Lecture 2 - Preview

Maxwell's Equations

The fundamental forms of Maxwell's equations are

$$\nabla\cdot\vec{E} = \rho/\epsilon_0 \quad \text{and} \quad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\cdot\vec{B} = 0 \quad \text{and} \quad \nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

where $\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$ (continuity equation)

The fields exert forces on charged particles

$$\vec{F} = q\vec{E}(\vec{x}) + q\vec{v}\times\vec{E}(\vec{x}).$$

- Note how these equations differ from the field equations of electrostatics and magnetostatics.

§ Faraday's Law $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

or, $\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$

or $emf = -\frac{d\Phi}{dt}$ (PHY 184)

§ Maxwell's displacement current

$$\vec{J}_D = \epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0(\vec{J} + J_D)$$

displacement "current"

current due to charged particle flow

$$\nabla\cdot\vec{E} = \rho/\epsilon_0 \quad \text{and} \quad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\cdot\vec{B} = 0 \quad \text{and} \quad \nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

- In practice, we only use these equations for isolated charges and currents

$$\rho(\vec{x}) = \sum_{i=1}^{N} Q_i\,\delta^3(\vec{x}-\vec{x}_i) \qquad \text{units} \quad C/m^3$$

$$\vec{J}(\vec{x}) = \sum_{i=1}^{N} Q_i\,\vec{v}_i\,\delta^3(\vec{x}-\vec{x}_i) \qquad C\frac{m}{s}\frac{1}{m^3} = \frac{A}{m^2}$$

- Self consistency

$$\nabla\cdot(\nabla\times\vec{B}) = 0$$

identity $\nabla\cdot(\nabla\times\vec{F}) = 0$ for any $\vec{F}(\vec{x})$

So we must have $\nabla\cdot\left(\mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right) = 0$

$$= \mu_0\left(\nabla\cdot\vec{J} + \frac{\partial}{\partial t}\epsilon_0\nabla\cdot\vec{E}\right)$$

$$= \mu_0\left(\nabla\cdot\vec{J} + \partial\rho/\partial t\right)$$

$= 0$ by the continuity equation

Conservation of charge

Volume δV

Consider the charge Q inside δV.

$$\frac{dQ}{dt} = -\oint_{\text{surfaces}} \vec{J}\cdot d\vec{A} \qquad \text{conservation of charge}$$

$$= -\int_{\delta V} (\nabla\cdot\vec{J})\,d^3x \qquad \text{(Gauss's theorem)}$$

$$= +\int_{\delta V} \frac{\partial\rho}{\partial t}\,d^3x \qquad \text{(continuity equation)}$$

$$= \frac{d}{dt}Q \quad \checkmark$$

Conservation of charge is equivalent to the continuity equation.

Macroscopic matter

↳ we'll need to make approximations because we can't follow all the atomic particles independently

Chap. 6 : dielectrics

Chap. 9 : magnetic materials

Maxwell's Equations with Matter

$$\nabla \cdot \vec{D} = \rho_{free} \quad \text{and} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

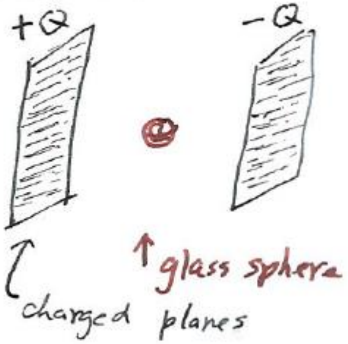
$$\nabla \cdot \vec{B} = 0 \quad \text{and} \quad \nabla \times \vec{H} = \vec{J}_{free} + \frac{\partial \vec{D}}{\partial t}$$

where $\vec{D} = \epsilon_0 \vec{E} + \underbrace{\vec{P}}_{\text{polarization}}$ and $\vec{H} = \frac{1}{\mu_0}\vec{B} - \underbrace{\vec{M}}_{\text{magnetization}}$

$\rho_{Free}(\vec{x})$ = free charge density, averaged over a small volume at $\vec{x}$

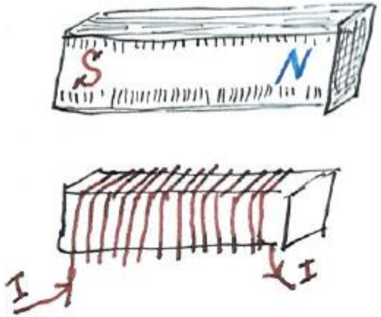
$J_{free}(\vec{x})$ = free current density, averaged similarly

Examples



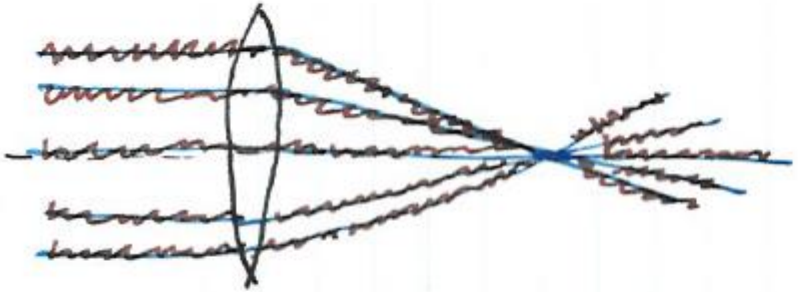
How does the glass sphere affect the electric field?

(Chap. 6)



Why is a bar magnet like a solenoid?

(Chap. 9)



What causes the refraction of the electromagnetic waves?

(Chapter 13)

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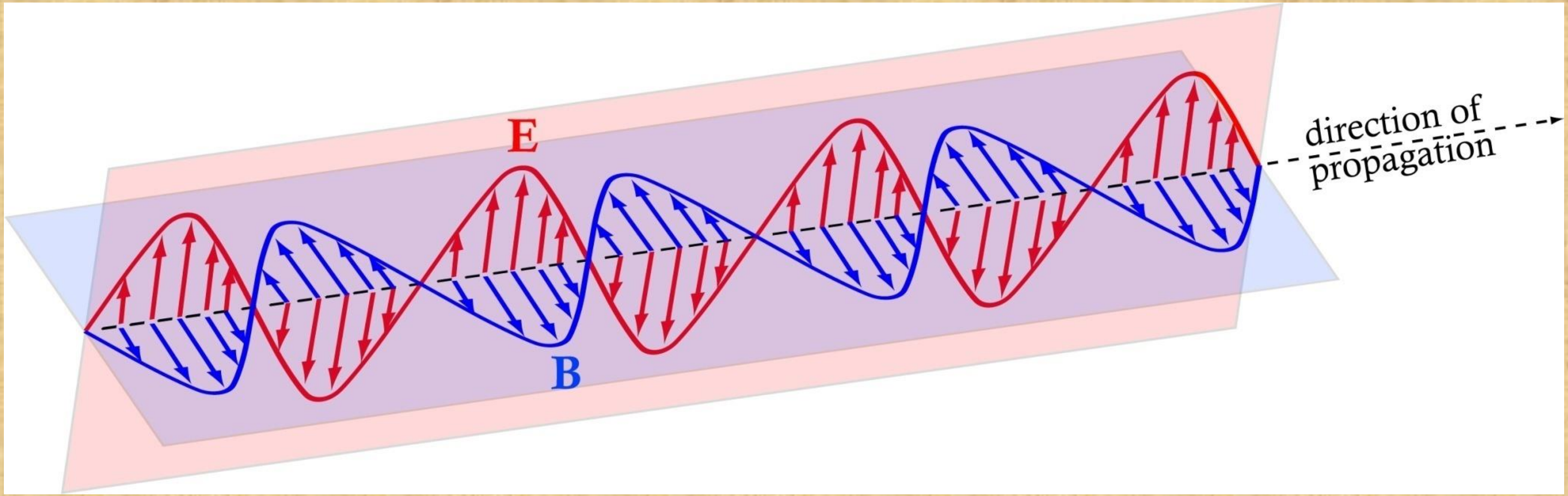

Maxwell's theory of light.

How do electromagnetic waves propagate in space?

How is the propagation affected by materials?

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Relativity (Chapter 12)

Einstein asked: How is the propagation of light described by Maxwell's equations?

Does an aether exist?

If not, how does light depend on the frame of reference of the observer? How do the $\vec{E}$ and $\vec{B}$ fields depend on the frame of reference?

These questions led Einstein to the theory of special relativity.

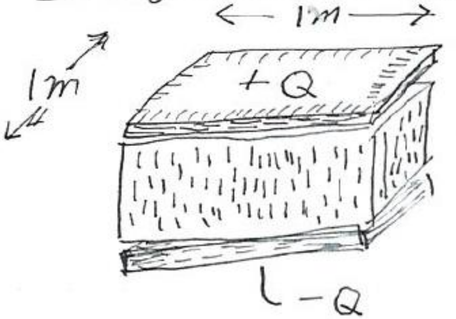
The COVARIANT FORM of ELECTROMAGNETISM

$$\sum_{\nu=0}^{3} \frac{\partial F^{\mu\nu}}{\partial x^{\nu}} = \mu_0 J^{\mu}$$

$$\sum_{\nu=0}^{3} \frac{\partial G^{\mu\nu}}{\partial x^{\nu}} = 0$$

This unified theory gives the ultimate appreciation of electromagnetism.

Example



$+Q = 0.5\ \mu C$

$\epsilon = 80\epsilon_0$ in the dielectric

$-Q = -0.5\ \mu C$

(A) What is $\vec{D}(\vec{x})$ between the plates?

(B) What is $\vec{E}(\vec{x})$ between the plates?

(give the values with units)

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C}{Vm}$$

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