Homework Set #1 due Wed Jan 18

/1/ (a) First, determine the electric field due to a uniformly charged sphere (charge Q and radius a) centered at the origin. (b) Sketch a graph of $E_r(r)$ versus r. (c) Calculate $\cdot\nabla\cdot\mathbf{E}$.

/2/ Consider a point dipole located at the origin. The dipole moment is $p\hat{\mathbf{k}}$. The electrostatic potential is $V(r,\theta)=p\cos\theta/(4\pi\varepsilon_0 r^2)$.

(a) Calculate the electric field as a function of position $\vec{\mathbf{x}}$ and express the result in polar coordinates.

(b) Calculate $\nabla\,\mathbf{E}$.

/3/Consider a small current element I **dl** located at the origin. The magnitude of **dl** is δ and the direction is $\hat{\mathbf{k}}$; so $\mathbf{dl}=\delta\,\hat{\mathbf{k}}$. The magnetic field at $\vec{\mathbf{x}}$ due to the current element is

$$\delta\vec{\mathbf{B}}(\vec{\mathbf{x}})=\frac{\mu_0 I\delta}{4\pi}\,\frac{\hat{\mathbf{k}}\times\hat{\mathbf{r}}}{r^2}\qquad\text{(Biot Savart)}$$

(a) Calculate the curl of **δB(x).**

(b) Calculate the divergence of **δB(x).**

/4/ Consider a uniformly polarized dielectric sphere with radius a. The polarization field is $\mathbf{P(x)}=P\,\hat{\mathbf{k}}$ for $r\le a$.

(a) Sketch the *electric field lines*, both inside and outside the sphere. Your sketch should be qualitatively accurate and include the directions.

(b) Let $E_r(r,\theta)$ be the r-component of **E** at position (r,θ,ϕ). From the fields calculate the discontinuity,

$$g(\theta)=\lim_{\varepsilon\to 0}\left[E_r(a+\varepsilon,\theta)-E_r(a-\varepsilon,\theta)\right]$$

(c) Compare $g(\theta)$ and $\sigma_B(\theta)$ (= the surface charge density).