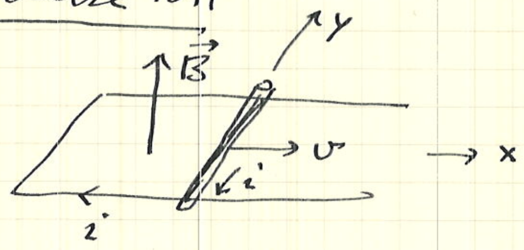
Homework Assignment #3

Exercise 10.1



(A) $\mathcal{E}MF = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$= v_x B l$

$i = v_x B l / R$

(B) $\vec{F} = l\, \vec{i} \times \vec{B} = -\frac{v_x B^2 l^2}{R} (\hat{e}_x)$

(C) $m \frac{dv_x}{dt} = -\frac{B^2 l^2}{R} v_x = -m\gamma v_x$

$v_x(t) = v_0 e^{-\gamma t}$ where $\gamma = B^2 l^2 / mR$

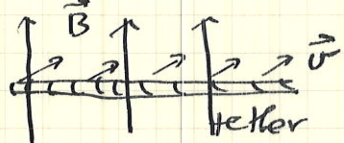
$x(t) = \frac{v_0}{\gamma}(1 - e^{-\gamma t})$

(D) Energy dissipated $= \int_0^\infty i^2(t) R\, dt$

$= \left(\frac{Bl}{R}\right)^2 v_0^2 R \int_0^\infty e^{-2\gamma t} dt = m\gamma v_0^2 \frac{1}{2\gamma}$

$= \frac{1}{2} m v_0^2$ = initial kinetic energy.

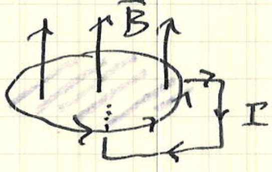
Exercise 10.2 (The space shuttle tether experiment)



$\mathcal{E}MF = \int (\vec{v} \times \vec{B}) \cdot d\vec{l} = vBl$

$\mathcal{E}MF = 4800$ volts

Exercise 10.3 (The Faraday disk generator)

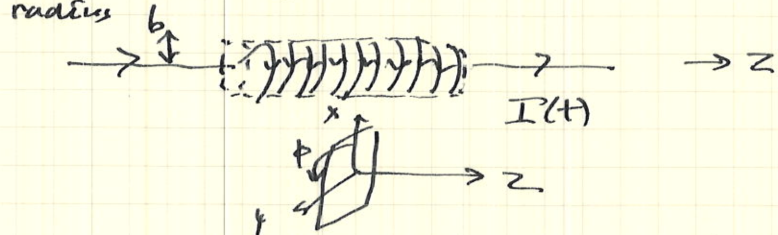


$\mathcal{E} = \frac{1}{2} \omega B a^2$

$I = \frac{\mathcal{E}}{R} = \frac{\omega B a^2}{2R}$

$I = 7.9$ mA

Exercise 10.6 ($\vec{E}$ in a solenoid)



(A) The magnetic field is $\vec{B} = \begin{cases} \mu_0 n I(t)\, \hat{k} & \text{inside} \\ 0 & \text{outside} \end{cases}$

By symmetry, $\vec{E} = E_\phi(r,t)\,\hat{\phi}$

Faraday's law : $\oint \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$

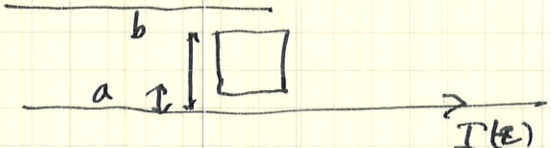
Inside $(r<b) \Rightarrow E_\phi \cdot 2\pi r = -\mu_0 n \frac{dI}{dt}\pi r^2$

$$E_\phi = -\frac{1}{2}\mu_0 n \frac{dI}{dt}\, r \quad (r<b)$$

Outside $(r>b) \Rightarrow E_\phi \cdot 2\pi r = -\mu_0 n \frac{dI}{dt}\pi b^2$

$$E_\phi = -\frac{1}{2}\mu_0 n \frac{dI}{dt}\, b^2/r \quad (r>b)$$

Exercise 10.8



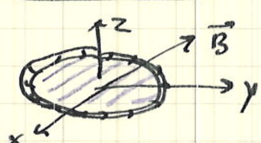
outward flux $= \Phi = \int_a^b \frac{\mu_0 I}{2\pi r}(b-a)\,dr$

$$= \frac{\mu_0}{2\pi}(b-a)\ln\frac{b}{a}\, I_0 \cos\omega t$$

The induced current (counterclockwise) is

$$I_{in} = \mathcal{E}/R = \frac{1}{R}\left(\frac{-d\Phi}{dt}\right) = \frac{\mu_0}{2\pi}(b-a)\ln\frac{b}{a}\frac{I_0}{R}\,\omega\sin\omega t$$

Exercise 10.9



(A) The flux in the z direction is

$$\Phi = \int \vec{B}\cdot d\vec{A} = \frac{B_0}{\sqrt{2}}(\hat{j}+\hat{k})(1-e^{-\lambda t})\cdot\hat{k}\,\pi a^2$$

$$= \frac{\pi a^2 B_0}{\sqrt{2}}(1-e^{-\lambda t})$$

The counter clockwise current is

$$I = \frac{\mathcal{E}}{R} = \frac{-1}{R}\frac{d\Phi}{dt} = -\frac{\pi a^2 B_0 \lambda}{\sqrt{2}R}e^{-\lambda t}$$

(B)