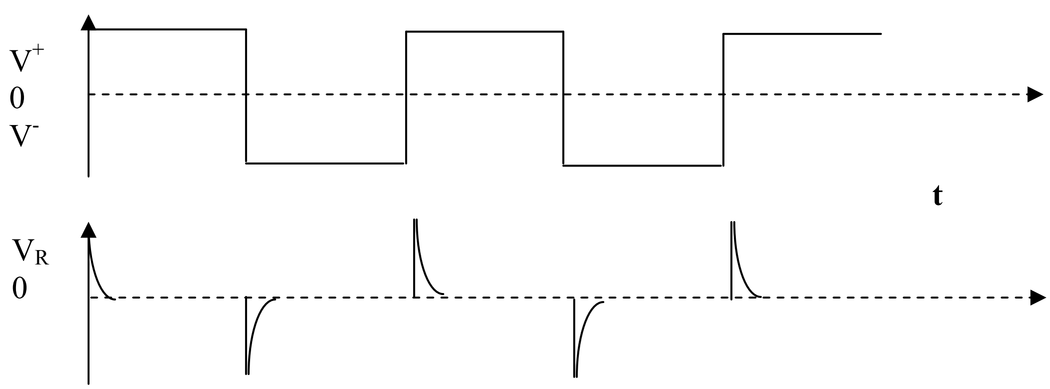

Figure 5: Input and Output of Differentiator.

Now build the differentiator circuit, again replacing the battery and switch by a square wave generator rather than actually using a switch. Set the square wave frequency first to 20 Hz and then to 200 Hz, and observe the resistor voltage in each case.
Sketch the derivative of a square wave. How does the output of the differentiator circuit compare to that derivative?

Experiment 5. RC high-pass filter.

The high-pass filter is simply the differentiator circuit above, but with source replaced by a sine oscillator, allowing to measure the response at a single frequency.

The transfer function is in this case

$$H(\omega) = j\omega\tau/(1+j\omega\tau).$$

Show that in the limit of high frequency H = 1.

Calculate the frequency, $f=\omega/(2\pi)$, of the "half-power point."

Calculate the phase shift at this frequency.

Build the circuit and find the frequency for half power. Use the DPO to find the phase shift at that frequency and compare with calculations.

Next, carry out the "half-voltage" calculations and measurements, as for the RC low-pass filter.

RL Circuits

Within this part of the lab you will use a 27 mH inductor and resistors.

Experiment 6. Real inductors – the unfortunate truth

Use an ohmmeter to measure the DC resistance of the inductor. Write the answer in your lab notebook.

Experiment 7. Real inductors – arcs and sparks

Set up the circuit below.

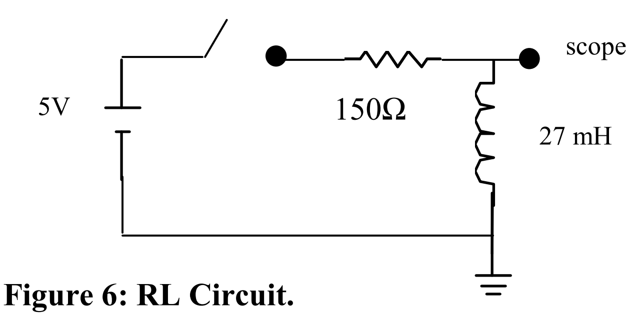

Figure 6: RL Circuit.

Once equilibrium is established, after the switch is closed, there remains a voltage across the inductor. Why should this be?

Disconnect the power supply abruptly and carefully watch the voltage across the inductor. Connect, disconnect, connect, disconnect… You should observe voltage spikes that exceed the original supply voltage. How can this be? How can you get more voltage from the inductor than the power supply voltage? Is there a violation of a Kirchoff law? Is there a violation of conservation energy?

Experiment 8. RL differentiatior

Replace the power supply and switch above with a square-wave generator.

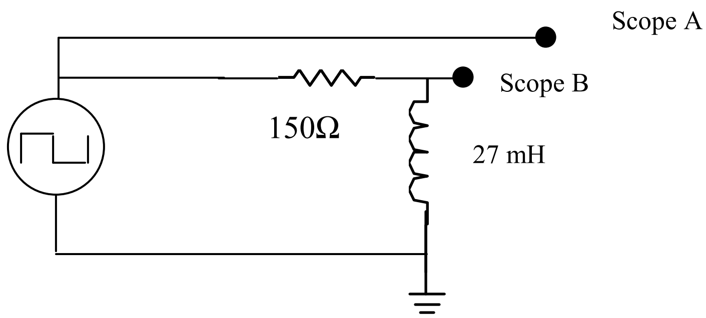

Figure 7: RL Differentiator.

Calculate time constant $\tau = L/R$. Within R remember to include the resistance intrinsic to the inductor as well as a 50Ω resistance contributed by the pulser. Measure the time constant on the DPO and compare with the calculated value.

Experiment 9. RL integrator

Design an RL integrator and verify its operation on the DPO.