

RLC Circuits

Note: Parts marked with * include calculations that you should do before coming to lab.

In this lab you will work with an inductor, a capacitor, and a resistor to demonstrate concepts of low-pass, bandpass, and high-pass filters, amplitude response, phase response, power response, Bode plot, resonance and Q.

Series RLC Circuits

*1. Simple filters

Figures 1 (a), (b), and (c) show low-pass, bandpass, and high-pass filters. Write the transfer function $H(\omega)$ for *each* of these filters, showing the ratio V_{out}/V_{in} as a

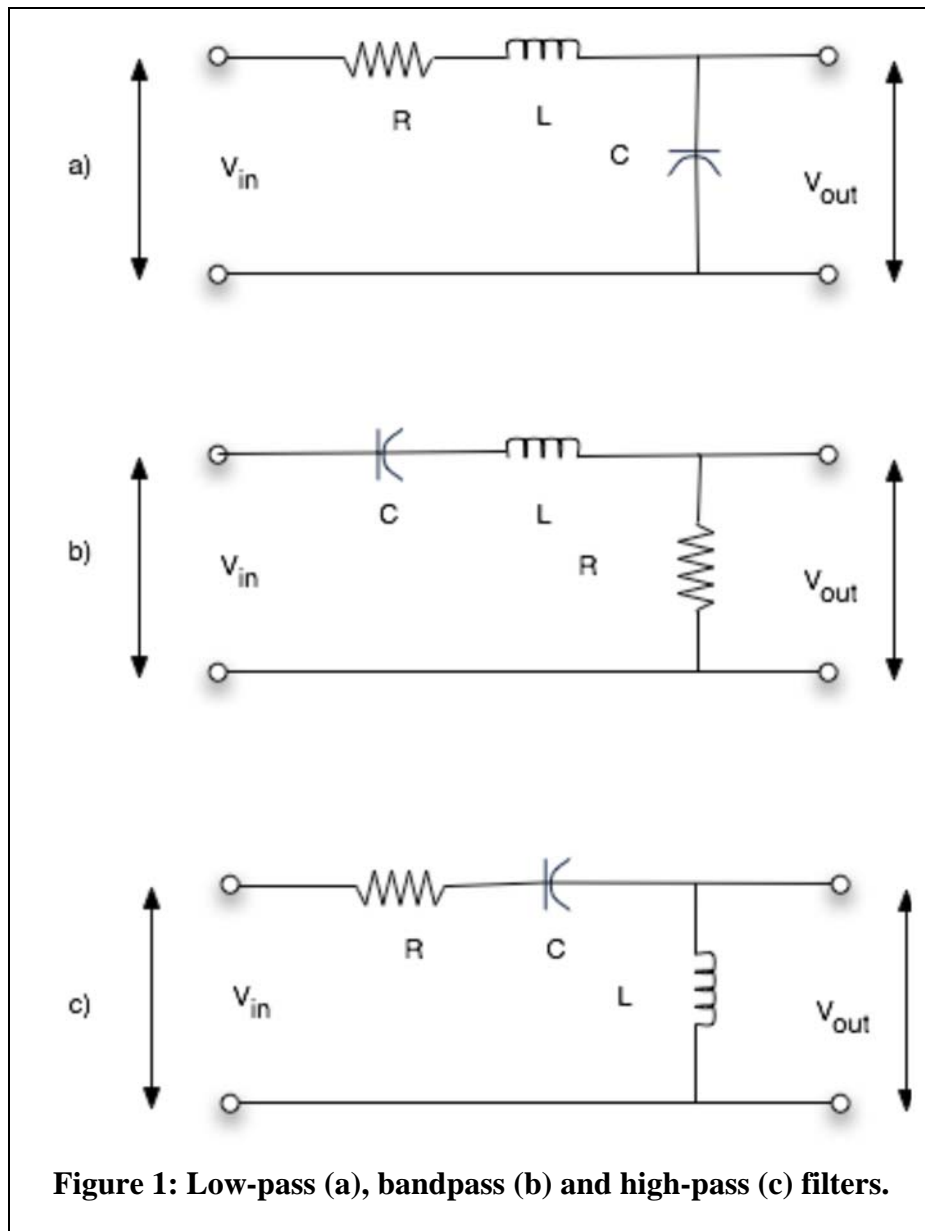


Figure 1: Low-pass (a), bandpass (b) and high-pass (c) filters.

function of the angular frequency ω of the input voltage.

*2. *The low-pass filter calculations*

Show that the low-pass filter in Fig. 1 (a) above has a power response function:

$$|H(\omega)|^2 = \frac{\omega_0^4}{(\omega_0^2 - \omega^2)^2 + \omega^2(R/L)^2}, \text{ where } \omega_0 = 1/\sqrt{LC}.$$

* Explain why this is a low-pass filter by finding the limits at $\omega = 0$ and $\omega = \infty$.

* Does a resonance occur near $\omega = \omega_0$? Explain why yes or why not.

The half-power points are the angular frequencies ω where the value of $|H(\omega)|^2$ is reduced to half the value at resonance. For this circuit, the half power points are

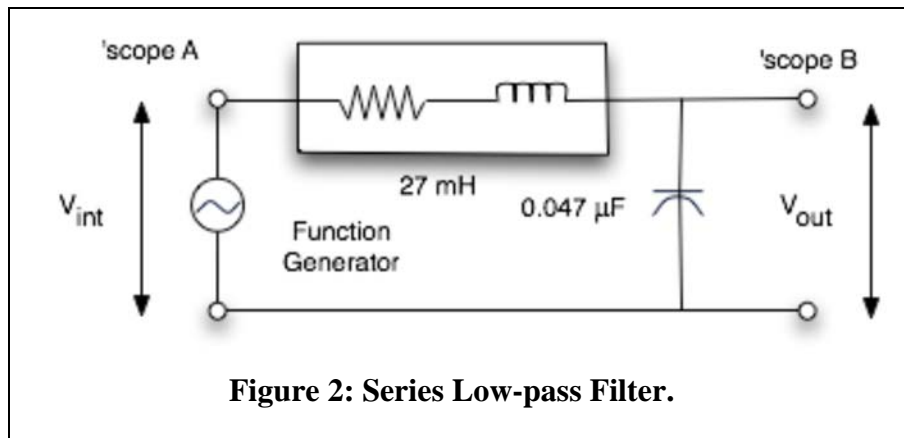
$$\omega_1 = \omega_0 - \frac{R}{2L} \text{ and } \omega_2 = \omega_0 + \frac{R}{2L}.$$

* The difference between half-power frequencies is the bandwidth of the resonance. The Q of the resonance is equal to the resonance frequency divided by the bandwidth. Show that $Q = \omega_0 L/R$.

3. *The low-pass filter experiment*

Set up the series low-pass filter shown below:

Notice that there is no discrete resistor. The resistor in this circuit represents the



resistance of the inductor plus any resistance contributed by the Function Generator. Normally the Function Generator has an output impedance of 50Ω . Verify that this is the case by measuring the resistance of the function generator output with a meter while the generator is turned on but the output voltage is set to the minimal setting of 10 mV_{PP} . Measure the resistance of the inductor, add the value contributed by the Function Generator and use this sum in your calculations below. (*If you would like to do your calculations ahead of the lab, use 55Ω as inductor's resistance.)

Calculate the resonance frequency and further measure it by changing the oscillator frequency. You may find it convenient here to attach your DPO probes to both the V_{in} and V_{out} and to use either "Measure" or "Cursor" functions to assess amplitudes of the voltages, their phase difference and frequency.

Calculate and measure the ratio of input and output voltages at the resonance. You should find that the output voltage is greater than the input! Explain how a passive circuit like this can give a voltage gain. Is this a violation of conservation of energy?

Using the measured resonance frequency and effective resistor value, *calculate* the Q . Vary the oscillator frequency to find the half-power frequencies and determine the Q from the measurements. (*Note:* By definition, at the half-power frequencies the output voltage is smaller than the output at resonance by a factor of $1/\sqrt{2}$.)

Measure the ratio of input and output voltages for very low frequency, about 1% of the value at resonance. From the transfer function you expect them to be the same. Are they? What is the phase shift at very low frequency?

4. Reduced Q

Reduce the Q of the filter by adding a $150\ \Omega$ resistor in series with the inductor. Measure the resonance frequency. Do you expect that frequency to be changed? Is it? Calculate and measure the Q for this circuit.

5. Bode Plot

Take out the $150\ \Omega$ resistor, going back to the original low-pass filter and increase the output voltage of the generator to at least $8\ V_{PP}$, to ensure that high-frequency output voltages beat any noise. Measure the output voltages when the input frequency is $20\ \text{kHz}$ and when the input frequency is $40\ \text{kHz}$. Use these measured values to show that the high frequency response of the filter decreases at a rate of $-12\ \text{dB}$ per octave, i.e. the output decreases by a factor of 4 when the frequency doubles, see the discussion at the end of this write-up.

Use the function generator and DPO to measure a Bode plot for this filter. The first part of the Bode plot is the magnitude of the response, expressed in dB as a function of decimal logarithm of frequency, at sample frequencies between $10\ \text{Hz}$ and $50\ \text{kHz}$. The second part is phase (expressed in degrees or radians) as a function of logarithm of frequency, $10\ \text{Hz}$ to $50\ \text{kHz}$.

6. Bandpass Filter

Connect the same components as in Figure 3:

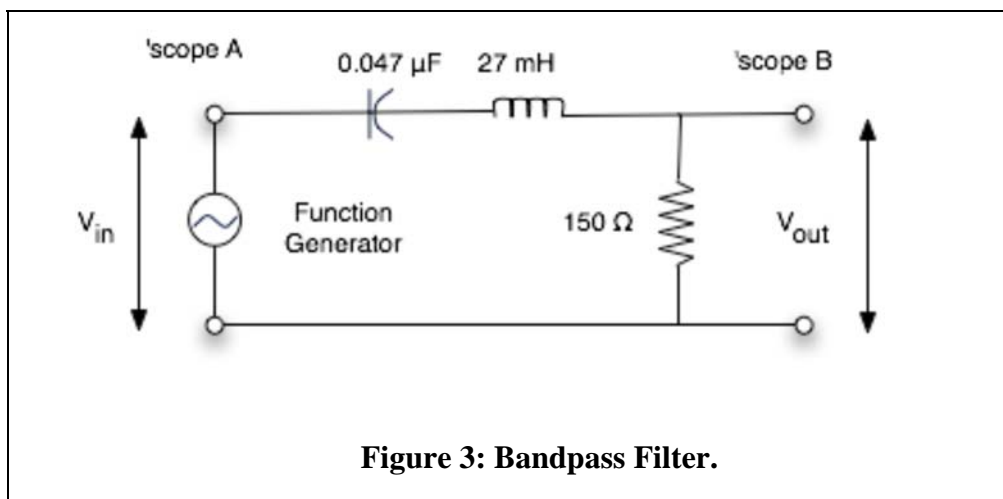
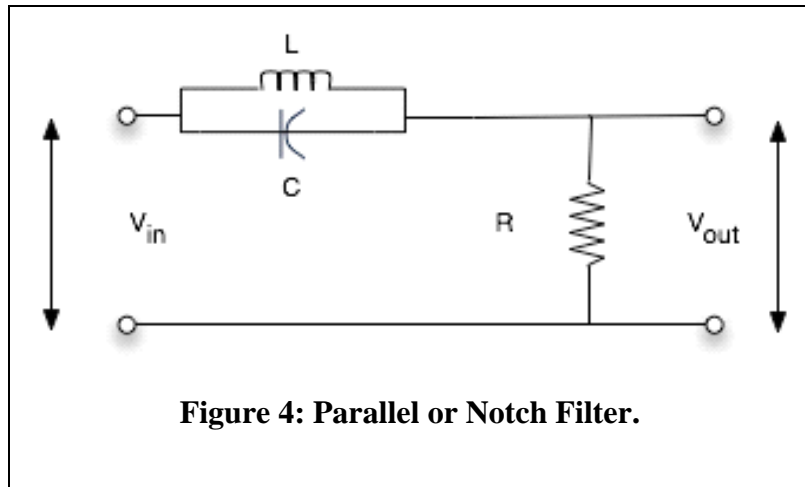


Figure 3: Bandpass Filter.

Use the DPO to show that the above is a bandpass filter. Print DPO displays showing input and output signals for frequencies below, within and above the transmitted band. Measure the resonance frequency. Compare with the resonance frequency of the low-pass



filter above. Measure the Q. Compare with the Q of the low-pass filter above.

* Show from the transfer function that the amplitude response at high frequency is -6dB/octave , namely the output decreases by a factor of 2 when the frequency doubles. Measure the amplitude response at 20 kHz and 40 kHz to check for -6dB/octave .

7. Parallel RLC Circuits

*As an example of a parallel circuit, consider the filter Figure 4 and calculate its transfer function.

Explain why this is a notch filter. What is the frequency of the notch? Use $L = 27\text{ mH}$, $C = 0.047\text{ }\mu\text{F}$ and $R = 150\text{ }\Omega$. Measure the depth of the notch by comparing the response at the bottom of the notch with the response at low or high frequency. Why doesn't the response actually go to zero at the bottom of the notch? Print DPO displays, exhibiting input and output signals, for frequencies below, at and above the notch.

Asymptotic notation

A filter can be described by its asymptotic frequency dependence. Although the transfer function may be a complicated complex function of frequency, the asymptotic characteristic is simple. For example, a low-pass filter may have a transfer function that is inversely proportional to frequency in the limit of high-frequency. We say then $H(\omega) \propto \omega^{-1}$. In general $H(\omega) \propto \omega^n$, where n is a negative number for a low-pass filter.

In the asymptotic limit, a filter has a gain characteristic of $n \cdot 20$ decibels per decade (dB/decade).

Proof: The gain characteristic in dB is

$$A_{\text{dB}} = 20 \text{ dB} \log_{10}|H(\omega)| .$$

If ω increases by a factor of 10 (one decade), then the change in gain is

$$\Delta A_{\text{dB}} = 20 \text{ dB} \log_{10}|H(10 \cdot \omega)/H(\omega)| ,$$

and this is just

$$\Delta A_{dB} = 20 \text{ dB} \log_{10}[10^n] = n*20 \text{ dB} .$$

Similarly, the asymptotic dependence can be given in dB/octave. Whereas a decade stands for a factor of 10 in frequency, an octave stands for a factor of 2 in frequency. If the asymptotic frequency dependence is, again, $H(\omega) \propto \omega^n$, then this is just

$$\Delta A_{dB} = 20 \text{ dB} \log_{10}[2^n] \approx n*6 \text{ dB} .$$